GBT Memo No. 40

NATIONAL RADIO ASTRONOMY OBSERVATORY
Green Bank, WV

MEMORANDUM

March 6, 1990

To: GBT Memo Series

From: Roger D. Norrod

Subj: **Increase in Distance between Large and Small Subreflectors**

We have been asked to increase the distance between the large (M1) and small (M2) subreflectors, to allow M2 to be pivoted into place without moving M1. Figure 1 shows the current design. Table 1 gives numerical parameters for both M1 and M2.

The parameter M_d (see Table 1) is given by:

$$M_d = \frac{\overline{F_1 I_1}}{\overline{F_0 I_1}}$$

so,

$$\overline{F_0 I_1} = \frac{15.099}{2.633} = 5.735 \text{ meters}$$

$$\overline{F_0 I_2} = \frac{13.143}{4.332} = 3.034 \text{ meters}$$

and,

$$\overline{I_1 I_2} = 5.735 - 3.034 = 2.701 \text{ meters} = 106 \text{ inches}$$

Lee King requested we increase $\overline{I_1 I_2}$ by at least 30 inches. This memo will discuss how this could be done and point out the repercussions.

The design equations for offset reflector geometry can be arranged in various forms. The equations I use were taken from [1] and relate physical antenna parameters in a reasonable way.

The equation for the paraboloid in Figure 2 is given by:

$$x^2 + y^2 = -4f\,(z - f) \qquad (1)$$

R is usually chosen to achieve a desired aperture size, f on the basis of electrical performance, and Y_C at the minimum for no blockage. Once these three parameters are set, the coordinates of I_L and I_H can be determined from (1), and the angles Θ_0 and Θ^* found.

It has been shown that the offset dual reflector geometry can be arranged such that circles of rays transmitted by the secondary feed are transformed by the dual reflectors to concentric circles on the paraboloid aperture, and that a rotationally symmetric feed pattern produces a rotationally symmetric aperture distribution. This also guarantees that a feed with no cross-pol gives rise to an aperture field with no cross-pol. Figure 3 shows the required geometry. The subreflector axis (F_0F_1) is tilted at an angle β with respect to the paraboloid axis (F_0V_0), and β can be found by solving

$$Y_C = 4fe \sin \beta/(1 + e^2 - 2e \cos \beta) \qquad (2)$$

where e is the eccentricity of the subreflector. In order to illuminate the main reflector, the feed axis must be tilted by the angle α with respect to the ellipsoid axis. For the transformation yielding zero cross-pol to hold, α and β must satisfy:

$$\tan \alpha = \frac{(1 - e^2) \sin \beta}{(1 + e^2) \cos \beta - 2e}$$

Θ_H is the cone angle from F_1 defining the rays which illuminate the edge of the subreflector and then are reflected to the edge of the main reflector.

$$\tan (\Theta_H/2) = R \frac{(1 + e^2 - 2e \cos \beta)}{2f (1 - e^2)}$$

Note that C, the ellipsoid interfocal distance, does not enter into these equations. It serves as a scaling factor and can be used to vary the size of the subreflector, but does not change the angles α, β, or Θ_H. A program has been written in Pascal for the IBM-PC that will solve these equations. The program's input parameters are e, Y_C, f, R, and C. Table 1 was generated by this program, and it makes exploring options fairly painless.

Table 2 varies the system parameters to try to increase the distance $\overline{I_1I_2}$. Cases 1 and 2 are our current M1 and M2-1 designs. Let us assume that M1 is not changed because to move it further from F_0, it would get larger, which is undesirable economically. Cases 3 and 4 illustrate what happens when e is varied to bring M2 closer to F_0. Case 4 achieves the 30 inch (0.76 meter) increase in $\overline{I_1I_2}$ desired. Case 3 is an intermediate step that achieves a 17 inch increase.

The subreflectors of cases 3 and 4 are smaller and the effective focal lengths longer, both desirable trends. The undesirable trend is the decrease in Θ_H. The feed diameters change approximately as the inverse ratio of Θ_H, and the feed length approximately by the square of that ratio.

Tables 3A-3D list representative feed sizes for the four values of Θ_H from Table 2. These sizes are approximate, and do not include corrugations, flanges, etc., but are useful for comparisons. Comparing tables 3B and 3D, case 4, wideband feeds at 3 cm are 1.9 feet dia. x 6.1 feet long, whereas case 2 feeds are 1.5 feet dia. x 3.7 feet long. At 6 cm the feeds are 3.8 x 12.1 feet versus 3.0 x 7.4 feet. On balance, case 4 does not look particularly objectionable, especially if we are allowed to do all wavelengths longer than about 3 cm with M1.

