# Does the GBT's Plate Scale Change with Elevation?

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## 1 Introduction

As we start to build wide-field, multi-pixel array receivers for the GBT, one question that we believe has not been discussed sufficiently is whether or not the optics of the GBT change with elevation in a way that would make the plate scale of the telescope elevation dependent. If the plate scale changes with elevation, then the relative angular spacing between the feeds in the arrays would also change with elevation. As we'll show, a change in plate scale effects all multi-feed receivers with comparable number of pixels, not just high frequency receivers or those at the Gregorian focus. This memo explores what we need to know in order to determine whether we can expect a change in plate scale, an estimate of its magnitude and effects on our current and planned suite of receivers, as well as ways to empirically determine the magnitude of the effect. As we will show, our results suggest that we will need an array with a field of view that is larger than 75 beam widths before we need to worry about this effect.

## 2 Basic GBT Optics: Plate Scale and Effective Focal Length

For any telescope, the plate scale (in 'per meter) is given in the small angle approximation by:

$$Plate\ scale = {}^{\Delta\theta}/_{\Delta x} = 3437.7/EFL$$

Here, EFL is the effective focal length (in m) of the optics, $\Delta\Theta$ (in ') is the angular displacement in the sky produced for a displacement of $\Delta$x (in m) in the focal plane of the telescope. For a compound lens system like the Gregorian GBT,

$$EFL = M \cdot F_p$$

for M = the magnification of the secondary optics and $F_p$, the focal length of the primary. Thus, the question of whether the plate scale changes with elevation is really asking whether the telescopes magnification or the focal length of the primary change with elevation. The percentage change in either the plate scale or EFL is given by:

$$\frac{\Delta EFL}{EFL} = \frac{\Delta Plate\ Scale}{Plate\ Scale} = \frac{\Delta M}{M} + \frac{\Delta F_p}{F_p}$$

The measured plate scale for the GBT is 18.04'/m (Prestage and Balser, 2005), close to the expected value of 18.03'/m from the advertised EFL = 190m. The GBT's primary has $F_p$ = 60m, which implies for either the advertised EFL or the measured plate scale that M=3.167.

### A. *Expected Change in Magnification*

As the telescope moves in elevation, we know from the Finite Element Modeling (FEM) modeling that $F_p$ changes with elevation. But, can the magnification (M) of the subreflector also change with elevation?

The GBT's Gregorian secondary is an off-axis segment of a parent ellipsoid that has an eccentricity (e) of 0.528 and whose major axis is tilted by β=5.570° to the axis of the primary (e.g., Goldman 1997).

