

# Calibration and modeling of a dual-axis inclinometer

David H. Parker\*      Frederic R. Schwab†      John W. Shelton‡

National Radio Astronomy Observatory, Green Bank, WV§

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## Abstract

Methods are described to calibrate and model a dual-axis inclinometer, using a single-axis sine bar, by rotating the inclinometer axially on the sine bar; and using a cross-coupled polynomial expansion model. A simple combination of angle blocks and fixturing is used to generate the axial rotations. An example of how the method was used to calibrate the CST92 radio telescope, panel corner setting tools, for the Robert C. Byrd Green Bank Telescope (GBT) is described.

*Keywords: inclinometer, tilt meter, calibration, radio telescope*

## 1 Introduction

Setting the surface of the Robert C. Byrd Green Bank Telescope (GBT) was complicated by the fact that it is a 100 meter offset paraboloid, i.e., it is a portion of a 208 meter parent paraboloid. The 2004 reflector panels were carefully measured on a coordinate measurement machine (CMM) and best fit to the desired setting on the telescope[1]. Custom instrumentation was developed to adjust cardinal points on four adjacent panel corners in order to match the corner settings using the CMM measurements[2, 3, 4, 5, 6]. Similar methods have since been adapted for the Sardinia Radio Telescope (SRT)[7].

The instrument rotational orientation was established along radial lines to the virtual vertex of the parent paraboloid by an alignment telescope, and digital indicators established the distance from the instrument to each panel corner. Due to the offset design, the need to set the telescope surface at a nominal elevation angle (in order to minimize the gravitational error near the center of the operating range),

and the practical limitations of working on the inclined surface, the telescope was placed in the “bird-bath” position (65.864 degrees above the horizon) to set the surface.

In any elevation angle, other than 90 degrees (pointing to zenith), the tangent surface tilts are compound angles and are unique at every point on the surface. Therefore, it was necessary to reference the instrument to the gravity vector at every location, i.e., one axis of the instrument could not be simply leveled as one could do with a symmetric antenna pointing to the zenith. Rather than mechanically adjusting the instrument to each unique compound tilt (in order to match the local surface tangent) before reading the digital indicators; an electronic dual-axis inclinometer, along with the digital indicators, was interfaced to a handheld computer and corrections to the actual “off tangent” digital indicator readings, were made by the software. Thus, the mechanical tilt of the instrument was not critical and the computer corrected the digital indicator readings to virtual readings made from an instrument mounted tangent to the reflector surface. The desired setting accuracy of the panels was  $\pm 25 \mu\text{m}$ , so a precision calibration of the inclinometer was required.

Typical dual-axis inclinometers (tilt meters) are simply constructed by stacking two single-axis inclinometers with output signals  $V_x$  and  $V_y$ . They are supplied with the two single-axis calibration sheets. However, calibration of the individual inclinometers is insufficient for measuring compound tilts due to secondary interactions between the two tilt angles, e.g., if an inclinometer is leveled and then tilted about the orthogonal axis, in general, it will not maintain the same output signal. The accuracy of the orthogonality with which the two inclinometers are stacked is also a source of systematic error.

## 2 Compound tilt generation

Clearly, the two inclinometers must be calibrated as a mated pair, and as a function of compound tilts,

\*dparker@nrao.edu

†fschwab@nrao.edu

‡jshelton@nrao.edu

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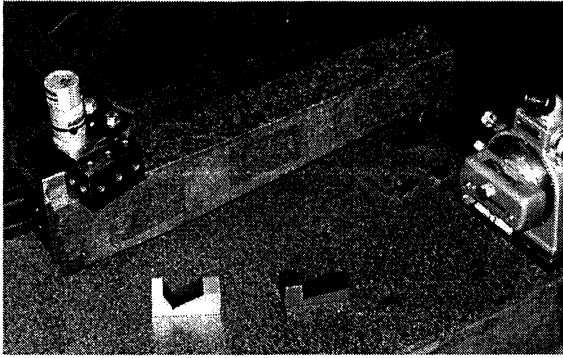


Figure 1: Inclinometer on sine bar for calibration. Note angle blocks in foreground and gage block under right end of sine bar.

for precision work. The obvious method for generating compound tilts is by using a compound sine bar. However, it can be time consuming to generate a rich data set. A simpler, and faster, method is to use a single-axis sine bar in concert with axial rotations of the instrument.

Assume the instrument is placed on a sine bar, as shown in Figure 1, with the  $x$ -axis parallel to the rotational axis of the sine bar and the  $y$ -axis orthogonal such that  $x \times y = z$  where  $z$  points up. By tilting the sine bar by an angle  $\phi$ , compound tilts are generated as a function of a rotation  $\theta$  about the  $z$ -axis (positive for ccw rotation). It can be shown that the equivalent tilts, in the instrument coordinate system, can be modeled as

$$\gamma_x = \arcsin\{\sin(\phi) \cos(\theta)\} \quad (1)$$

$$\gamma_y = \arcsin\{\sin(\phi) \sin(-\theta)\}. \quad (2)$$

### 3 Output signal model

As a first approximation, the output signals  $V_x$  and  $V_y$  should be a linear function of the sine of the angles. If we assume

$$\sin(\gamma_x) = f(V_x, V_y) \quad (3)$$

$$\sin(\gamma_y) = g(V_x, V_y) \quad (4)$$

then we can approximate the sine of the angles as a function of the output signals with functions such as

$$\begin{aligned} \sin(\gamma_x) = & a_0 + a_1 V_x + a_2 V_y + a_3 V_x^2 + a_4 V_x V_y \\ & + a_5 V_y^2 + a_6 V_x^3 + a_7 V_x^2 V_y + a_8 V_x V_y^2 \\ & + a_9 V_y^3 \end{aligned} \quad (5)$$

$$\begin{aligned} \sin(\gamma_y) = & b_0 + b_1 V_x + b_2 V_y + b_3 V_x^2 + b_4 V_x V_y \\ & + b_5 V_y^2 + b_6 V_x^3 + b_7 V_x^2 V_y + b_8 V_x V_y^2 \\ & + b_9 V_y^3. \end{aligned} \quad (6)$$