As mentioned above, the subreflector can be reduced in size by decreasing C, without changing e, α, β, or Θ_H. Table 2, case 5, achieves the 17 inch $\overline{I_1I_2}$ increase of case 3 by moving the M2 feed 1.6 meters in front of the M1 feeds. Case 6 achieves the 30 inch increase by decreasing C to 8.2 meters. The attractive feature is that the subreflector has been made smaller and moved closer to F_0 without making the feeds larger. A practical problem with this approach is that the M2 feeds and receivers will block the M1 feeds and would probably have to be physically moved when using the M1 feeds. Also, one working idea for supporting the prime focus feeds might have interference problems if the subreflector feeds are moved closer to the subreflectors.

It is possible to effectively reduce C using curved reflectors, while keeping the feeds in convenient locations, as sketched in Figure 4. Theoretically, if edge diffraction is negligible, mirrors M4 and M3 can transfer a perfect image of the feed pattern from F2′ to F2. Also, theory indicates that the system M2, M3, and M4 could be designed to achieve zero cross-pol in the main reflector aperture. However, the cost of two extra curved reflectors is probably not negligible, and we would have to be concerned with the feed spillover, possible baseline effects, and other hidden traps. I do not feel qualified to bet the farm on such a system without serious analysis and the opinions of electro-magnetic experts.

There is a possible performance penalty when C is reduced. As was mentioned in GBT Memo 29, offset feeds do not properly illuminate the main reflector if the subreflector is not enlarged. This effect probably gets worse as C is reduced, but we have no numerical analysis yet to indicate the seriousness of the problem. However, the problem depends on the absolute offset angle, not the number of beamwidths scanned, so I suspect it is not a serious concern for the frequency range of M2.

In summary, it appears feasible to reduce the distance $\overline{F_0I_2}$, and M2 to approximately 3 meters, by reducing Θ_H or C, or a combination of both.

REFERENCE

[1] Reflector and Lens Antennas, C. J. Sletten, Editor. Artech House, 1988.

TABLE 1

Parameters for Current M1 and M2-1.

GREGORIAN OFFSET ANTENNA 02/22/90 21:47:46
Current M1

1) Eccentricity = 0.528 2) Yc = 54.000
3) Focal Length = 60.000 4) Radius = 50.000
5) Ellipsoid Focal Length = 11.000

Alpha =	17.898781	Beta =	5.569959
Mag =	3.166423	Feed Semi-Angle =	14.992858
B =	379.970705	D =	14.228535
Theta * =	39.005231	Theta o =	42.823536

Subreflector	7.553	by	7.948	Y Range	-7.262	to	-0.329
Main Reflector	100.000	by	109.659	F1 to I1	=	15.099	

Theta C = 48.455491 Rho C = 72.150 Md = -2.633200
Equivalent Paraboloid: f0 = -189.985 i0 = -0.000000

GREGORIAN OFFSET ANTENNA 02/22/90 21:48:08
Current M2-1

1) Eccentricity = 0.680 2) Yc = 54.000
3) Focal Length = 60.000 4) Radius = 50.000
5) Ellipsoid Focal Length = 11.000

Alpha =	10.246437	Beta =	1.956730
Mag =	5.209655	Feed Semi-Angle =	9.145532
B =	625.158570	D =	6.394464
Theta * =	39.005231	Theta o =	42.823536

Subreflector	4.073	by	4.332	Y Range	-4.009	to	-0.173
Main Reflector	100.000	by	109.659	F1 to I1	=	13.143	

Theta C = 48.455491 Rho C = 72.150 Md = -4.332353
Equivalent Paraboloid: f0 = -312.579 i0 = -0.000000

TABLE 2

Parameters to Increase the Distance I_1I_2

Case	e	C	Subreflector Size	F_1I_1	F_0I_1	F_e	Θ_H
1	0.528	11	7.55 x 7.95	15.1	5.74	190	14.99
2	0.680	11	4.07 x 4.30	13.1	3.03	313	9.15
3	0.714	11	3.5 x 3.7	12.8	2.59	357	8.00
4	0.740	11	3.1 x 3.3	12.6	2.27	400	7.16
5	0.680	9.4	3.5 x 3.7	11.2	2.59	313	9.15
6	0.680	8.2	3.0 x 3.2	9.8	2.27	313	9.15