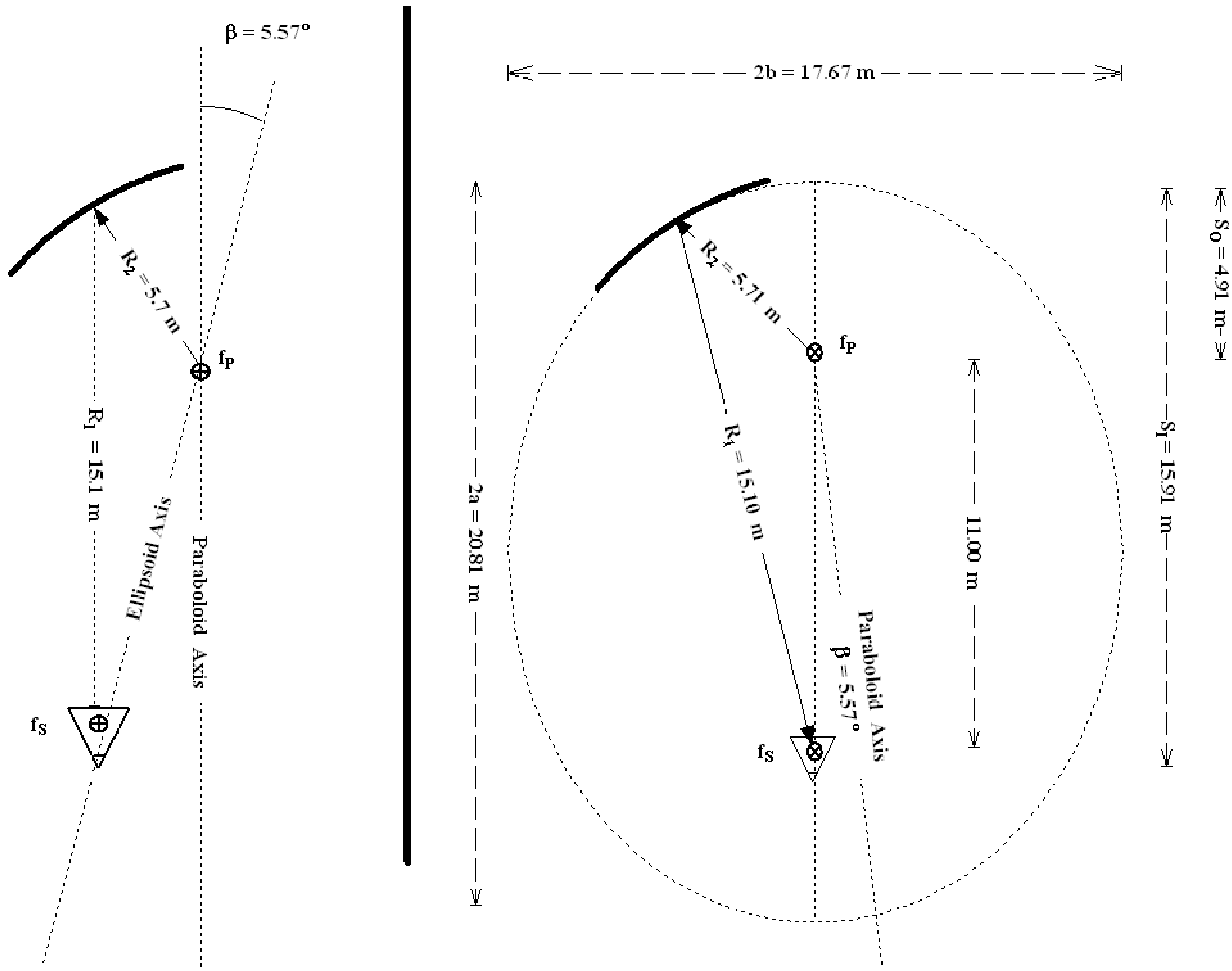


Figure 1: Sketch of subreflector optics (Not to Scale)

For any ellipsoid, the distance that the light travels from one focus ($f_p$) to any point on the ellipsoid and then on to the other focus ($f_s$) is equal to the twice the semi major axis (a) of the ellipse. That is, 2a= R2+R1. The distances R1 and R2 in Fig 1 are taken from Goldman (1997). Using the values of R1, R2, and e, one can use analytical geometry (e.g., Beyer, 1978) to determine the other dimensions given in the right hand side of Fig. 1.

The magnification of any ***on-axis*** lens system is $S_I/S_o$ where $S_o$ is the distance between the object (or $f_P$ in the case of our compound lens) and the primary plane of the lens or mirror and $S_I$ is the distance between the primary plane of the lens or mirror and the image formed by the lens or mirror. These are not the distances $R_1$ or $R_2$ but, rather the distances of the two foci of the ellipse to the vertex of the ellipsoid. For an ellipsoid, $S_0$=a(1-e) and $S_I$=a(1+e).

If the GBT optics where of an on-axis design, M would equal (1+e)/(1-e) = 3.237, which is slightly larger than the value 3.167 given above. For the GBT, one must compensate for the tilt β between the axes of the paraboloid and the ellipsoid and instead use the off-axis equations for magnification (e.g., Rusch et al, 1990):

$$M = \frac{1 - e^2}{1 + e^2 - 2 \cdot e \cdot cos(\beta)}$$

Using β=5.57° gives the advertized M=3.167. In the case β=0, the above simplifies to the on-axis equation.

Unlike a paraboloid, an ellipsoid can only focus rays from one focus to the other. If the focal point of the primary isn't exactly coincident with a focus point of the ellipsoid, or if the phase center of the feed isn't located at the other focus of the ellipsoid, then there will be a loss of gain that will be seen as a reduction in aperture efficiency. (Another consequence we will ignore here is a shift in pointing.) As the GBT tilts in elevation and the structure differentially deforms with gravity, we can assume there might be changes in the angle β as well as a change in the nominal 11m distance between the primary's focal point and the feed's phase center.

We cannot adjust the shape of the ellipsoid so as to make the foci of the ellipse coincide with both the primary's focal point and feed. Instead, we use an elevation-dependent focus tracking algorithm that shifts and tilts the subreflector so as to locate the two foci of the ellipsoid into a compromised position that is close to the definitive locations and that produces the best possible gain as a function of elevation. Since the primary focal point and feed are not located at the two foci of the ellipsoid for many elevations, we see an elevation-dependent gain change at the higher observing frequencies. The slight change in the geometry of the location of the secondary should produce a change in magnification that manifests itself as a change in plate scale.

