GBT Memo No. 31

Keck Observatory Technical Note 291

Panel Surfaces for the Green Bank Telescope

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Introduction

NRAO is planning to build a 100m diameter radio telescope to replace the 300 foot telescope that collapsed in 1988. They plan to make it an off axis design to improve the noise performance of the system. At present, the primary mirror design is that it be parabolic with a radius of curvature k = 120m and the parabola vertex will be close to one edge of the mirror. For purposes here, I will assume that the most extreme point on the mirror is 100m off axis. This is roughly true, and modest variations are not central to the points made in this note.

The primary is expected to be made of a mosaic of surface panels, roughly 2m in characteristic diameter. Trapezoidal panels are traditional for radio telescopes, but are not required for any astronomical reasons. Panels are expected to be made by glueing aluminum sheets to a backup structure while pressed against a pre-machined master mold. Evidently a major consideration in the size and shape of the panels is the cost and complexity of making these master molds. The panel surface accuracy expected is about 50μm rms, and the telescope is being designed to operate at wavelengths as short as 3mm. At this wavelength, panel errors would thus be roughly λ/30 rms wavefront.

In this note I calculate the surface shape of the panels and assess how rapidly they vary as a function of off axis distance. I conclude that for a primary of this size and for these relatively small panels, they can be adequately described as quadratic surfaces, and that radial displacements of a panel (or the master mold) by distances well over one meter are negligible. Thus a given master mold can be used for making surface panels with appreciably different off axis distances.

Surface Shape

The equations for the surface shape of a panel are derived in Keck Observatory Report 91 (Nelson and Temple-Raston, 1982). We assume the primary is a conic specified by a radius of curvature k, and a conic constant K. It's surface height is given by

$$Z(X,Y) = \frac{1}{K+1}\{k - [k^2 - (K+1)S^2]^{1/2}\} \quad (1)$$

$$S = (X^2 + Y^2)^{1/2}$$

where X and Y are the global coordinates and the vertex of the mirror is at (X,Y,Z) = (0,0,0). For a parabola, K = -1.

Assume a given panel has it's center at X_c, Y_c, Z_c. We can make a series expansion of the above expression in a local coordinate system (x,y,z) centered on a given panel with an off axis distance $R = [X_c^2 + Y_c^2]^{1/2}$ and arranged so the local coordinate system xy plane is tangent to the panel surface. In general this expansion can be written in cylindrical coordinates as

$$z(\rho,\theta) = \Sigma\, \alpha_{mn}\rho^m \cos n\theta + \beta_{mn}\rho^m \sin n\theta \quad m \geq n \geq 0,\ \text{m-n even.} \tag{2}$$

where $\rho = [x^2 + y^2]^{1/2}/a$ and θ are polar coordinates in the xy plane. θ is measured counterclockwise from the x axis. We have for convenience normalized the radial coordinate by the radius a of the panel. By orienting the x y coordinate system so x is radially outward, we can set $\beta_{mn} = 0$. For simplicity we assume the panels are circular with a radius a. With these conventions, expressions for α_{mn} are derived in KOR 91 and reproduced below. In these equations, $\varepsilon = R/k$.

$$\begin{aligned}
\alpha_{20} &= \frac{a^2}{k}\left[\frac{2 - K\epsilon^2}{4(1 - K\epsilon^2)^{3/2}}\right] && (focus)\\
\alpha_{22} &= \frac{a^2}{k}\left[\frac{K\epsilon^2}{4(1 - K\epsilon^2)^{3/2}}\right] && (astigmatism)\\
\alpha_{31} &= \frac{a^3}{k^2}\left[\frac{K\epsilon[1 - (K+1)\epsilon^2]^{1/2}(4 - K\epsilon^2)}{8(1 - K\epsilon^2)^3}\right] && (coma)\\
\alpha_{33} &= \frac{a^3}{k^2}\left[\frac{K^2\epsilon^3[1 - (K+1)\epsilon^2]^{1/2}}{8(1 - K\epsilon^2)^3}\right] && \\
\alpha_{40} &= \frac{a^4}{k^3}\left[\frac{8(1+K) - 24K\epsilon^2 + 3K^2\epsilon^4(1 - 3K) - K^3\epsilon^6(2 - K)}{64(1 - K\epsilon^2)^{9/2}}\right] && \left(\begin{matrix}spherical\\aberration\end{matrix}\right)\\
\alpha_{42} &= \frac{a^4}{k^3}\left[\frac{K\epsilon^2[2(1+3K) - (9+7K)K\epsilon^2 + (2+K)K^2\epsilon^4]}{16(1 - K\epsilon^2)^{9/2}}\right] && \\
\alpha_{44} &= \frac{a^4}{k^3}\left[\frac{K^2\epsilon^4[1 + 5K - K\epsilon^2(6 + 5K)]}{64(1 - K\epsilon^2)^{9/2}}\right] &&
\end{aligned} \tag{3}$$

For a parabola (K = -1) these equations simplify to

$$\begin{aligned}
\alpha_{20} &= \frac{a^2}{k}\left[\frac{2 + \epsilon^2}{4(1 + \epsilon^2)^{3/2}}\right]\\
\alpha_{22} &= -\frac{a^2}{k}\left[\frac{\epsilon^2}{4(1 + \epsilon^2)^{3/2}}\right]\\
\alpha_{31} &= -\frac{a^3}{k^2}\left[\frac{\epsilon(4 + \epsilon^2)}{8(1 + \epsilon^2)^3}\right]\\
\alpha_{33} &= \frac{a^3}{k^2}\left[\frac{\epsilon^3}{8(1 + \epsilon^2)^3}\right]\\
\alpha_{40} &= \frac{a^4}{k^3}\left[\frac{3\epsilon^2(8 + 4\epsilon^2 + \epsilon^4)}{64(1 + \epsilon^2)^{9/2}}\right]\\
\alpha_{42} &= \frac{a^4}{k^3}\left[\frac{\epsilon^2(4 - 2\epsilon^2 - \epsilon^4)}{16(1 + \epsilon^2)^{9/2}}\right]\\
\alpha_{44} &= -\frac{a^4}{k^3}\left[\frac{\epsilon^4(4 - \epsilon^2)}{64(1 + \epsilon^2)^{9/2}}\right]
\end{aligned} \tag{4}$$

These expansion coefficients are exact, and for the surfaces of interest in this problem, higher order terms in the series expansion (2) are negligible. Note that all the coefficients go to zero as ε becomes large. Thus, as one expects, extremely far off axis panels become flat.

In Figure 1 we show a plot of α_{20} as a function of off axis distance R assuming a = 1m and k = 120m. It is typically about 3mm and decreases with off axis distance. This term is axisymmetric and often called power or focus.

In Figure 2 we see a plot of α_{22} as a function of off axis distance. Its typical value is about 400μm, modest compared to the shortest wavelength, but not negligible. Thus one cannot describe the individual panels as simple spheres (as one could adequately do if only α_{20} were significantly non zero). This quadratic term is often called astigmatism. Since its rms size is under 250μm, to this tolerance, one can approximate the panels as spheres.

In Figure 3 we see a plot of α_{31} the next leading term, called coma. The typical amplitude is about 5μm, and is negligible for the surface tolerances desired for this telescope.

Figures 4 through 7 show α_{33} (tricorner), α_{40} (spherical aberration), α_{42}, and α_{44}. All these terms are negligible for this telescope, being below 1μm in amplitude.

From examining these expansion coefficients, we see that even for moderate changes in the telescope panel parameters (k, a, R) a panel surface can be adequately characterized by the two quadratic terms α_{20} and α_{22} (power and astigmatism). Thus the panel shapes are rather simple and can be expressed to within about 1μm rms as

$$z(\rho,\theta) = \alpha_{20}\rho^2 + \alpha_{22}\rho^2\cos 2\theta. \qquad (5)$$

Surface Shape Variation With Off Axis Distance

Since the mirrors are to made against master molds it is of interest to determine how many molds will be needed, in particular whether one needs a different mold for each distinct panel. With traditional trapezoidal panels arranged in rings with a fixed off axis distance, one might expect to build a mold for each ring of panels, and thus require roughly 50 molds for 2m panels in this off axis configuration. With hexagonal panels, there may be roughly 1200 distinct off axis distances for the panels and in principle one needs a mold for each panel type. In practice if two panels differ from each other in surface shape by a sufficiently small amount, then they can be made from the same mold.

