

# Exploring the Measured and Theoretical Noise of the GBT Spectrometer

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*October 30, 2009*

## 1 Abstract

The proposers guide for the GBT and our documentation for the GBT Spectrometer provide values for the sampling sensitivity of the 3- and 9-level modes. I ran various tests to try to determine whether the measured sensitivities match those in our documentation. The measurements suggest that the 3-level mode, at least for 12.5 MHz bandwidths, provides the sensitivity we have been using. However, the 9-level mode seems to have noticeably less sensitivity. A bad default power level does not seem to be the cause of the loss in efficiency for 9-level. Furthermore, neither the 3- or 9-level modes provide the theoretical increase in sensitivity when multiple samplers with identical inputs are averaged together.

## 2 Introduction

GBT observers must generate some measure of either the time they need for an experiment or a measure of the sensitivity they will achieve in a certain amount of time. To do this, we have been providing the observers with a sensitivity calculator that uses the theoretical algorithm for determining the noise of an experiment. With the GBT spectrometer, one significant quantity used in this calculation is the theoretical sensitivity of the different level sampling modes of the GBT Spectrometer. The quantity known as the K1 factor in the GBT proposer's guide is inverse sensitivity.

This memo explores whether or not the theoretical K1 values agree with what we can measure. The motivation for this work comes from our current efforts to generate a new version of the sensitivity calculator. Also, we have recently been asked by Rich Lacasse to explore the change in sensitivity with the various sampling levels with the GBT Spectrometer, which he hopes will help him with his work on the ALMA correlator. I will also explore whether averaging different samplers that have exactly the same input reduces the noise in the amount predicted by various text books.

## 3 Observations and Data Reduction

On Sept 16, as part of a receiver checkout, I collected data using the 12.5 MHz, 3 and 9-level modes of the GBT Spectrometer to determine the various statistical properties of the different sampling levels. The input signal was the IF Rack noise source. The data I collected used the following Astrid scripts and configuration files (which I've added comments to for those who are unfamiliar with Astrid):

```
# Load in the hardware configuration
execfile("/users/rmaddale/Config3vs9.turtle")
```

```
# A 'source catalog', though I never really use it
Catalog(fluxcal)
objName = "3C295"
```

```

# Configure for 3-level, 12.5 MHz, 4-spectral-window
Configure(config3Level)

# Set the attenuators in the IF Rack, then those in the Converter Rack
# The power levels into the Spectrometer will be the default for 3-level
Balance()

# Take four observations in this mode, each lasting 120 seconds
Track(objName, None, 120.)
Track(objName, None, 120.)
Track(objName, None, 120.)
Track(objName, None, 120.)

# Switch to 9-levels, everything else should be the same. Set attenuators
# for default levels for 9-levels, repeat observations
Configure(config9Level)
Balance()
Track(objName, None, 120.)
Track(objName, None, 120.)
Track(objName, None, 120.)
Track(objName, None, 120.)

```

The hardware setups were defined as:

```

config3Level = ""
# Set up routing between IF Rack noise source and Spectrometer

receiver = "NoiseSource"
obstype = "Spectroscopy"
backend = "Spectrometer"

# Set Spectrometer for 4 dual-polarization, 4 spectral windows, 12.5 MHz, 3-levels
# 16384 channels
nwin = 4
bandwidth = 12.5
spect.levels = 3
nchan = "low"

# Need to specify a total-power switching mode
swmode = "tp"

# Set the desired power levels in the IF Rack so as to put the power
# levels in the Converter rack at a mid-range level
iftarget=0.1

# Use 5-second dump times with a 1-sec switch cycle. Scan durations are set
# in the observing script at 120 sec.
tint = 5
swper = 1.0

# The remaining parameters are here just to make astrid happy
restfreq = 750, 750, 750, 750
deltafreq = 0,0,0,0

```

```

swtype = "none"
swfreq = 0.0, 0.0
vlow = 0
vhigh = 0
vframe = "topo"
vdef = "Radio"
noisecal = "lo"
pol = "Circular"
"""