Luckily, the recent introduction of correcting the shape of the primary with elevation using the results of holographic observations has a number of beneficial side effects with regards to the secondary optics. The loss in gain by not having the two foci of the ellipse coincident with the primary's focal point and the feed will show up in the holographic maps of the primary surface as broad, low-order aberrations. The results of the holography experiments actually have embedded in them the corrections that need to be made to the shape of the primary so as to ensure that β and the distance between the focus of the primary and feed remain constant with elevation. Thus, to the degree that the holography observations correct the low order aberrations as a function of elevation, we can assume that the orientation and location of all aspects of the secondary optics remain constant. Therefore, we can assume that M does not change with elevation.

## *B. Expected Change in the Focal Length of the Primary*

With the elimination of a change in M as a possible contributor to a plate scale change, we are left with the influence that a change in the focal length of the primary will have on the plate scale. The FEM of the structure suggests that $\Delta F_p$ is ~0.010 m (Wells & King, 1995). We can also make some inferences using empirical results.

When using a prime focus receiver, we use an elevation-dependent focus tracking curve (which is not the same curve used for Gregorian observation) that maximizes the telescope's gain, and thus positions the phase center of the receiver's feed at the location of $f_p$ (Balser, Ghigo, Maddalena, and Langston, 2001). The amount by which the focus tracking curve adjusts the location of the feed is a combination of the change in the focal length of the primary plus any other structural deformations that changes the distance between the vertex of the primary and the mounting point of the receiver on the feed arm. Thus, we could determine the change in the primary focal length if we knew what fraction of the prime-focus focus-tracking curve came from other structural deformations. For this memo, we consider the bending of the feed arm as probably the most significant contributor to the focus tracking curve other than a change in $F_p$.