TABLE 3A

FEED DESIGN

INPUTS:

FEED HALF BEAM ANGLE, a 14.99 = 0.26 RADIANS

Representative Feed Sizes, Wide-band Corrugated Horns

Spherical Phase Error, DEL = 0.75

THETA * / THETA f = 0.80

Flare Angle, THETA f = 0.33 = 18.74 DEGREES

FEED DIA / LAMBDA = 9.09

FEED LENGTH / LAMBDA = 14.15

LAMBDA,cm	DIA,M	LEN,M	DIA,FT	LEN,FT
3	0.27	0.4	0.89	1.39
4	0.36	0.6	1.19	1.86
6	0.55	0.8	1.79	2.79
15	1.36	2.1	4.47	6.96
20	1.82	2.8	5.97	9.29
30	2.73	4.2	8.95	13.93

Representative Feed Sizes, Narrow-band Corrugated Horns

Spherical Phase Error, DEL = 0.20

Ke = 4.00

Flare Angle, THETA f = 0.16 = 9.29 DEGREES

FEED DIA / LAMBDA = 4.92

FEED LENGTH / LAMBDA = 15.25

LAMBDA,cm	DIA,M	LEN,M	DIA,FT	LEN,FT
3	0.15	0.5	0.48	1.50
4	0.20	0.6	0.65	2.00
6	0.30	0.9	0.97	3.00
15	0.74	2.3	2.42	7.50
20	0.98	3.0	3.23	10.00
30	1.48	4.6	4.85	15.01

TABLE 3B

FEED DESIGN

INPUTS:

FEED HALF BEAM ANGLE, a 9.15 = 0.16 RADIANS

Representative Feed Sizes, Wide-band Corrugated Horns

Spherical Phase Error, DEL = 0.75
THETA * / THETA f = 0.80
Flare Angle, THETA f = 0.20 = 11.44 DEGREES
FEED DIA / LAMBDA = 14.98
FEED LENGTH / LAMBDA = 37.77

LAMBDA,cm	DIA,M	LEN,M	DIA,FT	LEN,FT
1	0.15	0.4	0.49	1.24
2	0.30	0.8	0.98	2.48
3	0.45	1.1	1.47	3.72
4	0.60	1.5	1.97	4.96
6	0.90	2.3	2.95	7.43
9	1.35	3.4	4.42	11.15

Representative Feed Sizes, Narrow-band Corrugated Horns

Spherical Phase Error, DEL = 0.20
Ke = 4.00
Flare Angle, THETA f = 0.10 = 5.72 DEGREES
FEED DIA / LAMBDA = 8.01
FEED LENGTH / LAMBDA = 40.17

LAMBDA,cm	DIA,M	LEN,M	DIA,FT	LEN,FT
1	0.08	0.4	0.26	1.32
2	0.16	0.8	0.53	2.64
3	0.24	1.2	0.79	3.95
4	0.32	1.6	1.05	5.27
6	0.48	2.4	1.58	7.91
9	0.72	3.6	2.36	11.86

TABLE 3C

FEED DESIGN

INPUTS:

FEED HALF BEAM ANGLE, a	8.00	=	0.14 RADIANS

Representative Feed Sizes, Wide-band Corrugated Horns

Spherical Phase Error, DEL =	0.75		
THETA * / THETA f =	0.80		
Flare Angle, THETA f =	0.17	=	10.00 DEGREES
FEED DIA / LAMBDA =	17.15		
FEED LENGTH / LAMBDA =	49.37		

LAMBDA,cm	DIA,M	LEN,M	DIA,FT	LEN,FT
1	0.17	0.5	0.56	1.62
2	0.34	1.0	1.13	3.24
3	0.51	1.5	1.69	4.86
4	0.69	2.0	2.25	6.48
6	1.03	3.0	3.38	9.72
9	1.54	4.4	5.06	14.58

Representative Feed Sizes, Narrow-band Corrugated Horns

Spherical Phase Error, DEL =	0.20		
Ke =	4.00		
Flare Angle, THETA f =	0.09	=	5.01 DEGREES
FEED DIA / LAMBDA =	9.15		
FEED LENGTH / LAMBDA =	52.41		