For our panels, differences that are small compared to 50μm are negligible since that is the panel fabrication tolerance. As an example we will consider the range of off axis distances for which the surface of a panel changes by no more than 10μm rms.

If we assume for simplicity that the surface panels are circular, it is straightforward to calculate the rms surface σ difference from a flat. This is given by

$$\sigma = \left[\frac{\alpha_{20}^2}{12} + \frac{\alpha_{22}^2}{6} + \frac{\alpha_{31}^2}{72}\right]^{1/2} \qquad (6)$$

where we assume only the first three terms in the expansion are important. In the preceding section we showed that in fact, only the first two terms are likely to be of interest.

Figure 8 shows the rms surface difference between two panels whose off axis distances differ by one meter. From this one can see that for the entire mirror, shifts of one meter cause changes of at most 7μm rms. Thus a mold can be used for panels within at least a range of ±1m. For panels close to the optical axis, a single mold can be used for a range in off axis distance from 0 to 14m with a surface error of under 10μm. The details of how few molds one needs depends on the exact size and shape of the panels (I assumed 2m diameter circular panels here) and on the tolerance one requires (I assumed 10μm rms), but it is clear that for reasonable size panels of any shape, one needs substantially less than 50 molds.

Conclusions

We have shown that for the modest sized panels planned for the GBT, their surfaces can be adequately represented as a locally quadratic surface. Although they are not all identical, panels with different off axis distances differ only slightly from each other in their surface shape. For the GBT, panels differing in off axis distance by 1m differ from each other by under 10μm rms. As a result of this, it may be practical to make hexagonal or other shaped panels with the same modest number of molds as one would need with trapezoidal panels.