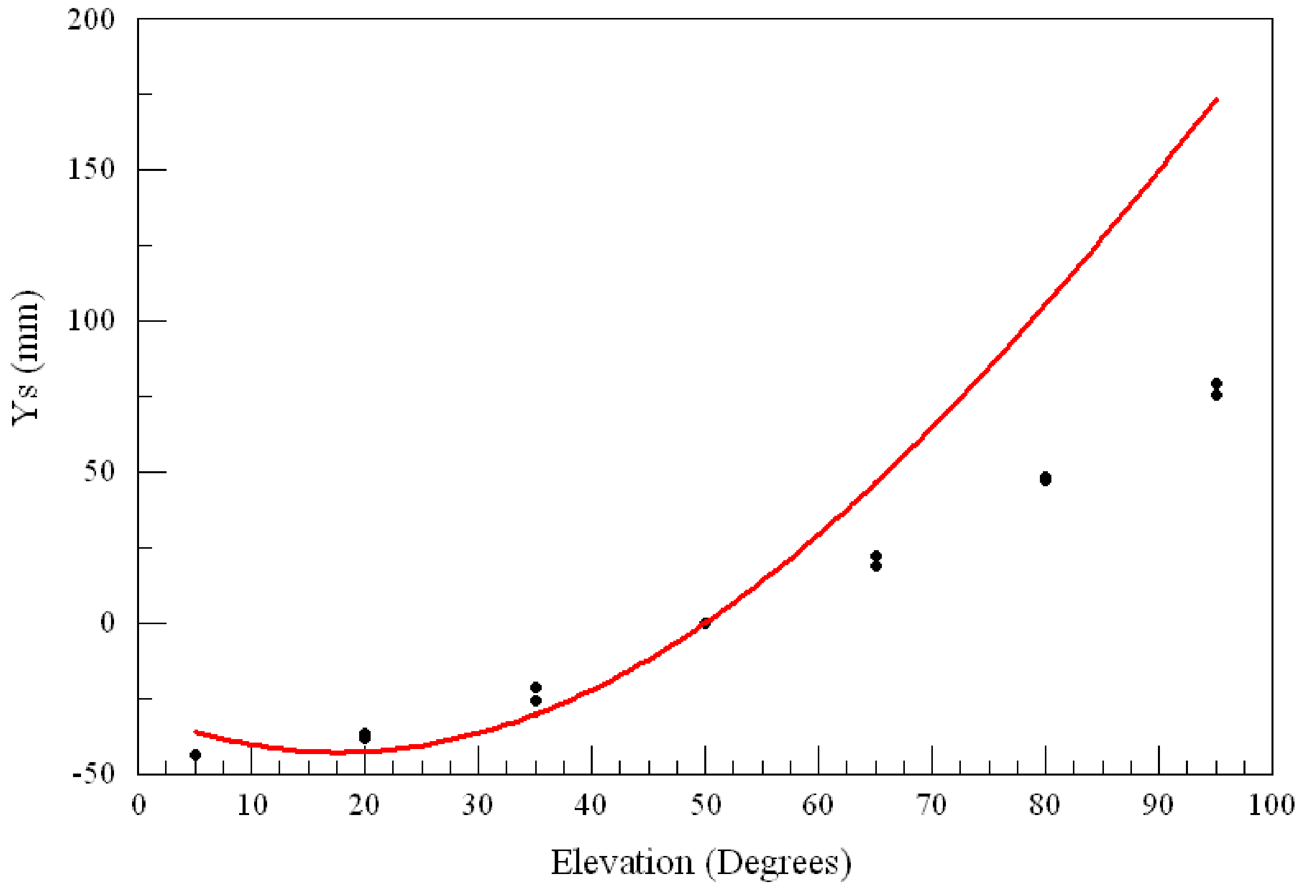


**Figure 2: Focus Tracking Curve for Prime Focus and Measured Change in Feed Arm Length**

The GBT metrology group measured the bending of the feed arm as a function of telescope elevation. Such observations were performed in 2000 which, Ghigo and Maddalena (2001) projected along the Y-axis of the subreflector's motion (i.e., $Y_s$ from Goldman 1997). What we require here is the projection of the metrology data along the Y-axis of the prime focus ($Y_p$ from Goldman 1997). The rotation of Ys with respect to the axis of the paraboloid is 36.7° while for $Y_p$ this angle is 45.5°. Because of the low accuracy to which we know the prime focus tracking curves, and because we need here only a rough estimate of $\Delta F_p$, we will ignore the small difference in the rotation angles of the $Y_s$ and $Y_p$ coordinate systems and assume the projection of the feed arm along $Y_s$ given by Ghigo and Maddalena is close enough to what one would get for a projection along $Y_p$.

Fig. 2 overlays the empirically determined focus tracking curve for prime focus with the interpretation of the metrology results of Ghigo and Maddalena. We have removed an offset of 969 mm from the focus tracking curve so as to better illustrate the differences between the two curves.

From the mechanics of the prime focus drive systems we know that the $Y_p$ axis values increase as the receiver is moved away from the primary. An increasing value for $Y_s$ in Ghigo and Maddalena corresponds to an expansion of the length between the vertex and the subreflector. Thus, the orientations of the two curves in Fig 2 are in the same way. An estimate of the amount by which the focal length of the primary may be changing is the difference between these two curves, which at most seems to be about 75 mm over a usable elevation range.

### *C. Estimate for the Maximum Percentage Change in EFL and Plate Scale*

Since we can assume that any change in M with elevation has been eliminated by the combined efforts of our focus tracking curve and the holographic adjustments of the surface of the primary, we are left with a change in the focal length of the primary as the probable remaining source of a change in EFL and plate scale:

$$\frac{\Delta EFL}{EFL} = \frac{\Delta Plate\ Scale}{Plate\ Scale} = \frac{\Delta F_p}{F_p}$$

Note that, with the elimination of M as a source of an elevation-dependent change in EFL, this equation is also appropriate for prime focus array receivers.

The FEM suggests that the maximum value for $\Delta F_p$ is 0.010 m, giving an expected plate scale change of 0.00017. The empirical focus tracking/metrology results suggest a substantially larger value of $\Delta F_p$=0.075 m which gives a plate scale change of $\Delta F_p/F_p$= 0.0013.

## 3 Implications of the Determined Change in Plate Scale

A change in plate scale means that one has to correct the calculated sky position of the outer beams of a multi-feed receiver by a scaling factor that changes with elevation. The functional form of this correction is not yet known, so we would need to develop both empirical ways of measuring the effect and software algorithms that would use these empirical results to produce accurately determined sky positions for all the feeds in an array at all elevations. Of course, it would be a saving in labor if we could decide that the effect of a plate scale change is small and ignorable.