LAMBDA,cm	DIA,M	LEN,M	DIA,FT	LEN,FT
1	0.09	0.5	0.30	1.72
2	0.18	1.0	0.60	3.44
3	0.27	1.6	0.90	5.16
4	0.37	2.1	1.20	6.88
6	0.55	3.1	1.80	10.32
9	0.82	4.7	2.70	15.48

TABLE 3D

FEED DESIGN

INPUTS:

FEED HALF BEAM ANGLE, a   7.16   =   0.12 RADIANS

Representative Feed Sizes, Wide-band Corrugated Horns

Spherical Phase Error, DEL =	0.75		
THETA * / THETA f =	0.80		
Flare Angle, THETA f =	0.16	=	8.95 DEGREES
FEED DIA / LAMBDA =	19.17		
FEED LENGTH / LAMBDA =	61.60		

LAMBDA,cm	DIA,M	LEN,M	DIA,FT	LEN,FT
1	0.19	0.6	0.63	2.02
2	0.38	1.2	1.26	4.04
3	0.57	1.8	1.89	6.06
4	0.77	2.5	2.52	8.08
6	1.15	3.7	3.77	12.13
9	1.72	5.5	5.66	18.19

Representative Feed Sizes, Narrow-band Corrugated Horns

Spherical Phase Error, DEL =	0.20		
Ke =	4.00		
Flare Angle, THETA f =	0.08	=	4.48 DEGREES
FEED DIA / LAMBDA =	10.22		
FEED LENGTH / LAMBDA =	65.32		

LAMBDA,cm	DIA,M	LEN,M	DIA,FT	LEN,FT
1	0.10	0.7	0.34	2.14
2	0.20	1.3	0.67	4.29
3	0.31	2.0	1.01	6.43
4	0.41	2.6	1.34	8.57
6	0.61	3.9	2.01	12.86
9	0.92	5.9	3.02	19.29