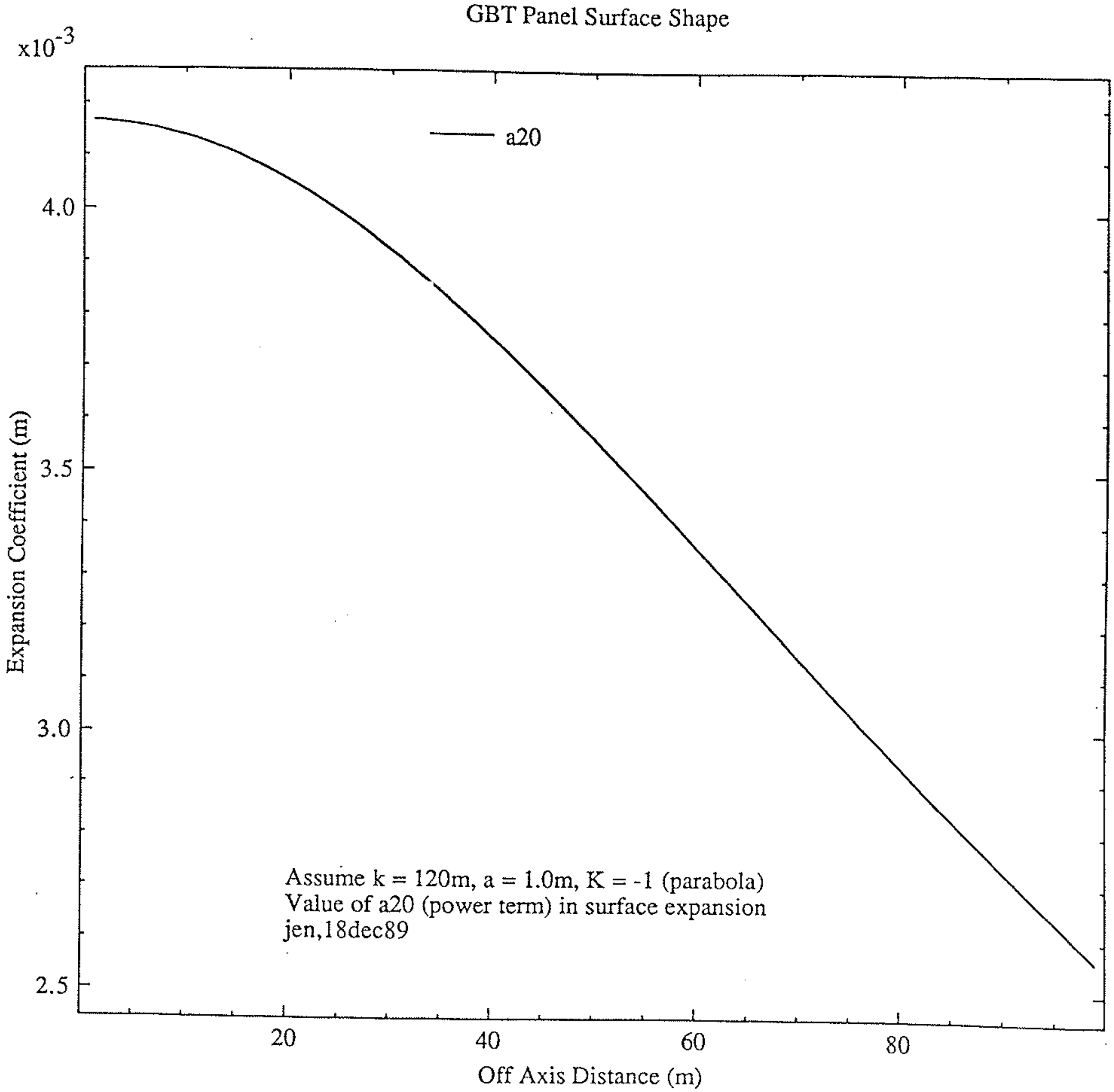

Figure 1. α_{20} vs off axis distance R for the GBT

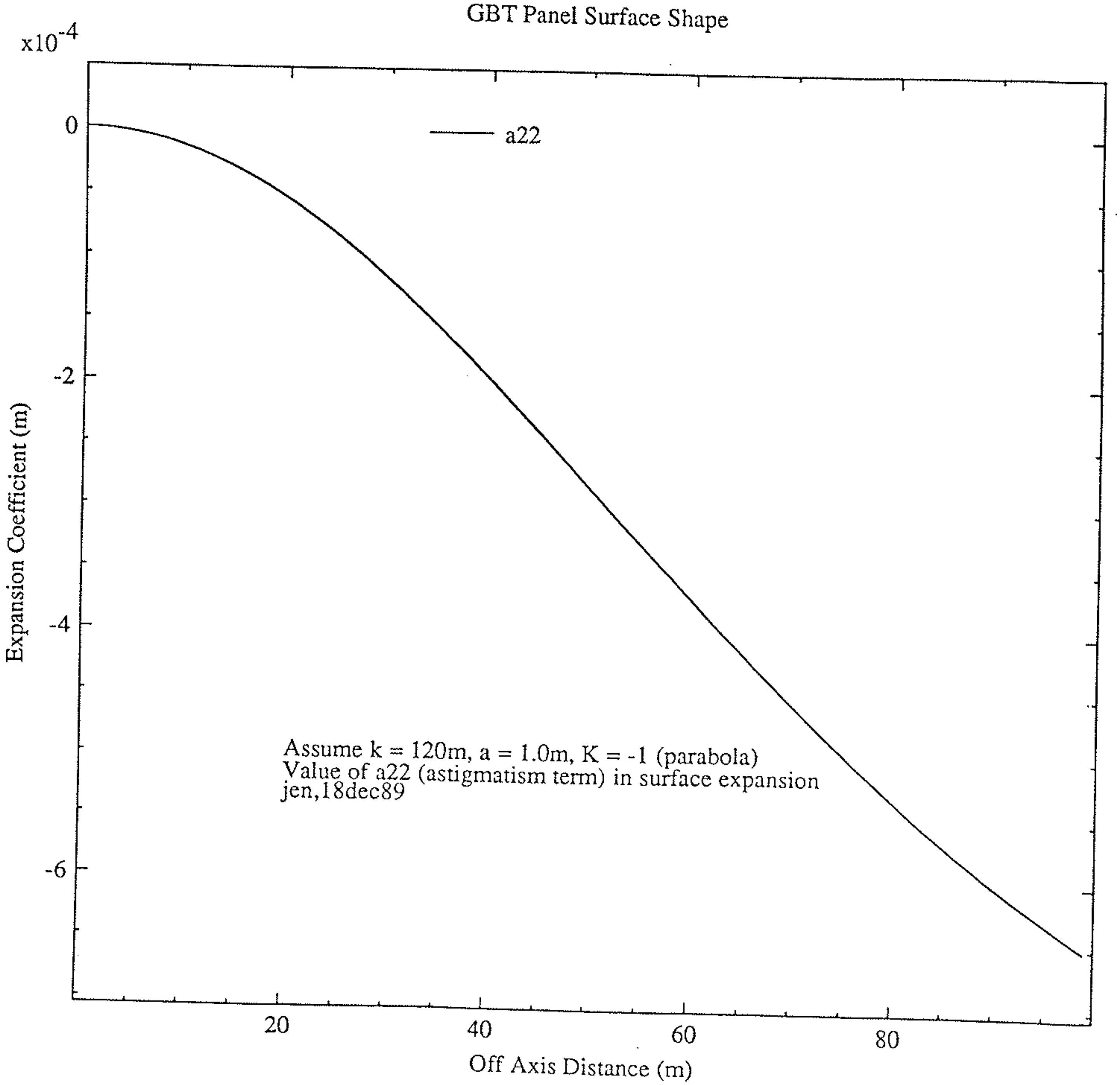

Figure 2. α_{22} vs off axis distance R for the GBT

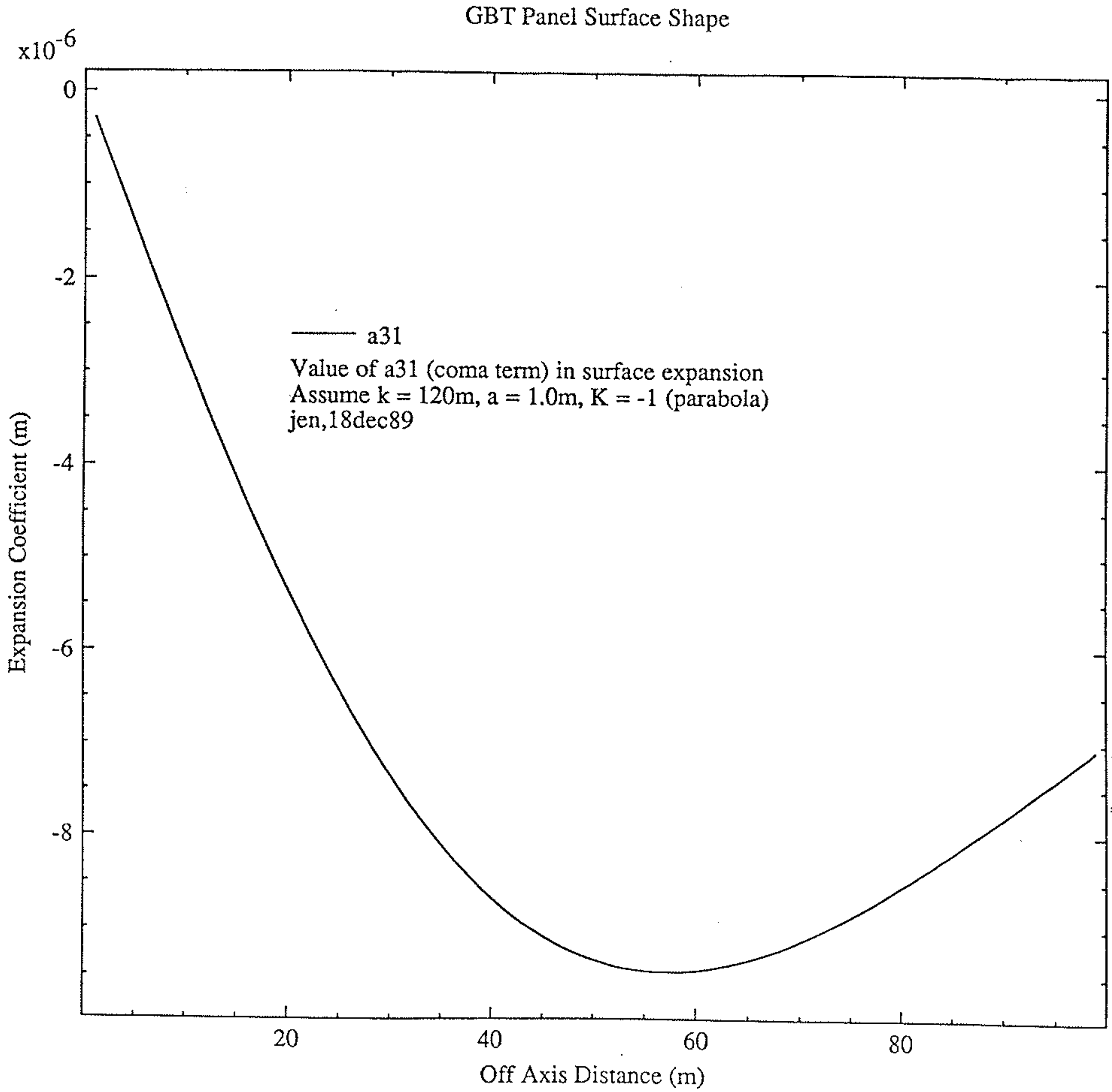

Figure 3. α_{31} vs off axis distance R for the GBT