The acceptable magnitude of the effect on the pointing of off-axis feeds is given by:

$$\frac{FOV/2}{FWHM} \cdot \frac{\Delta F_p}{F_p} < \frac{\Delta\varphi}{\varphi}$$

Where FOV is the angular full field of view of the array receiver, FWHM is the full-width, half-maximum beam width of a feed, and $\Delta\varphi/\varphi$ is the allowable fractional error one desires in the pointing of the outer pixels in the array. Note that the ratio FOV/FWHM is independent of observing wavelength for the same degradation in efficiency due to off-axis coma, and thus, the effect of $\Delta F_p/F_p$ on the relative pointing within an array is independent of the observing wavelength for an array.

We'll use $\Delta\varphi/\varphi$=0.05 as the maximum allowable fractional pointing error across an array. In order to help us decide whether we need to worry about plate scale changes, we will use a value of $\Delta F_p/F_p$= 0.0013, the larger estimate given in section 3.c. Thus, the field of view for an array must be > 75 FWHM before we violate our pointing accuracy criteria. With beams spaced 2.5 FWHM apart, this corresponds to an array with ~1000 pixels. We conclude that the effect of an elevation-dependent plate scale is completely ignorable for our current and planned suite of receivers, which have relatively small fields of view.

# 4 Summary and Suggestions

With regards to an elevation-dependency of plate scale with elevation, we have shown that:

- We need not worry about elevation dependent changes in the magnification produced by the subreflector
- The effect is probably dominated by the elevation-dependent changes in the focal length of the primary.
- The change in focal length of the primary with elevation can be roughly determined from our empirical focus tracking curve and metrology data to be about 0.075 m while the FEM suggests 0.010 m.
- This change in focal length would produce pointing offsets for the outer pixels in an array that are more than 5% of a beam width when the field of view of the array exceeds 75 beam widths. For an array with feeds spaced 2.5 beam widths apart, one would need an array with about 1000 feeds before this affect becomes important.
- These results apply to all array receivers, regardless of observing frequency or whether they are at prime focus or the Gregorian focus.
- Our current and planned suite of receivers need not worry about elevation dependent plate scale changes due to their small field of views.

Once we start building large arrays, we should test whether the results of this memo hold true. One traditional way of measuring the effect would be to point on a central pixel of an array using an astronomical point source, then immediately follow this with a second pointing observation that uses an outer pixel. One repeats this pair of observations as a source rises and sets and differences the offsets from the two pointing observations to determine if there is a trend in the differences with elevation.

This experiment seems rather intensive and time consuming. At the high frequencies where we will have such large arrays, a variable atmosphere and the time delay between the two pointing might results in a misinterpretation of an atmospheric effect or thermal deformations as a plate scale change. At high frequencies, sources tend to be weak and pointing residuals a significant fraction of a beam width.

Better would be fast, high signal-to-noise observations that measures all feeds simultaneously. One such experiment is to pass the array in elevation across the full extent of the moon and determine the relative time each pixel encountered the sharp edge of the moon. Average these results with the relative time that each pixel leaves the edge of the moon. The moon is a very bright (>220K) object that once per month will pass through an elevation range of 0 to 80°. One need not know the accurate position of the moon or anything about the curvature of the moon's limb. By averaging the relative time that the beams fall on and leave the moon eliminates to first order any lack of knowledge with respect to the moon's position. Averaging the results also cancels out the delay in timing due to the curvature of the moon's limb. Repeat the experiment in azimuth, and repeat as a function of elevation. In addition to determining feed location and any variability in plate scale, such observations would allow us to determine if an array receiver rotates as a function of elevation, such as should occur at some level from the known sideways twisting of the feed arm with elevation.

If the effect is ever determined to be non negligible, then the software system that determines the relative pointing between feeds could model a change in plate scale, as well as a rotation of the receiver, using a rotation matrix. For example, one would have for each feed i values of beam offsets in elevation and cross elevation (El_Offset_i, XEl_Offset_i) that were determined at some nominal elevation To determine the true El and XEl offset:

$$\begin{bmatrix} True\_XEl\_Offset\_i \\ True\_El\_Offset\_i \end{bmatrix} = \begin{bmatrix} a11(elev) & a21(elev) \\ a12(elev) & a22(elev) \end{bmatrix} \cdot \begin{bmatrix} XEl\_Offset\_i \\ El\_Offset\_i \end{bmatrix}$$

The elements in the matrix would be the same for all feeds and would be functions of elevation (e.g., $a11 = a11\_c \cdot cos(elev) + a11\_d \cdot sin(elev)$ where a11_c and a11_d are empirically-determined constants). If, for example, there is only a plate scale change, then the off axis terms of the matrix would be zero and the on-axis terms would describe a scaling factor to derive the true beam offset from a traditional static offset.

# 5 References

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