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ATMOSPHERIC LIMITATIONS ON THE GBT LASER RANGING SYSTEM

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In order to achieve the necessary precision in the pointing of the Green Bank Telescope we are considering a system using laser ranging, with the lasers operating at a wavelength of a few hundred nanometers. There is then a question as to what limitations on the measurement are presented by the atmospheric fluctuations in the transmission path. I will discuss the possible size of the component of the total error that arises because of atmospheric turbulence.

There is an extensive literature dealing with the accuracy of laser ranging measurements. The paper that I have found the most useful is by C. S. Gardner (Applied Optics, 15, 2539, 1976). He discusses the estimation of the instantaneous random path deviation ΔL and of the mean-square path deviation. Equation 20 of the paper summarizes the result:

$$\langle \Delta L^2 \rangle = 26.3\ C_n^2\ L_o^{5/3}\ Le \qquad (1)$$

In this expression, $\langle \Delta L^2 \rangle$ is the mean-square path deviation of the measurement for a round trip between the ranging site and target at a single wavelength, L_o is the outer scale of turbulence, L_e is the effective pathlength, and C_n^2 is the refractive index structure function. All lengths are measured in meters, and C_n^2 is given in units of $m^{-2/3}$. The quantity L_o varies with altitude, and according to Gardner can be taken to be the lesser of 100 m or 1/3-1/5 the height above the ground. Further, for horizontal paths L_e should be approximately equal to L, the total pathlength that the laser beam traverses. Thus the problem of estimating the accuracy of the laser ranging technique reduces to estimating the properties of the refractive index structure function C_n^2.

The atmospheric refraction is approximately a linear function of the ratio of the atmospheric pressure to the temperature. In the context of refractive index variation, the variation in temperature dominates. Thus the value of the refractive index structure function C_n^2 is often inferred by making a measurement of the temperature structure function C_t^2 using for example an accurate differential thermometer.

There is also a large body of literature on the characteristics of the quantity C_n^2. A useful summary is that given by D. L. Walters and K. E. Kunkel (J. Optical Society America, 71, 397, 1981). Generally C_n^2 is a strong function of height above the ground, of time of day, and of the nature of the terrain over which the propagation test is made. To illustrate this, I reproduce from Walters and Kunkel four figures. The first shows that the variation of C_n^2 with height goes as $h^{-1.16}$. Figure 2 shows the diurnal variation of the structure function measured at a height of 9 m above some mountainous terrain. Figure 3 shows similar data for a desert environment, while Figure 4 shows how going even to 33 m above the ground will improve the atmospheric stability.

Figure 2 presents measurements made at three mountains in New Mexico having heights above sea level between 1.5 and 2.5 km. The data suggest that over a vegetation-covered area in a temperate climate the value of C_n^2 will fall to $5x10^{-15}$ $m^{-2/3}$ at night. If L_e is taken to be 200 m, and L_o a typical value of 10 m, then equation (1) predicts that the roundtrip path length can be measured with an rms uncertainty of 35 micrometers, an acceptably small value. However, over a desert during the day the value of C_n^2 might rise to at least $3x10^{-13}$ $m^{-2/3}$, in which case the rms of the laser ranging measurement will approach 300 micrometers.

I have trouble guessing what the structure function for our particular case will be. Much of the path for both the pointing and the surface adjustment systems will lie more than a few meters above the ground, so that if C_n^2 falls with height as indicated in Figure 1 and Figure 4, the rms of the ranging measurement should be 50 micrometers or less even in the daytime. However, the measurement is being made over a metal structure, at least for the surface adjusting system, so that almost certainly C_n^2 will not fall as rapidly with height as suggested by the figures. On the other hand, the painted telescope structure might cause less thermal convection than the dry sand case I have used above. The C_n^2 for wet fields for example is about 2.5 times smaller than for dry sand. The C_n^2 for wet fields and overcast sky is a full order of magnitude less that of dry sand under full solar illumination.

Another source of uncertainty is the applicability of the turbulence theory, especially the question of the scale lengths. It is however encouraging that there are a few measurements over path lengths of about 100 m which give results that generally agree with the theoretical expectations (cf. Matsumoto and Tsukahara, Applied Optics, 23, 3388, 1984).