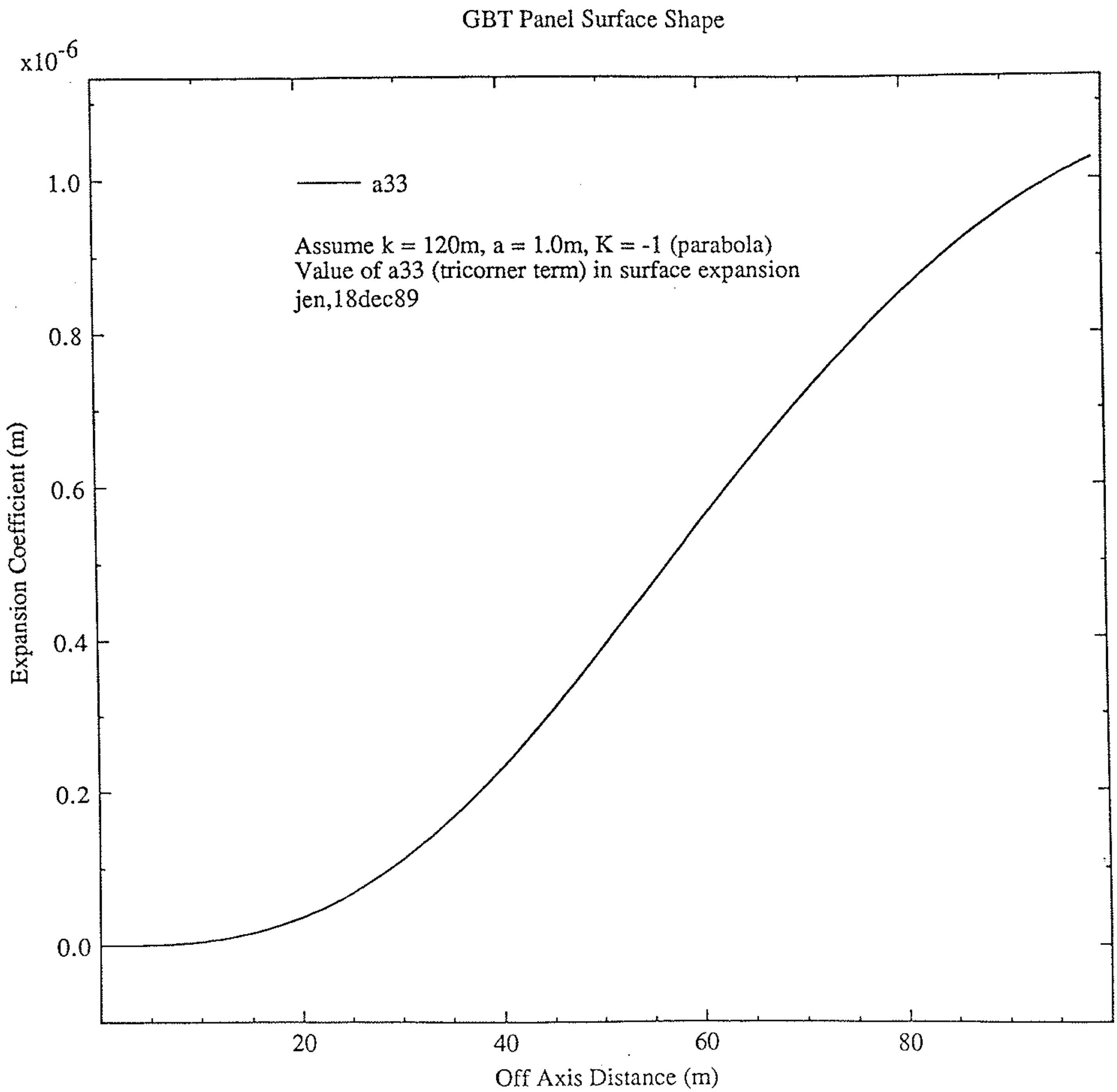

Figure 4. α_{33} vs off axis distance R for the GBT

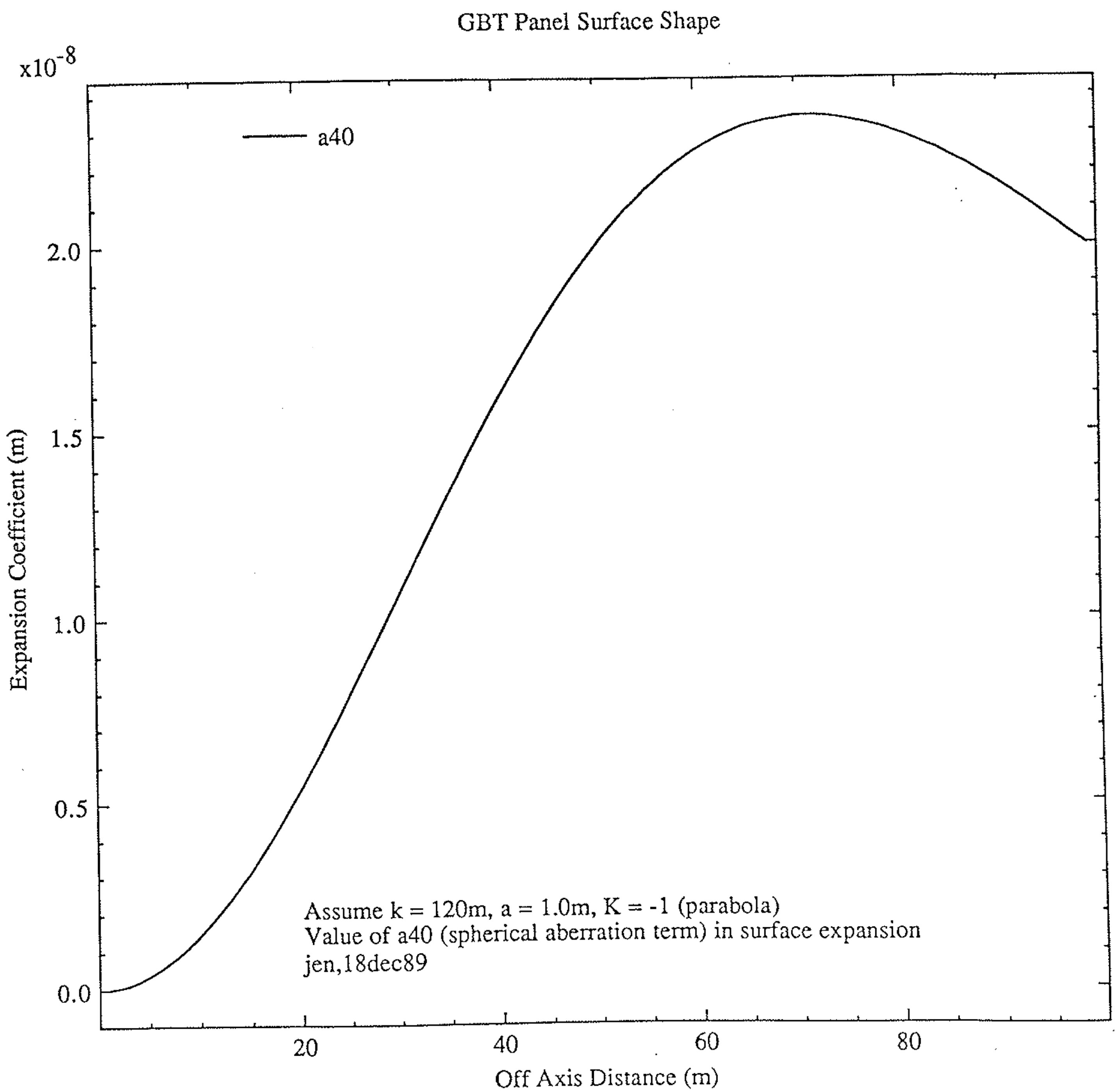

Figure 5. α_{40} vs off axis distance R for the GBT

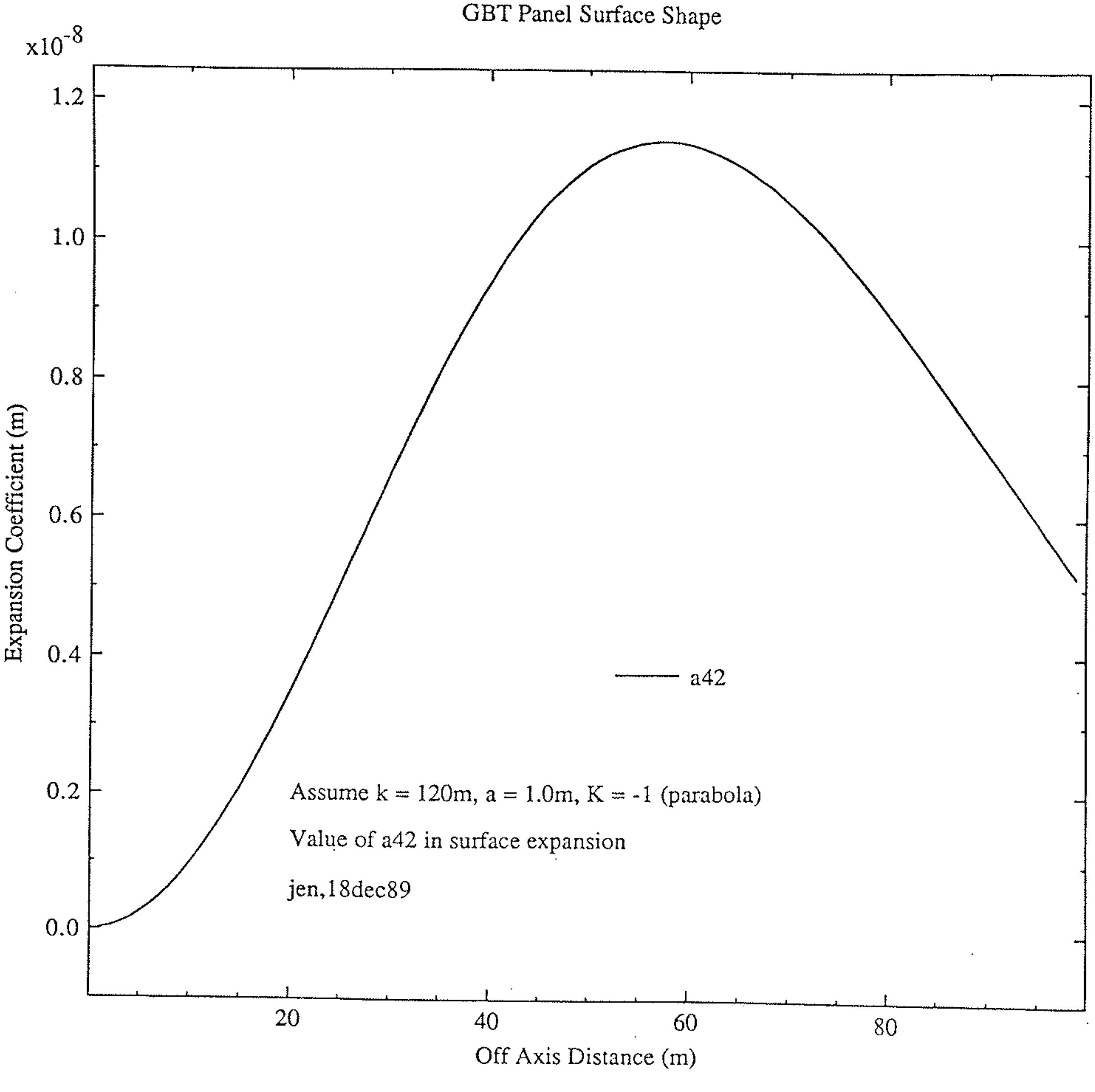

Figure 6. α_{42} vs off axis distance R for the GBT