```

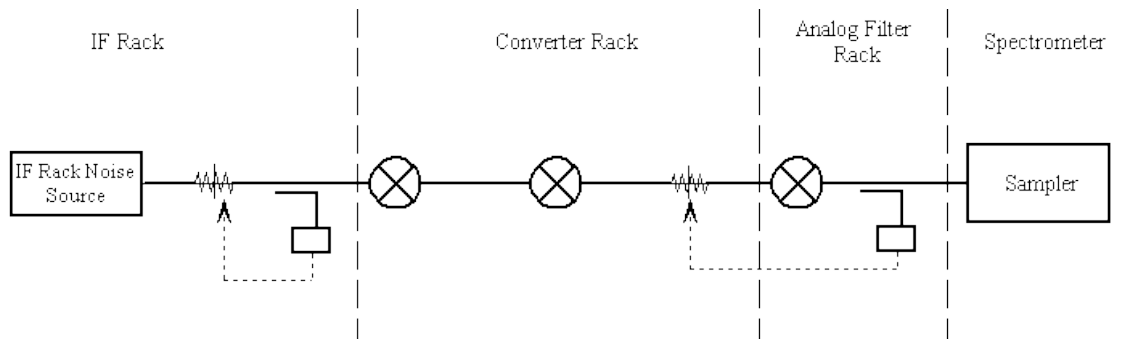
```

# Now, the 9-level configuration. All is the same except for 9-levels, which will give
# us 4096 channels
Config9Level = """
receiver = "NoiseSource"
obstype = "Spectroscopy"
backend = "Spectrometer"
nwin = 4
bandwidth = 12.5
spect.levels = 9
nchan = "low"
swmode = "tp"
lftarget=0.1
tint = 5
swper = 1.0
restfreq = 750, 750, 750, 750
deltafreq = 0,0,0,0
swtype = "none"
swfreq = 0.0, 0.0
vlow = 0
vhigh = 0
vframe = "topo"
vdef = "Radio"
noisecal = "lo"
pol = "Circular"
"""

```

Note that the use of 3C295 was merely a convenient way of executing the Astrid scripts. Since I was using the IF Rack noise source and never was looking at the sky, whether we were pointing at any object was totally moot. The relevant parts of the hardware setup are illustrated by Fig. 1.

Figure 1



Note that the noise source is split in the IF Rack and then a further 4 times in the Converter Rack, giving eight paths through the Converter Rack, Analog Filter Rack, and into the spectrometer. If these were on-the-sky observations, I would be specifying a configuration of four 'dual polarization' spectral windows, for a total of eight samplers, all with the same center (and bogus) frequency. But, the IF Rack noise source feeds what would be the dual polarization samplers in standard observing, so, in fact, I have eight redundant data streams.

The *iftarget* level specifies the power level I desired right after the attenuators in the IF Rack. The value of the *iftarget* was determined empirically so as to give a proper power levels throughout the system. The **Balance()** command in the observing file first sets the attenuators in the IF Rack so as to obtain the desired *iftarget* as seen by the IF Rack power meters. **Balance()** then uses the power meters in the Analog Filter Rack, plus the duty cycles as measured by the Spectrometer, to set the attenuators in the Converter Rack. In this way, the power level as seen by the Spectrometer samplers would be appropriate for the sampling mode. Thus, if I were to double the *iftarget* value, the attenuators in the IF Rack would be 3 dB lower but the attenuators in the Converter Rack would be 3 dB higher.

Most of the other parameters in the configuration are there just to make Astrid happy and do not affect the actual observing. In retrospect, I could have used 'tpnocal' as a switching mode so as to speed up the data reduction but the choice of 'tp' was of no major concern.

The data are associated with the TSCAL\_16SEP09 project. This data set also includes data from the receiver checkout. The scans of interest are:

32-35 : 3-level  
36-39 : 9-level

The data were converted from lags to spectra by the SDFITS filler, which also performs the van Vleck corrections. The data reduction was done in GBTIDL and were simple (scan2-scan1)/scan1 using the scripts:

```
; Retrieves a spectrum for a particular sampler. Averages all integration
; in that spectrum
pro rawget, scan, sampler
; Create an accumulator for purposes of averaging integrations
ac = {accum_struct}
accumclear, ac
; Find the records in the data file. Each record holds a single integration, phase
ints = getchunk(count=count, scan=scan, sampler=sampler)
; Average up the integrations
for i=0, (count-1) do begin
    dcaccum,ac, ints[i],weight=1
end
accumave,ac, p0
; Put the average into the primary data container
set_data_container, p0, buffer=0
end
; Perform (scan2-scan1)/scan1 for a desired sampler
pro temp, scan1, scan2, sampler
rawget, scan1, sampler
copy, 0,1
rawget, scan2, sampler
subtract & divide
end
```

The rest of the analysis consisted of using the GBTIDL 'stats' command on the central 80% of individual scan pairs or using 'stats' on averages of various combinations of scan pairs.