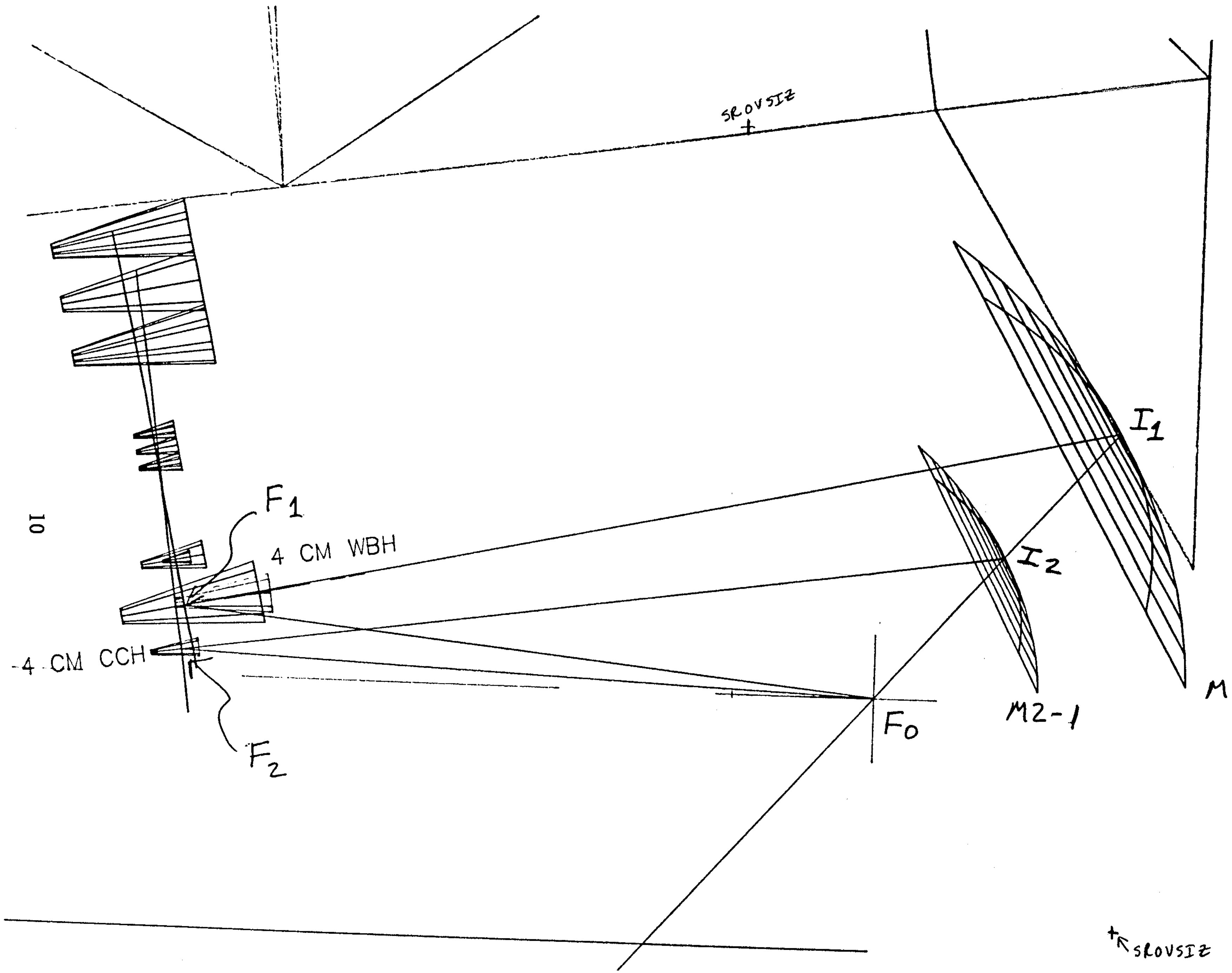

FIGURE 1. Current design, showing both subreflectors.