It should be noted that equation (1) contains no allowance for the error arising because of an incorrect measurement of the constant component of the total path length. John Payne (Review of Scientific Instruments, 44, 304, 1973) has discussed this briefly. Over the round trip distance of 200 m, the "excess path" under standard conditions will be approximately 60 mm (cf. the useful discussion in Chapter 13 of Interferometry and Synthesis in Radio Astronomy, by Thompson, Moran, and Swenson). The temperature and pressure will therefore each have to be measured to an accuracy of 0.1 percent in order to reduce the error of total path to 60 micrometers. This may be difficult in practice. If it turns out to be so, it will be necessary to estimate the characteristics of the ranging path either by monitoring a path of fixed distance or by using a two-color laser.

I conclude that the proposed ranging system will not be limited by the affects of atmospheric turbulence during the night. It perhaps will meet the desired accuracy of 50 micrometers during cloudy days, especially if there is a slight breeze. The system will not be able to reach this goal during full sunlight, unless some way can be found to overcome the level of turbulence that I anticipate will be present. Thus thermally-induced surface errors will not be corrected at these times. Finally I note that the system will have difficulty even in calm air when there is fog or cloud.

It would be valuable to measure some C_n^2 values in Green Bank, especially near the 140-ft itself, so that the estimates made above can be refined. The series of tests proposed for the lasers will give these data, particularly if one of the ranging instruments is mounted on the deck or lower structure of the 140-ft.

Should the tests show that the refractive index structure function for the Green Bank site has very large values, the ranging system may be forced to use two-color lasers. These devices use the fact that the refractive index is slightly different at different optical wavelengths to reduce the error that arises in the fluctuations of refractive index along the propagation path. It appears that over distances as short as 200 m an improvement of a factor of ten can be achieved (Gardner 1976).

Increased accuracy can not be readily achieved merely by increased integration. The studies of Matsumoto and Tsukahara show that the variance of the phase error decreases only as the -1/3 power of time, for averaging times less than about one minute.

I wish to thank L. D'Addario for helpful comments on an earlier version of this note.

Figure Captions

Figure 1. Altitude dependence of the refractive index structure function measured for midday and clear weather conditions. (from Walters and Kunkel Fig. 1)

Figure 2. Seasonally averaged values of the refractive index structure function measured at a distance of 8 m above a mountain surface. (from Walters and Kunkel Fig. 7)

Figure 3. Seasonally averaged values of the refractive index structure function measured at a distance of 9 m above a desert surface. (from Walters and Kunkel Fig. 5)

Figure 4. As in Figure 3, except the measurements were made at a distance of 33 m above the desert. (from Walters and Kunkel Fig. 6)