I need to mention a few details on the data reduction script. I needed **rawget** since the built-in **gettp** won't work with data taken with the IF Rack noise source. Because the SDFITS filler also can't supply the usual values for *IFNUM* and *PLNUM* (respectively, the spectral-window and polarization number), **rawget** requires specifying the actual sampler names (A9, A10, A13, A14, B17, B18, B21, B22). The use of **getchunk** automatically includes both the diode on and off data of the *tp* switching mode specified in the configuration. Note that the actual diode phase makes no difference since we again are using the noise source, which has no equivalent of a calibration diode.

For normal astronomical observing, the **temp** command would include a multiplication by a System Temperature ( $T_{\text{sys}}$ ). Here, we're interested in comparing the theoretical and measured values for

$$\frac{\sigma}{T_{\text{sys}}} = \frac{K1}{\sqrt{K2 * t_{\text{eff}} * BW / N_{\text{chan}}}}$$

where  $\sigma$  = the noise in the difference spectra,  $K1$  = inverse sensitivity of the selected Spectrometer mode,  $BW$  = the Spectrometer's bandpass,  $N_{\text{chan}}$  = the number of channels in that bandpass and  $t_{\text{eff}}$  = the effective integration time -- see the GBT Proposer's Guide for details). Since the default is that the SDFITS filler should not perform any windowing of the data,  $K2$ , the correlation between Spectrometer channels, should be 1.22 (and at least the same value for 3- and 9-level sampling). The results of **temp** are exactly in the desired units of  $\sigma/T_{\text{sys}}$ . To simplify representation of the results, I have multiplied all  $\sigma/T_{\text{sys}}$  by  $10^6$ .

#### 4 Measured and theoretical noise from three and nine-level sampling

For each sampling level, for scan pair, and for each sampler within that scan pair, I determined the measured values for  $\sigma/T_{\text{sys}}$  using the above 'temp' command followed by the appropriate use of the GBTIDL 'stats' command.

**Table 1: Measured Values for  $10^6 \cdot \sigma/T_{\text{sys}}$**

Scans	Sampler								Mean and Standard Deviation
	A9	A10	A13	A14	B17	B18	B21	B22	
	3-Level								
32-33	5379	5382	5378	5404	5344	5415	5408	5391	5387.7±22.5
34-35	5264	5305	5299	5305	5295	5287	5297	5345	5299.6±22.6
	9-Level								
36-37	2373	2362	2370	2377	2360	2379	2371	2375	2371.0±6.6
38-39	2391	2385	2386	2391	2384	2391	2397	2396	2390.0±4.9

Within each scan pair, the noise level of each sampler is extremely consistent, as shown by the extremely small standard deviations. The agreement implies that the ratio of system temperatures and calibrated data produced by various samplers may be better than 0.5%.

Yet, note that the different scan pairs for the same level of sampling produce mean noise levels that differ by over 3 standard deviations. I find these changes in  $\sigma/T_{\text{sys}}$  perplexing for a couple of reasons. Scan pairs were taken back-to-back and as quickly as is possible with no intervening hardware adjustments (e.g., attenuators settings). And, the data reduction and units I am using are mostly immune to drifts in the power generated by the noise source, while the drifts in the measured input power were relatively small anyway (0.01%/min for 9-level, 0.04%/min for 3-level). These results could mean that some flaw in the Spectrometer or SDFITS filler can generate  $T_{\text{sys}}$  values that fluctuate with every scan by 1 to 1.5% percent from their true values. If I'm correct, this could imply that for some types of observing, the Spectrometer cannot produce scan-to-scan calibration that is any better than this same percentage. This might be another symptom of the so-called DC-Offset problem where we cannot measure the same source intensity towards continuum sources.