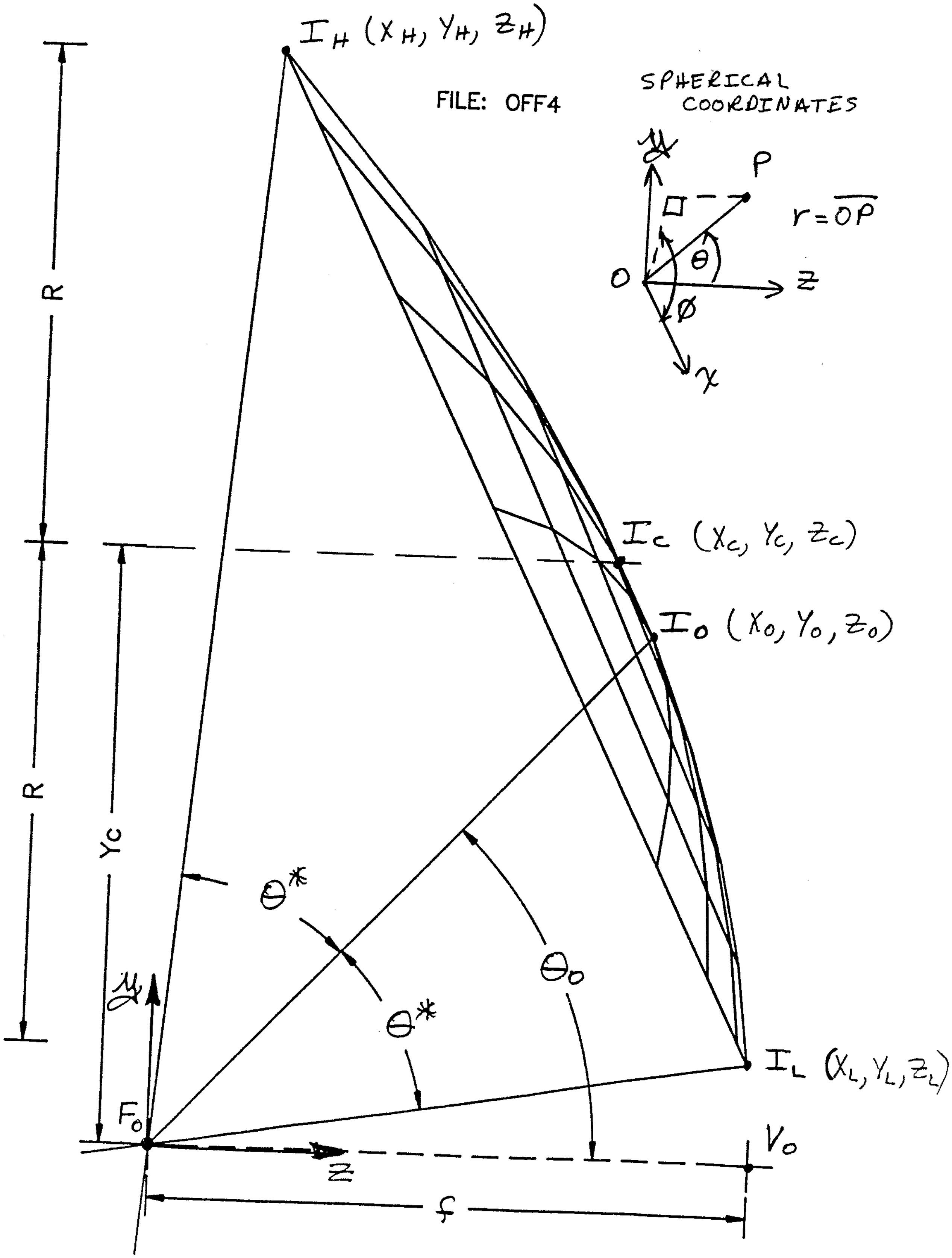

FIGURE 2. Geometry of main reflector.

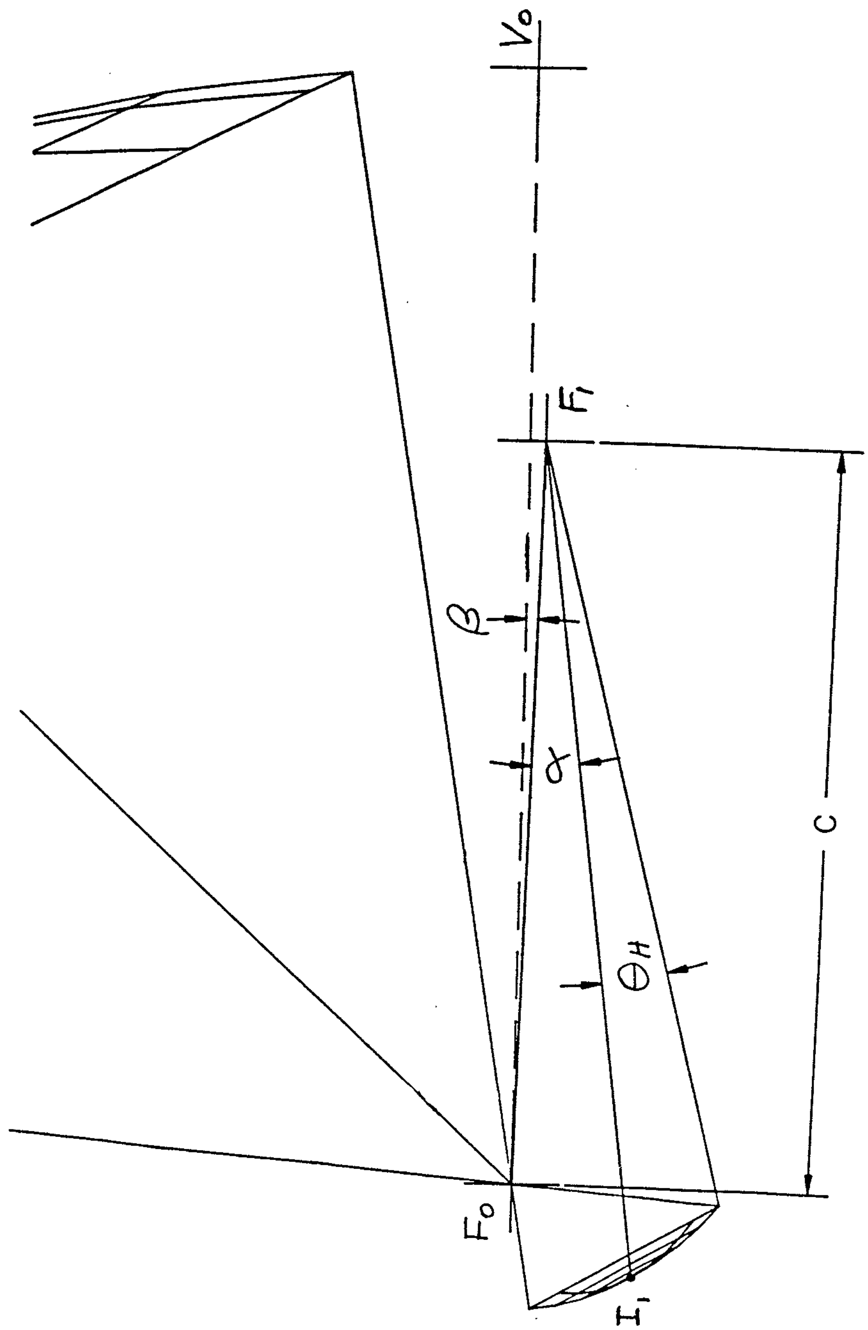

FIGURE 3. Subreflector geometry.