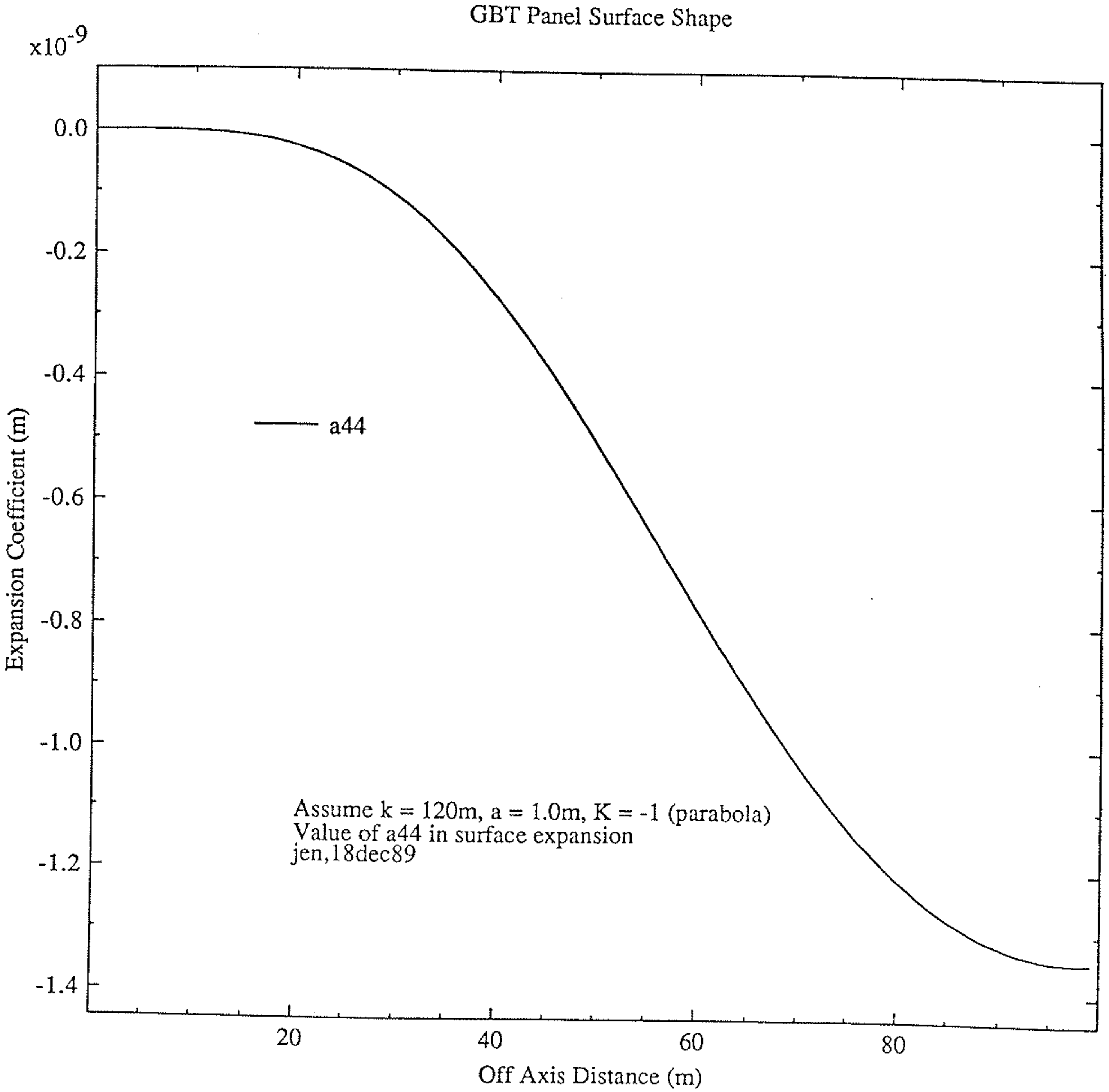

Figure 7. α_{44} vs off axis distance R for the GBT

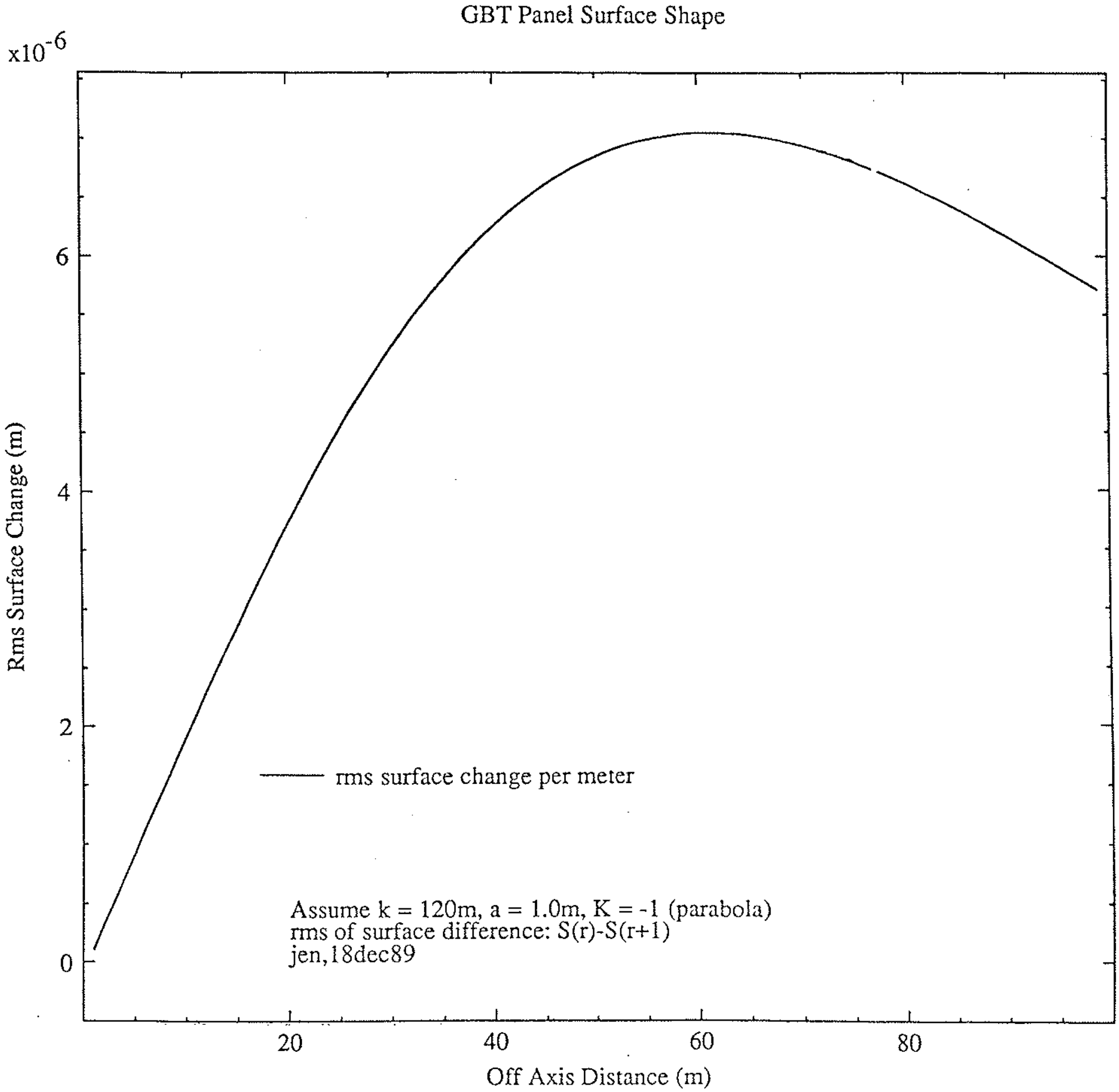

Figure 8. The rms difference between two panels differing by 1m in off axis distance is plotted as a function of off axis distance.