How do these measured values compare to theoretical estimates of the noise? For all observations,  $\text{BW}=12.5 \times 10^6$  Hz and, according to the GBT proposer's guide and most references on autocorrelation spectrometers,  $K2 = 1.22$  for both sampling modes. Although each scan was 120s long, due to 'blanking', the actual exposure time was 114.7s. Using the definition of  $t_{\text{eff}} = (t_{\text{on}} * t_{\text{off}}) / (t_{\text{on}} + t_{\text{off}})$ , I get  $t_{\text{eff}} = 57.35$ s. Using the number of channels for each type of data, and the values for  $K1$  from the GBT proposer's guide, the theoretical noise values are:

**Table 2: Theoretical vs. Measured Noise and K1**

Sampling Level	$N_{\text{chan}}$	Theoretical		Mean of Measured	
		K1	$10^6 \cdot \sigma / T_{\text{sys}}$	K1	$10^6 \cdot \sigma / T_{\text{sys}}$
3	16384	1.235	5345.3	$1.235 \pm 0.014$	$5344 \pm 62$
9	4096	1.032	2233.4	$1.100 \pm 0.006$	$2381 \pm 13$

It's nice to see that the theoretical  $\sigma$  and  $K1$  for 3-level sampling match the measured values to a high degree of accuracy (1%), implying that the Spectrometer behaves just as theory predicts for 3-level sampling.

But, the theoretical  $\sigma$  and  $K1$  for 9-level sampling is 11 standard deviations from the mean of all the measured values! The excess noise is probably not from bandpass drifts between the two observations, which would result in incomplete subtraction of the bandpass shape by the **temp** command -- an additional test with baseline fitting of a 24th order polynomial did little to reduce the measured  $\sigma/T_{\text{sys}}$ , and taking the power spectrum of the data showed nothing unusual.

Thus, nine-level data seems to be 6.6% noisier than is expected from theory. Essentially one could consider this as an error in the theoretical  $K1$  value, as if the  $K1$  value for 9-level is more like  $1.100 \pm 0.006$ , not the value of 1.032 quoted in the proposer's guide.

## 5 Improvement in noise when averaging samplers with identical input signal

In many text books on radio astronomy and autocorrelators, the point is made that one can decrease the noise of an observation by averaging together samplers that have identical input signals. One would expect that

averaging data from perfect samplers would not show a decrease in noise since the input signals are 100% correlated. The improvement in noise happens because of the inefficiencies of sampling to only a few quantized levels. Observers could in principle take advantage of this feature to shorten their observing time if they used redundant samplers whenever their experiment didn't require all of the GBT Spectrometer samplers.

After generalizing and rearranging equation 8.59 of "Interferometers and Synthesis in Radio Astronomy" (Thompson, Moran, and Swenson, 2001), the improvement in sensitivity after averaging n samplers is given by:

$$\frac{K1}{K1_n} = \sqrt{\frac{K1^2 \cdot n}{K1^2 + n - 1}}$$

where K1 is the inverse theoretical sensitivity for an individual sampler (as mentioned above) and K1<sub>n</sub> is the inverse sensitivity after averaging n samplers. For large n, K1/K1<sub>n</sub> approaches K1 (i.e., K1<sub>n</sub> approaches one, the value for a perfect, infinite-bit sampler).

I averaged the difference data for scan pair 32-33 (3-level) for sampler A9 with the difference data for the A10, A13,... B22 samplers. With each subsequent average I measured the incremental decrease in the measured noise. From this, I could then determine whether the decrement in theoretical noise agrees with the decrement in measured noise as n went from 2 to 3 to 4,... to 8. I then repeated the same steps using the 9-level scan pair 36-37. The following tables show the improvement in measured and theoretical noise for the two sampling modes.

Although the measured noise decreased with n, it did not do so nearly as much or as fast as predicted by theory. The progression in value for K1/K1<sub>n</sub> to what appears to be a saturation value that is well below the theoretically-predicted value of K1 suggests that an infinite number of samplers would not mimic the results from an infinite-bit perfect sampler. Currently, I am at a loss as to why the measured and theoretical decrements disagree by such a large amount.

**Table 3: 3-Level (Scans 32-33)**

Number of Samplers = n	Measured 10 <sup>6</sup> ·σ/T <sub>Sys</sub>	Improvement in σ/T <sub>Sys</sub>	
		Measured	Theoretical = K1/K1 <sub>n</sub>
1	5379	--	--
2	5168	1.041	1.099*
3	5088	1.057	1.139
4	5045	1.066	1.161
5	5025	1.071	1.175
6	5016	1.072	1.184
7	5013	1.073	1.191
8	5003	1.075	1.196

\* See the Q=3 entry of Table 8.1 in Thompson, Moran, and Swenson. The ratio of the beta=2 to 1 values agrees with the value I give here.