Thus, for the generated tilts  $\gamma_x(\phi, \theta)$  and  $\gamma_y(\phi, \theta)$  and measured output signals  $V_x(\gamma_x, \gamma_y)$  and  $V_y(\gamma_x, \gamma_y)$ , the  $a_i$  and  $b_i$  coefficients can be solved for using standard regression procedures.

### 4 Example

The model CST92 panel corner setting tool, developed for setting the surface on the GBT, required a high accuracy dual-axis inclinometer operating in the  $\pm 25$  degree range for compound tilts. This provided a gravity reference vector for the CST92, which was roughly attached tangentially to the parabolic reflector surface. This reference, in conjunction with 4 digital indicators which contact the corners of four panels, provides information to adjust the heights of the corners. See Figure 2.

Lucas Schaevitz model LSRP-30 biaxial inclinometers were selected for the instrument, and they were calibrated in the Green Bank metrology lab using this technique. The LSRP-30 inclinometer has a cylindrical base, so the first issue was to mount the inclinometer on a base which could be referenced to the CST92 mounting. This was satisfied by mounting the inclinometer on a machined block, with a ground flat reference side, which defined the reference axis. The mount screws were then sealed in order to insure the inclinometer/block remained a mated pair. Using a surface plate, the mated pair was then temporarily mounted on a precision 2-3-4 block, with the reference edge of the mated pair flush with one side of the 2-3-4 block, thus relaying the reference axis to the 2-3-4 block assembly.

The assembly was then placed on a 20 inch granite sine bar, which was placed on a leveled surface plate. Using 15-, 30-, and 45-degree angle blocks (pressed against the reference edge of the sine bar and various edges of the 2-3-4 block) to set the rotation about the vertical axis, and assuming the 2-3-4 block edges were square, the assembly was rotated through an entire rotation in steps of 15 degrees, i.e., 0, 15, 30, 45, 60, 75, ...345, 0. See Figure 1. By closing back to the initial zero position, repeatability was checked. The output signals of each channel were measured with a digital multimeter at each step. This was repeated for sine bar angles,  $\phi$ , of 15 and 25 degrees, or 75 data points. A Mathematica program was used for the data regression and calculation of the residuals[8, 9]. Typically, the standard uncertainty of the residuals, for the inclinometers we calibrated, was around 5 arc seconds.

If higher precision is required, additional higher order terms could be added to the equations, or the sources of the error could be modeled and the form

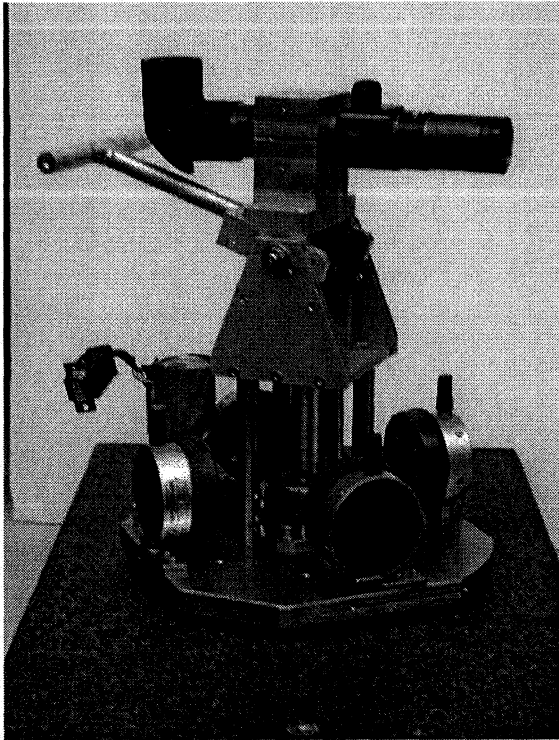


Figure 2: Model CST92 corner setting tool. Note the vertical cylinder dual-axis inclinometer on the left side of the assembly.

of the equations could be adjusted to better fit the sources of the errors. For example, the two axes are most probably not orthogonal. This could be built into the model and the angle could be solved for in the regression. The zero sine bar tilt data could also be used to model out a non-level surface plate.

#### 4.1 Calibration of the assembly

Care must be taken in mounting the calibrated inclinometer to the CST92. The inclinometer block reference edge was mounted to the CST92 and oriented with respect to the four indicator mounting holes (located on a square pattern) by placing dowels in two holes and pushing the inclinometer against a precision parallel resting against the two dowels, i.e., the inclinometer axis was forced to be parallel to the indicator axis. The alignment telescope was also set with respect to the indicator pattern. The remaining freedom of the four indicator zero settings was calibrated by placing the CST92 on a leveled surface plate as described by Ray[4].

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