THE OFF-AXIS EXPANSION OF CONIC SURFACES

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ABSTRACT

We derive a Taylor Series expansion for a general axisymmetric conic surface about an arbitrary point on the surface. The series is explicitly evaluated through 4th order. Such expansions are useful in evaluating the optical properties of segment surfaces used in the construction of segmented mirror telescopes. We show that the 4th order expansion is an adequate approximation for an extremely wide range of segment types.

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THE OFF-AXIS EXPANSION OF CONIC SURFACES

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1. Introduction

In order to evaluate or fabricate an optical surface, it is first necessary to define mathematically the shape of that surface. For the case of axisymmetric conics, the formulas are well known, but when one wishes to describe such a surface in a coordinate system not aligned with the axis of symmetry, analytic expressions are much more complex, and not readily available. In addition, when one wishes to assemble an axisymmetric optical surface from segments, it is desirable to have convenient expressions for the segment surface in a coordinate system that is centered on the segment rather than centered on the optical axis. In this report we will determine the coefficients of a Taylor series expansion representing a general conic in such a coordinate system.

2. Axisymmetric Conics

A general expression for a conic surface of revolution is given by

$$Z(X,Y) = \frac{1}{K+1}\left\{k - [k^2 - (K+1)S^2]^{1/2}\right\} \qquad (1)$$

$$= \frac{S^2}{2k} + (K+1)\frac{S^4}{8k^3} + (K+1)^2\frac{S^6}{16k^5} + \frac{5}{128}(K+1)^3\frac{S^8}{k^7} + \cdots$$

K = conic constant$(= -e^2,\ e = \text{eccentricity})$

k = radius of curvature

$S = (X^2 + Y^2)^{1/2}$

The geometry is indicated in Figure 1. For various values of K, this reduces to well known expressions

$K < -1$	Hyperboloid of revolution
$K = -1$	Paraboloid
$-1 < K < 0$	Prolate ellipsoid
$K = 0$	Sphere
$K > 0$	Oblate ellipsoid

The paraboloid expression can be found by taking the limit of equation (1) as $K \rightarrow -1$, to obtain

$$Z_{paraboloid} = \frac{S^2}{2k} \qquad (2)$$

For a sphere, equation (1) reduces to

$$Z_{sphere} = k - (k^2 - S^2)^{1/2} \tag{3}$$

or

$$Z_{sphere} = \frac{S^2}{2k} + \frac{S^4}{8k^3} + \frac{S^6}{16k^5} + \frac{5}{128}\frac{S^8}{k^7} + \cdots$$

3. Transformation to Off-axis Coordinates

We now consider the description of the conic as viewed from a coordinate system that is tangent to the surface at a general point away from the axis of symmetry. Figure 1 shows the geometry. We define R as the distance from the rotation axis to the new coordinate center; thus $R = (X_0^2 + Y_0^2)^{1/2}$. Without loss of generality we choose the new coordinate center on the Y axis.

We are interested in describing the surface of a segment of radius a in terms of the local coordinates x,y. We use the dimensionless polar variables $\rho = (x^2 + y^2)^{1/2}/a$ $\theta = \tan^{-1}(y/x)$. A convenient general form for the description of the surface is

$$z(\rho,\theta) = \sum_{m,n} \alpha_{mn}\rho^m \cos n\theta + \beta_{mn}\, \rho^m \sin n\theta \qquad m \geqslant n \geqslant 0,\ m - n \text{ even}. \tag{4}$$

By suitable selection of the coordinates, we can set $\beta_{mn} = 0$. Thus our objective is to derive expressions for the α_{mn}. The global coordinates (X,Y) are related to the local coordinates (x,y) by a rotation ϕ_0 and a translation Z_0.

$$\tan\phi_0 = \left.\frac{\partial Z}{\partial X}\right|_{X=R} = \frac{R}{[k^2 - (K+1)R^2]^{1/2}} \tag{5}$$

$$Z_0 = Z(X{=}R, Y{=}0) = \frac{1}{K+1}(k - [k^2 - (K+1)R^2]^{1/2})$$

$$X = x\cos\phi_0 - z\sin\phi_0 + R \tag{6a}$$

$$Y = y \tag{6b}$$

$$Z = x\sin\phi_0 + z\cos\phi_0 + Z_0 \tag{6c}$$

$$x = (X - R)\cos\phi_0 + \ + (Z - Z_0)\sin\phi_0 \tag{6d}$$

$$y = Y \tag{6e}$$

$$z = -(X - R)\sin\phi_0 + (Z - Z_0)\cos\phi_0 \tag{6f}$$

For compactness, we introduce the dimensionless variables

$$u = \frac{x}{k}\ ,\quad v = \frac{y}{k}\ ,\quad w = \frac{z}{k}\ ,\quad \epsilon = \frac{R}{k}\ ,\quad W = \frac{Z(X,Y)}{k} \tag{7a}$$

and the quantities

$$s \equiv \sin\phi_0 = \frac{R}{[k^2 - KR^2]^{1/2}} = \frac{\epsilon}{[1 - K\epsilon^2]^{1/2}}$$

$$c \equiv \cos\phi_0 = \left[\frac{k^2 - (K+1)R^2}{k^2 - KR^2}\right]^{1/2} = \left[\frac{1 - L\epsilon^2}{1 - K\epsilon^2}\right]^{1/2} \tag{7b}$$

$$L \equiv K + 1$$

$$W_0 \equiv Z_0/k \equiv (1/L)(1 - c\epsilon/s)$$

Now, using the variables in equation (7), we can rewrite equation (6c), evaluated at a point (X, Y) on the conic as

$$(1/L)(1 - [\,1-L(\,[uc-ws+\epsilon]^2 + v^2\,)]^{1/2}) = us+wc+W_0 \tag{8}$$

We wish to solve this equation for $w(u, v)$. First multiplying by L, subtracting 1, and squaring gives

$$1-L(\,[uc-ws+\epsilon]^2 + v^2) = (1 - L[wc+us+W_0])^2$$

Multiplying out the terms and collecting the coefficients of w yields

$$w^2[Lc^2 + s^2] + 2w[\,c(LW_0-1) - s\epsilon + (L-1)scu] \tag{9}$$
$$+ [\,(Ls^2 + c^2)u^2 + v^2 + 2(s(LW_0-1)+c\epsilon)u + (LW_0^2-2W_0+\epsilon^2)\,] = 0$$

Using the definitions in Equation 7 one can show that each of the last two sums in parenthesis vanishes. To further simplify Equation 9 we define the following quantities

$$\begin{aligned} f &\equiv (s/\epsilon)^2 g \\ g &\equiv -1/(\,Lc^2 + s^2\,) \\ h &\equiv (\epsilon/s)g \\ j &\equiv -(L-1)scg \end{aligned} \tag{10}$$