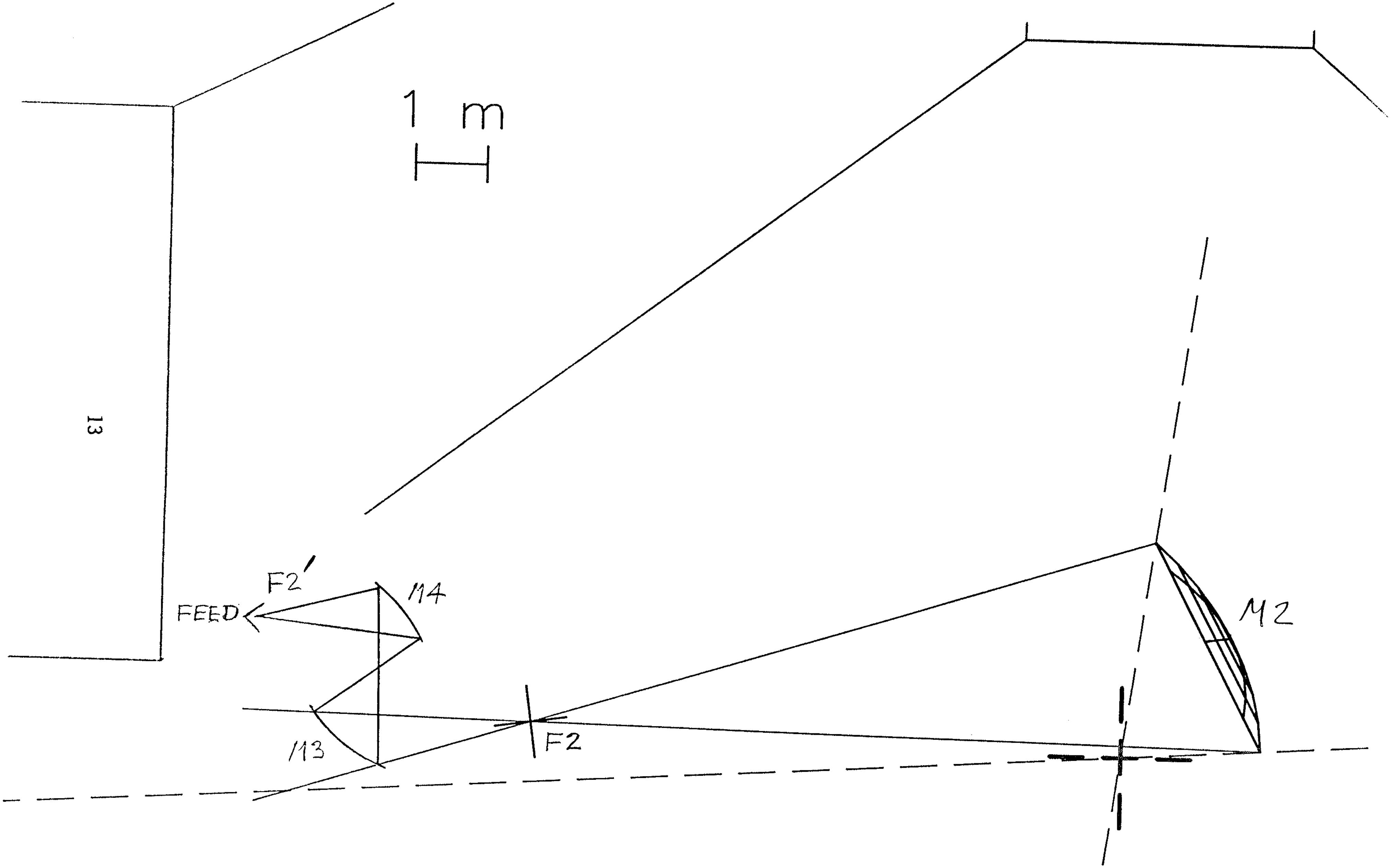

FIGURE 4. Optical transfer of focal point.