**Table 4: 9-Level (Scans 36-37)**

Number of Samplers = n	Measured $10^6 \cdot \sigma / T_{\text{Sys}}$	Improvement in $\sigma / T_{\text{Sys}}$	
		Measured	Theoretical = $K1/K1_n$
1	2373	--	--
2	2341	1.013	1.016
3	2334	1.016	1.021
4	2332	1.018	1.024
5	2328	1.019	1.025
6	2328	1.019	1.026
7	2329	1.019	1.027
8	2328	1.019	1.028

## 6 Exploring Input Power Levels

Fred Schwab proposed that the loss in efficiency for 9-levels in section 4 might be due to the monitor and control system setting up input power levels to the Spectrometer that were off from the optimum values. To explore this, I performed further tests in the 9- and 3-level (projects TSCAL\_23SEP09 and TSCAL\_24SEP09, respectively). In these tests, I used the same configuration files as given above but used the following example scripts:

```
# 3-level power level tests
execfile("/users/rmaddale/GBT/Commissioning_Tests/Spec9vs3/Config3vs9.turtle")
Catalog(fluxcal)
objName = "3C295"

# Configure and balance to default levels for 3-levels
Configure(config3Level)
Balance()

# Offset power by -14 dB in the IF Rack and leave the rest of the system alone
ChangeAttenuation("IFRack",-14.)
# Step in 2 db increments from -12 to +12 dB offsets
for i in range(0,13):
    ChangeAttenuation("IFRack",2.)
    Track(objName, None, 120.)
    Track(objName, None, 120.)

# Rebalance to default power levels and not step from +12 to -12 in 2 dB steps
Balance()
ChangeAttenuation("IFRack",14.)
for i in range(0,13):
    ChangeAttenuation("IFRack",-2.)
    Track(objName, None, 120.)
    Track(objName, None, 120.)
-----

# 9-level power level tests
execfile("/users/rmaddale/GBT/Commissioning_Tests/Spec9vs3/Config3vs9.turtle")
Catalog(fluxcal)
objName = "3C295"
```



```

Configure(config9Level)
Balance()

# Step from -12 to 12 dB is steps of 1 dB
ChangeAttenuation("IFRack",-13.)
for i in range(0,25):
    ChangeAttenuation("IFRack",1.)
    Track(objName, None, 120.)
    Track(objName, None, 120.)

# Rebalance to default levels and now step from +12 to -12 dB in steps of 1 dB
Balance()
ChangeAttenuation("IFRack",13.)
for i in range(0,25):
    ChangeAttenuation("IFRack",-1.)
    Track(objName, None, 120.)
    Track(objName, None, 120.)

```

Here, the experiment set up the default power levels into the Spectrometer and then removed 12 dB of attenuation in the IF Rack, leaving the attenuators unchanged in the Converter Rack. This essentially over feed the Spectrometer by about 12 dB more than its default power settings. Two 120-second observations were then made, after which 1 or 2 dB were added to the IF Rack attenuators and another pair of scans was taken, and so on until the IF Rack attenuators were 12 dB above the usual values. In this way, one can determine how K1 changes with input power levels.

The same data reduction scripts were used for every pair of observations to derive values of  $10^6 \cdot \sigma / T_{\text{sys}}$  as a function of IF Rack attenuator settings (and relative Spectrometer input power levels). The results for 3- and 9-level are shown in Figures 2 and 3.

In each figure, the upper plot is the measured value of  $10^6 \cdot \sigma / T_{\text{sys}}$  as a function of the IF Rack attenuator settings from their default settings. The left-hand y-axis of the upper plots represents the K1 value that corresponds to  $10^6 \cdot \sigma / T_{\text{sys}}$  for each spectrometer mode. The bottom plots represent the sampling efficiency ( $1/K1$ ) as a function of input power levels relative to the default settings. The blue horizontal lines are the values we have been quoting in our various memos and documents. The red curves represent the change in sampling efficiency with change in input power levels. The dotted vertical blue line is the default power level as automatically set by the monitor and control system. As the graphs show, the experiments were repeated (for as many times as was possible during maintenance). The reproducibility of results is rather impressive. The results for 0 dB offsets agree rather nicely with the results from the observations outlined in section 2, which were performed on a different day.