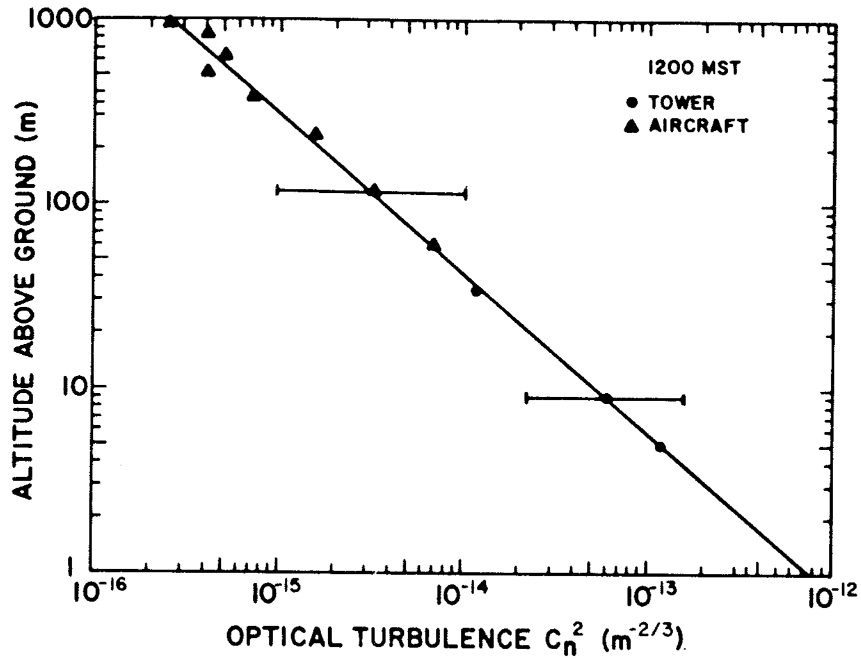

Figure 1

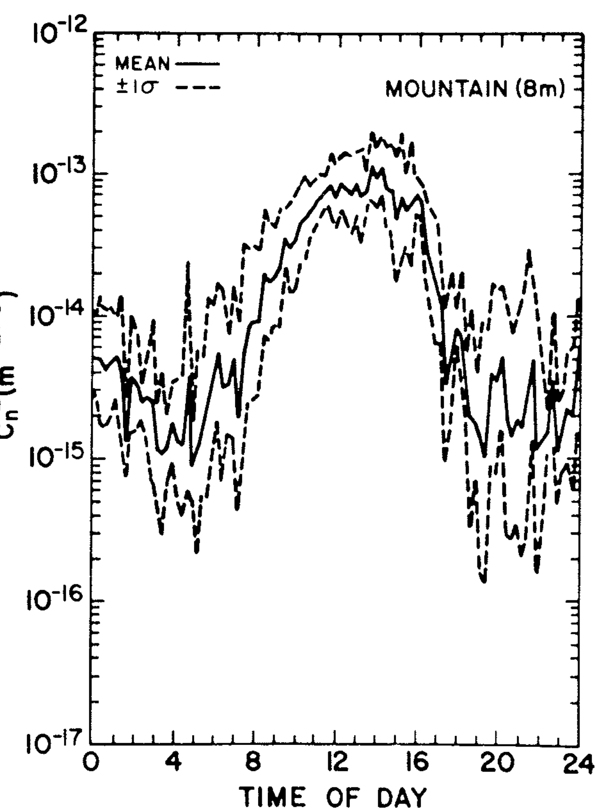

Figure 2

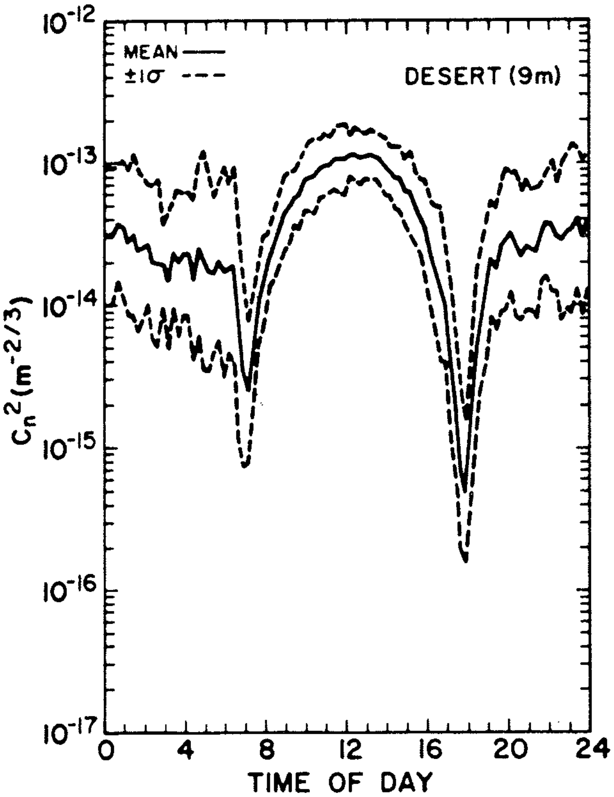

Figure 3

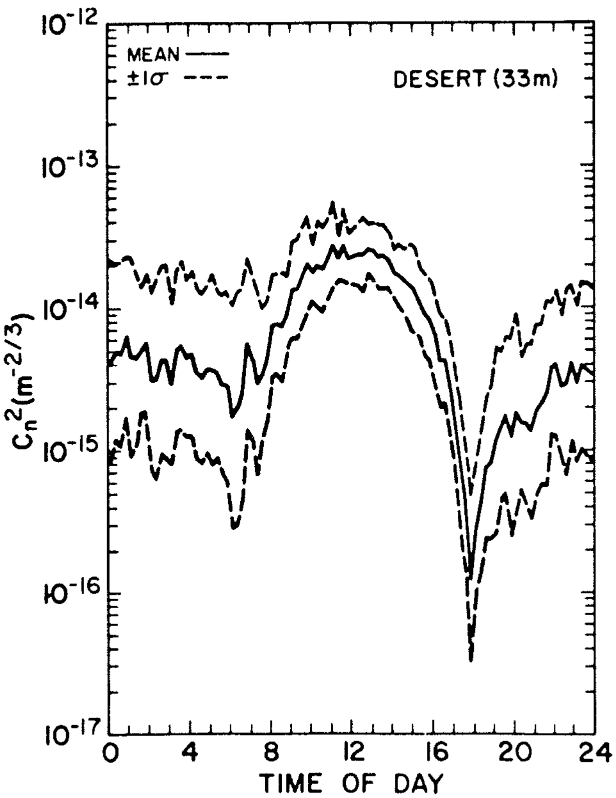

Figure 4