Using these we can rewrite the equation for w

$$w^2 + 2w(h + ju) - (fu^2 + gv^2) = 0 \tag{11}$$

The solution is

$$w = -(h + ju) + [(h + ju)^2 + fu^2 + gv^2]^{1/2} \tag{12}$$

4. Expansion

We now expand Equation 12 in a Taylor series. Since u and v are small, h is the largest term. We obtain after some algebra and regrouping of terms,

$$w = \frac{1}{2h}(fu^2 + gv^2) - \frac{j}{2h^2}(fu^3 + guv^2) + \tag{13}$$
$$\frac{1}{8h^3}[f(4j^2 - f)u^4 - g^2v^4 + 2g(2j^2 - f)u^2v^2] + \cdots$$

Conversion to polar coordinates is achieved with the following standard relations

$$\begin{aligned} u^2 &= \frac{(a\rho/k)^2}{2}(1 + \cos 2\theta) \\ v^2 &= \frac{(a\rho/k)^2}{2}(1 - \cos 2\theta) \\ u^3 &= \frac{(a\rho/k)^3}{4}(3\cos\theta + \cos 3\theta) \\ uv^2 &= \frac{(a\rho/k)^3}{4}(\cos\theta - \cos 3\theta) \\ u^4 &= \frac{(a\rho/k)^4}{8}(3 + 4\cos 2\theta + \cos 4\theta) \\ v^4 &= \frac{(a\rho/k)^4}{8}(3 - 4\cos 2\theta + \cos 4\theta) \\ u^2v^2 &= \frac{(a\rho/k)^4}{8}(1 - \cos 4\theta) \end{aligned} \tag{14}$$

Inserting these relations into Equation 13 and collecting the terms according to Equation 4 yields

$$\begin{aligned}
\alpha_{20} &= (a/k)^2 \frac{f+g}{2h} \\
\alpha_{22} &= (a/k)^2 \frac{f-g}{2h} \\
\alpha_{31} &= -(a/k)^3 \frac{b}{8h^2}(3f+g) \\
\alpha_{33} &= -(a/k)^3 \frac{j}{8h^2}(f-g) \\
\alpha_{40} &= (a/k)^4 \frac{1}{64h^3}[3f(4j^2-f) - 3g^2 + 2g(2j^2-f)] \\
\alpha_{42} &= (a/k)^4 \frac{1}{64h^3}[4f(4j^2-f) + 4g^2] \\
\alpha_{44} &= (a/k)^4 \frac{1}{64h^3}[f(4j^2-f) - g^2 - 2g(2j^2-f)]
\end{aligned} \tag{15}$$

Now plugging in the definitions from equations (7b) and (10) yields the final expressions for the coefficients.

$$\begin{aligned}
\alpha_{20} &= \frac{a^2}{k}\left[\frac{2-K\epsilon^2}{4(1-K\epsilon^2)^{3/2}}\right] && (\textit{focus}) \\
\alpha_{22} &= \frac{a^2}{k}\left[\frac{K\epsilon^2}{4(1-K\epsilon^2)^{3/2}}\right] && (\textit{astigmatism}) \\
\alpha_{31} &= \frac{a^3}{k^2}\left[\frac{K\epsilon[1-(K+1)\epsilon^2]^{1/2}(4-K\epsilon^2)}{8(1-K\epsilon^2)^3}\right] && (\textit{coma}) \\
\alpha_{33} &= \frac{a^3}{k^2}\left[\frac{K^2\epsilon^3[1-(K+1)\epsilon^2]^{1/2}}{8(1-K\epsilon^2)^3}\right] \\
\alpha_{40} &= \frac{a^4}{k^3}\left[\frac{8(1+K) - 24K\epsilon^2 + 3K^2\epsilon^4(1-3K) - K^3\epsilon^6(2-K)}{64(1-K\epsilon^2)^{9/2}}\right] && \left(\begin{matrix}\textit{spherical}\\ \textit{aberration}\end{matrix}\right) \\
\alpha_{42} &= \frac{a^4}{k^3}\left[\frac{K\epsilon^2[2(1+3K) - (9+7K)K\epsilon^2 + (2+K)K^2\epsilon^4]}{16(1-K\epsilon^2)^{9/2}}\right] \\
\alpha_{44} &= \frac{a^4}{k^3}\left[\frac{K^2\epsilon^4[1+5K-K\epsilon^2(6+5K)]}{64(1-K\epsilon^2)^{9/2}}\right]
\end{aligned} \tag{16}$$

In general,

$$\alpha_{mn} = o\left(\frac{a^m\epsilon^n}{k^{m-1}}\right)$$

For a sphere, $K = 0$, and equation (16) reduces to

$$\begin{aligned}
\alpha_{20} &= \frac{a^2}{2k} \\
\alpha_{40} &= \frac{a^4}{8k^3}
\end{aligned} \tag{17}$$

For a parabola, $K = -1$, and equation (16) reduces to

$$\begin{aligned}
\alpha_{20} &= \frac{a^2}{k}\left[\frac{2+\epsilon^2}{4(1+\epsilon^2)^{3/2}}\right] \\
\alpha_{22} &= -\frac{a^2}{k}\left[\frac{\epsilon^2}{4(1+\epsilon^2)^{3/2}}\right] \\
\alpha_{31} &= -\frac{a^3}{k^2}\left[\frac{\epsilon(4+\epsilon^2)}{8(1+\epsilon^2)^3}\right] \\
\alpha_{33} &= \frac{a^3}{k^2}\left[\frac{\epsilon^3}{8(1+\epsilon^2)^3}\right] \\
\alpha_{40} &= \frac{a^4}{k^3}\left[\frac{3\epsilon^2(8+4\epsilon^2+\epsilon^4)}{64(1+\epsilon^2)^{9/2}}\right] \\
\alpha_{42} &= \frac{a^4}{k^3}\left[\frac{\epsilon^2(4-2\epsilon^2-\epsilon^4)}{16(1+\epsilon^2)^{9/2}}\right] \\
\alpha_{44} &= -\frac{a^4}{k^3}\left[\frac{\epsilon^4(4-\epsilon^2)}{64(1+\epsilon^2)^{9/2}}\right]
\end{aligned} \tag{18}$$

5. Verification of the Expansion

Because the algebraic derivation of the above expressions is rather involved, we felt that an independent check of equation (16) was desirable. This has been done numerically. The procedure is based on the ease of transforming a single point on the conic from one coordinate system to the other, using equation (6). For each of a variety of cases, a grid of points (approx. 700) on the given surface was generated in the parent coordinate system, and each point was transformed to the local coordinate system using equation (6). This set of data points was then fit in a least squares sense with Zernike polynomials. Twenty eight polynomials (through 6th order) were used, and the resulting coefficients were then used to calculate the coefficients defined in equation (4). Conic constants ranging from −2 to +1 were used, along with a variety of off-axis distances. In all cases, the numerical results agreed with equation (16) to the expected accuracy of the numerical computation. Numerical errors were roughly 10^{-12} of the basic surface amplitude. Even the smallest terms were checked to accuracies better than 10^{-4}. Thus the correctness of the algebraic expressions has been confirmed numerically.

6. Examples

The hexagonal segments envisioned for the University of California Ten Meter Telescope have a segment radius a=0.9m. Thirty-six such segments make up the primary mirror. The f/1.75 primary has k=35m. Since the primary will be extremely close to a paraboloid, we will assume for this example that K=−1. Using equation (18) we calculate the expansion coefficients as a function of the off-axis distance. The results are shown in figure 2. The marks along the abscissa indicate the actual off-axis distances of the segments. As is true for all but the most extreme telescope configurations, astigmatism (α_{22}) is the dominant aberration, followed by coma (α_{31}). We have also included on the plot the root mean square deviation of the surface from the best fitting sphere. The sensitivity of the coefficients to the conic constant is indicated in figure 3. Here we show the coefficients over a range of K from −2 to +2. Note that for all coefficients the variation is smooth. In particular, we note that hyperbolic surfaces with conic constants near −1 are quite similar to parabolas.