For both 3- and 9-level sampling, the sampling efficiency is a maximum for power levels that are very close to their default values. A least-square fit to determine the power level at which one achieves the maximum sampling efficiency suggests that the error in the default power settings is  $\sim 0.5$  to  $0.7$  dB, which is well within the range of accuracy to which attenuators are being set. The results for 3-l3v3l match very well with the predicted change in sensitivity with input power but the 9-level results differ significantly and confirm the results in section 3 for zero dB power offsets. Thus, the low efficiencies for 9-level sampling at the optimum power setting (illustrated by the horizontal dashed green line in figure 3) are not caused by an error in the default power level settings.

Figure 2

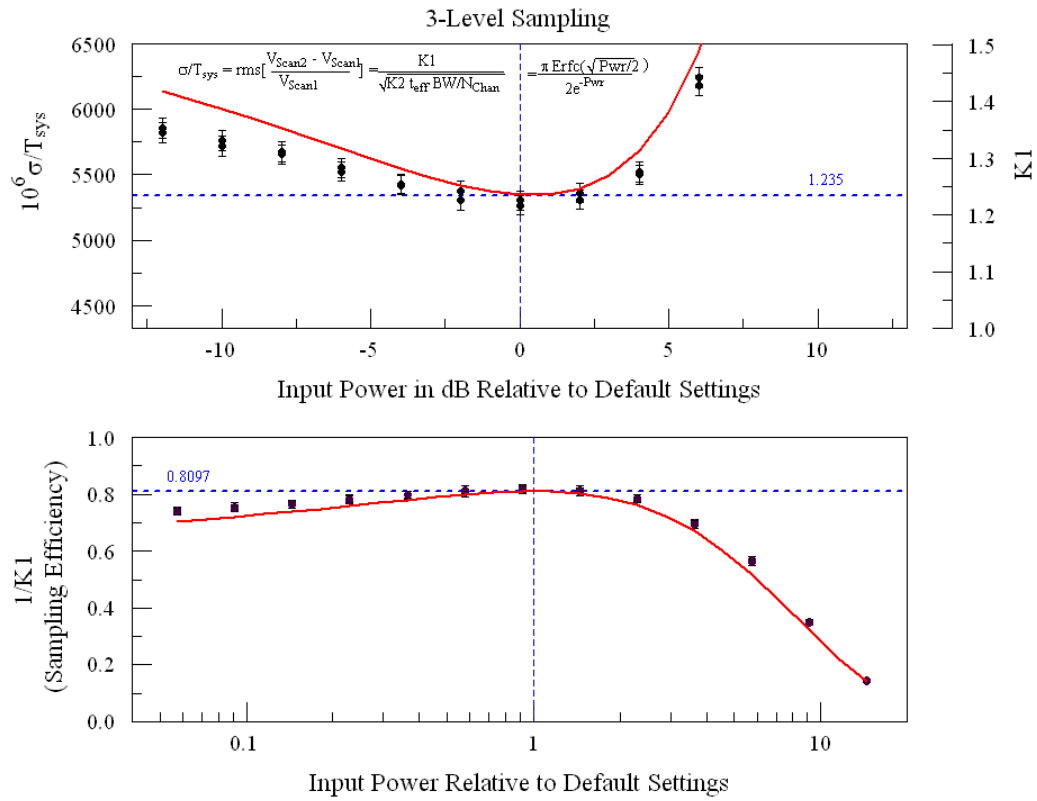
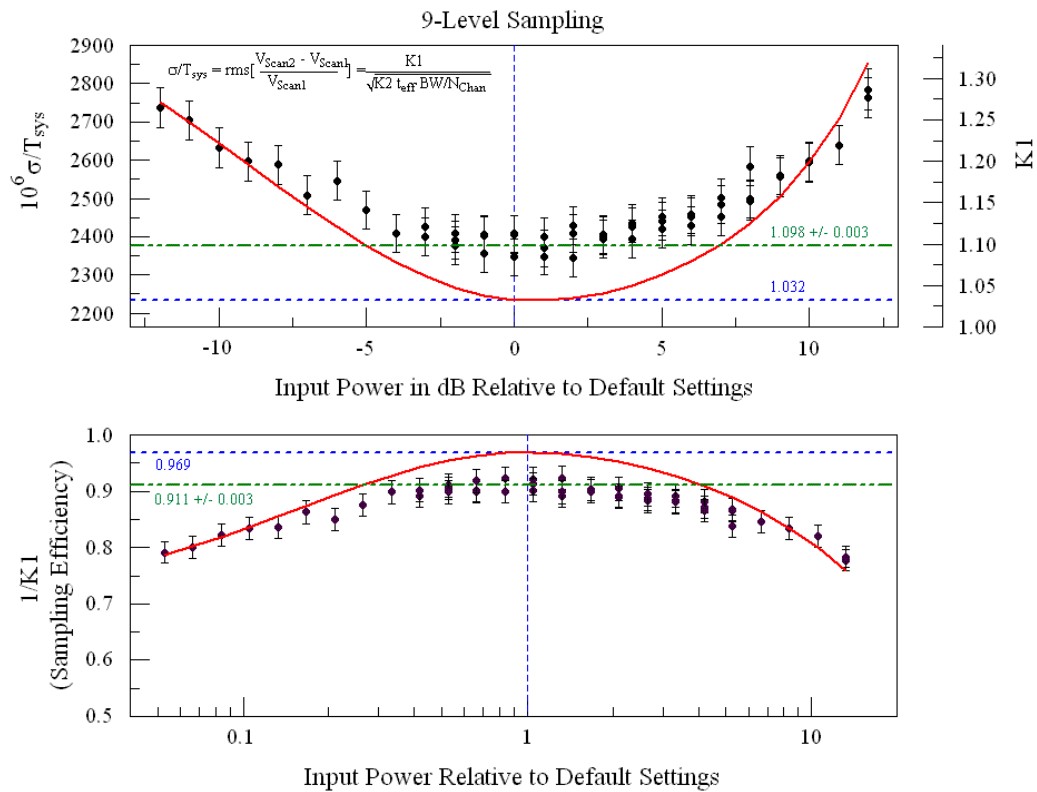