Acknowledgements

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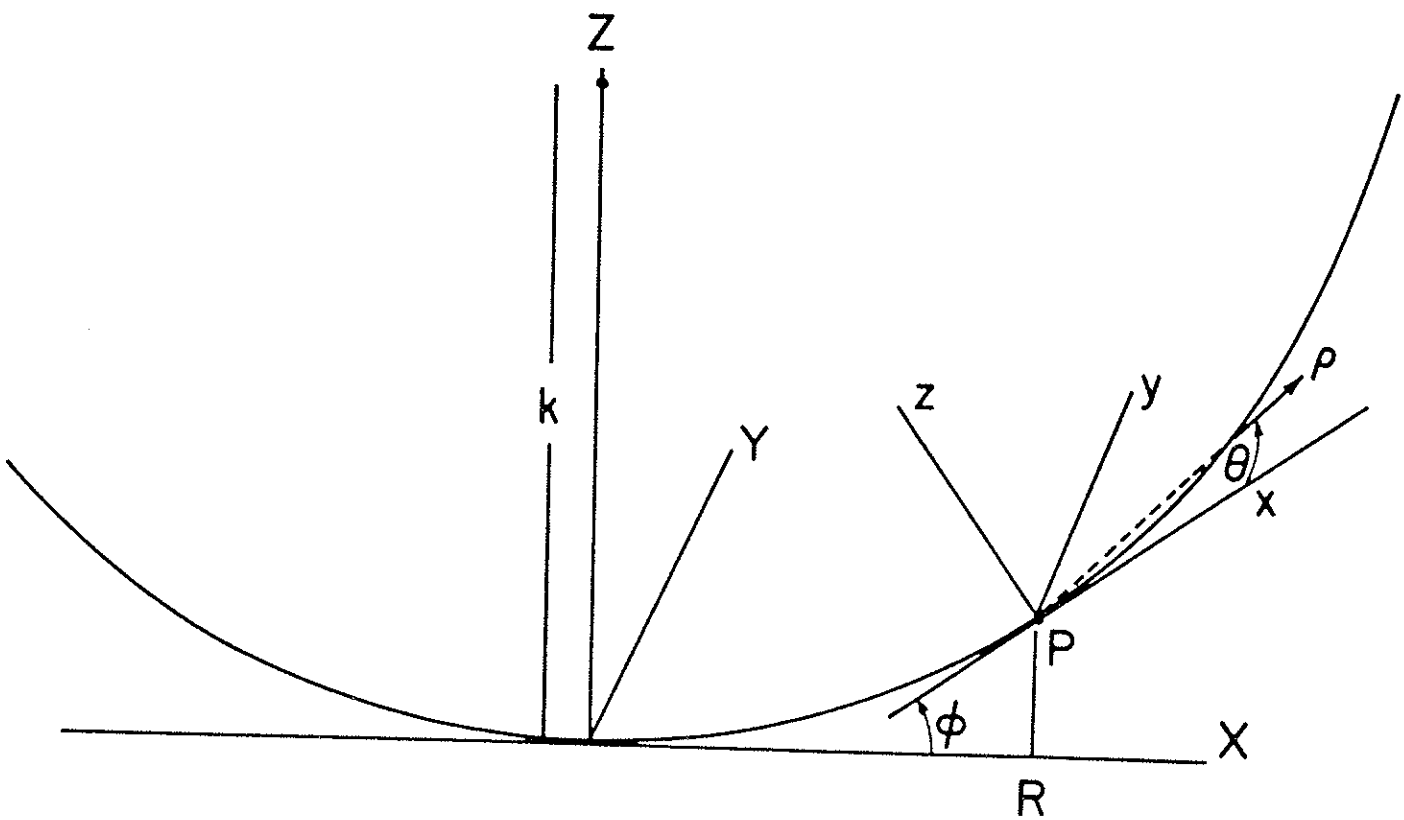

XBL 7912-13613

Fig. 1. Diagram defining global (X,Y,Z) and local coordinates (x,y,z = r,Θ,z) of mirror segment on conic.

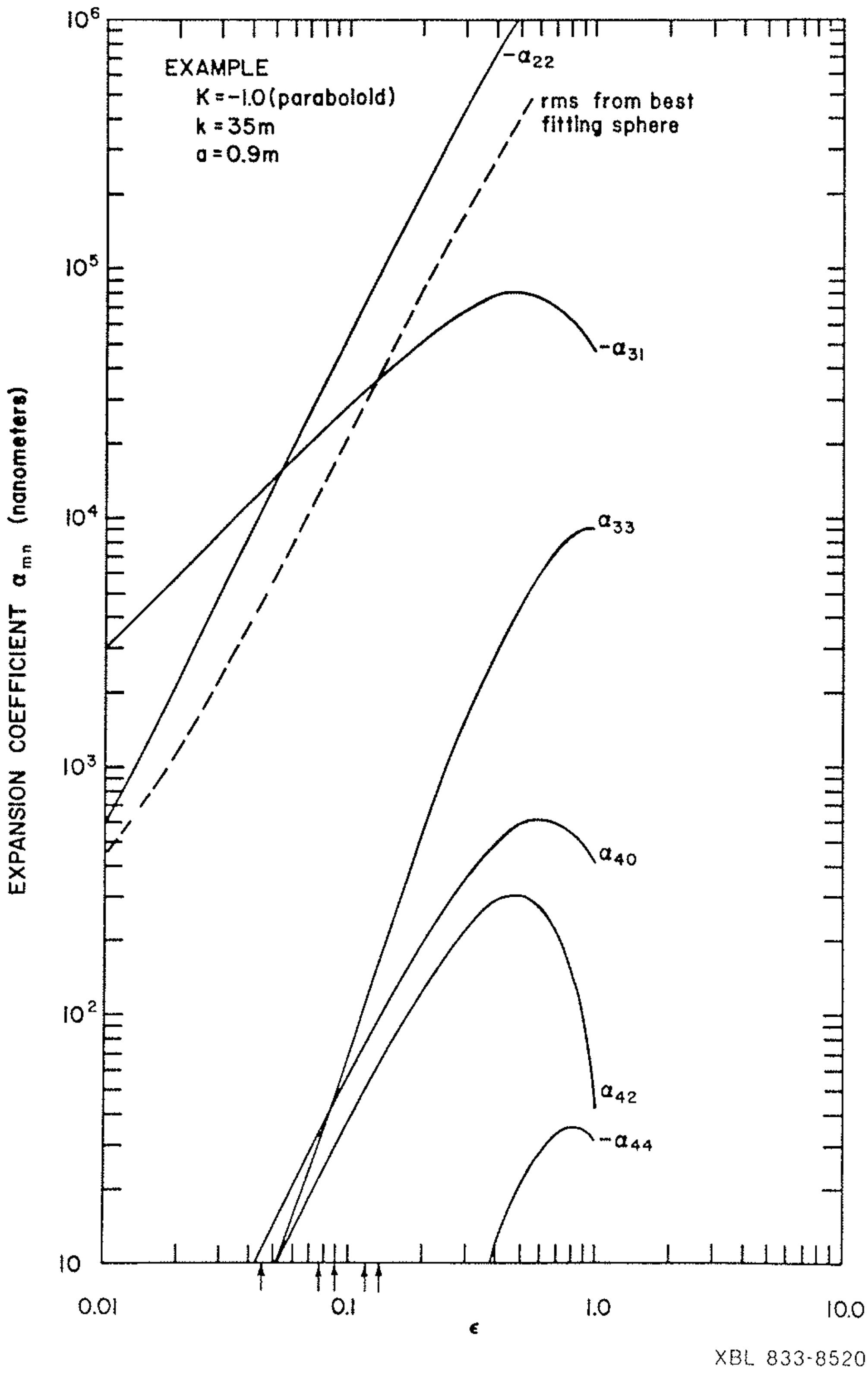

Fig. 2. Expansion coefficients describing the surface of an off axis section of a conic as a function of off-axis distance are shown. In this example the conic is a paraboloid, and a segment radius of a = 0.9m and a radius of curvature k = 35m are assumed. The arrows along the abscissa indicate the off axis distances for the segments of the UC Ten Meter Telescope. The focus term has been omitted. The dashed line gives the root mean square difference between the conic and the best fitting sphere.