Figure 3



## 7 Summary and Suggestions

These tests of the Spectrometer's sampling efficiency show more unexpected results than expected:

- As expected, 3-level sampling has a K1 value ( $=1/\text{sampling sensitivity}$ ) of 1.235.
- The change in sensitivity with input power levels is as expected for 3-level sampling.
- But, 9-level modes have a measured K1 value of  $1.100 \pm 0.003$  instead of 1.032, the value we have been advertizing in our documentation.
- For 9-level sampling, the change in sensitivity with input power is also not as expected
- For both sampling levels, averaging samplers with identical inputs does not increase the sensitivity as was expected. In fact, it does not appear that an infinite number of samplers will mimic the sensitivity one would achieve with an infinite-bit sampler.
- The sampler to sampler variations in K1 values is very low, suggestive that these measurements have an intrinsically high accuracy.
- The scan-to-scan variations in K1 are many times the sampler-to-sampler variations within a scan. This might be a manifestation of the often-reported scan-to-scan jump in  $T_{\text{sys}}$  values.

Further tests will be run as time allows. These might include:

- Test the 3- and 9-level sampling modes with a 50 MHz bandwidth.
- Examine how the different levels are being populated. If the levels are not being populated as theory predicts (for example, if the levels are not spaced evenly or as designed), then the 9-level mode may be acting like a mode that uses fewer levels, which would produce a lower sampling efficiency and higher K1 values. For example, using Fig. 2 of Thompson et al (2007), if the levels are either twice as wide or twice as narrow as the optimum value, we would have approximately the low quantization efficiency that is measured here.,
- Inject a CW signal to measure ratios of signal-to-noise.

## 8 References

Fisher, J.R., 2002, GBT Memo, "Autocorrelator Sampler Level Settings and Transfer Function".

GBT Proposers Guide (<http://www.gb.nrao.edu/nraoPST.htm>)

Thompson, A.R., Moran, J.M, and Swenson, G.W. Jr., 2001, "Interferometry and Synthesis in Radio Astronomy".

Thompson, A.R., Emerson, D.T., and Schwab, F.R. 2007, *Radio Science*, vol. 42, RS3022.

## 9 Acknowledgments

I appreciate all the comments and suggestions I have been receiving with regards to these findings. I especially appreciate the many suggestions and comments made by Fred Schwab and Rich Lacasse on the draft memo. This territory is unfamiliar enough that I would be at a loss where it not for the expertise of my collective colleagues.