HYSPEC MEMO NO. 2

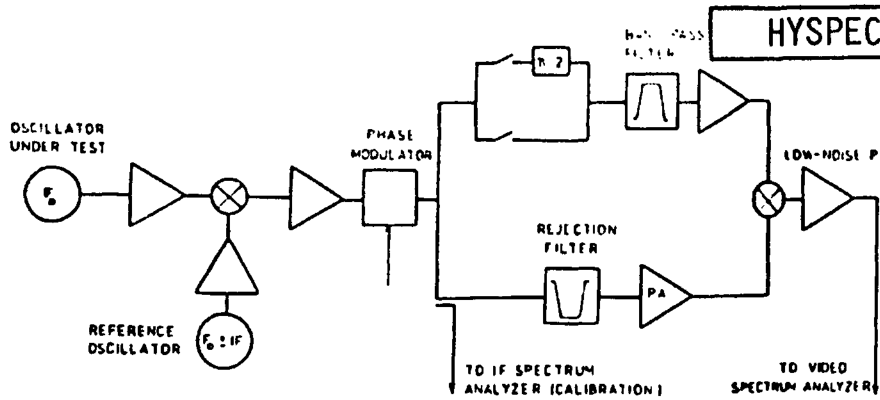

Fig. 8. Measurement system for high spectral purity SAW oscillators.

new system enables the measurement of the phase or amplitude noise density spectrum for offset frequencies from 2 kHz to 2 MHz from the carrier. The ground noise floor was measured to be −180 dBc/Hz for an input power of +13 dBm.

V. Conclusion

This study showed that the partial rejection of the carrier with a stopband filter is a way to artificially increase the phase and amplitude noises of a signal to levels which can be more easily measured. But this filtering operation modifies the spectra and induces a cross spectrum between phase and amplitude noises. It was demonstrated that the cross spectrum effect does not affect the phase and amplitude noise spectra for Fourier frequencies higher than 2 kHz, i.e., located outside the stopband of the filter. In this case only a weighting factor due to the filter is to be taken into account for recalculating the original spectra (before filtering) from the spectra which were measured after the filter.

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Analog-Filter Digital-Correlator Hybrid Spectrometer

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Abstract—A system is described for measuring the power spectrum and cross spectral density of wide-band (≥ 50 MHz) noise-like signals which occur, for example, in radio astronomy. The system utilizes a comb-filter bank followed by digital correlator processing of each filter output. The cost equation, design factors, and a sample system are described. For measurement of the spectrum at a large number of frequencies, the system cost of the hybrid system is shown to be much lower than the cost of spectrometers which utilize either a filter-bank or digital correlator alone.

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I. Introduction

It HAS been obvious for many years to designers of digital correlators that prefiltering by J filters per signal reduces the digital operations required per second by a factor of J. For spectral analysis of a single signal at bandwidths < 50 MHz and number of points < 5000, this step is unnecessary; logic is inexpensive and the accuracy of analog operations is a concern. However, for cross spectral analysis of several signals at very wide bandwidths, the cost of an all-digital machine can become prohibitive. This is the case for a contemplated millimeter-wave radio astronomy array with ~40 signals at GHz bandwidths.

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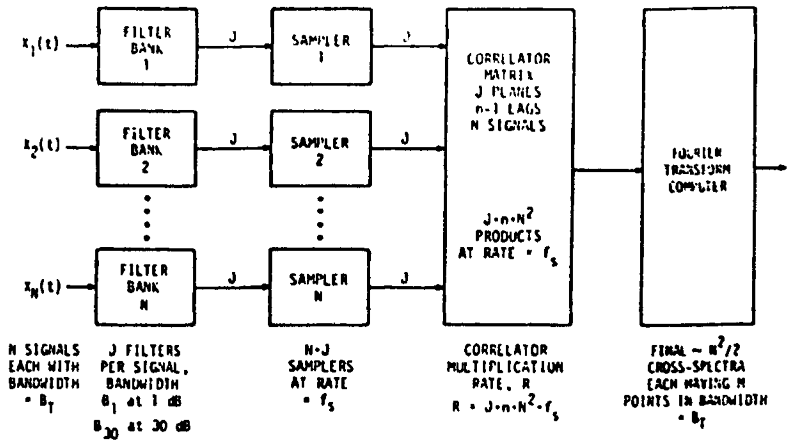

Fig. 1. Block diagram of hybrid filter-bank digital cross correlator spectrometer for processing an array of N signals.

For single-signal spectral processing, where a squaring operation rather than a multiplying operation is required, acoustooptical spectrographs [1] are very practical even for wide bandwidths. The system described in this report has both the advantage of ease of use in an array and flexibility of frequency scaling. Bandwidths narrower than the maximum bandwidth can be analyzed with increased resolution by a simple reassignment of the correlator channels in the system. A variation of the hybrid system described here is an analog filter bank followed by a digital, fast Fourier transform calculator and a cross multiplier; this has been described by Chikada *et al.* [2].

The purpose of this paper is not to suggest a new idea but to develop it. A cost equation for a hybrid system will be found; this leads to an optimum number of filters and minimum system cost. Special requirements on filter center frequency and shape factor imposed by the hybrid system will be discussed along with methods for realizing the filter-bank down-conversion system. Desirable system capabilities such as continuity of frequency points, filter overlap, and frequency diversity will be presented. Finally, a sample design suitable for millimeter-wave radio astronomy on a single telescope will be described.

II. Basic Relations and Cost Equation

A block diagram of a hybrid filter correlator system is shown in Fig. 1. The purpose of the system is to compute the cross-power spectra of N input signals, each having a bandwidth B_T. There will be $N(N+1)/2$ such cross-power spectra including the self-power spectra of the input signals. The desired frequency resolution b of the spectral measurement will be defined so adjacent windows cross at $2/\pi$ (~ 2 dB) points. If b is also the spacing between window centers, then the total number of independent points (this dictated the $2/\pi$ choice) in each spectra is $M = B_T/b$.

Each signal is first passed through a filter bank with J filters with 1-dB bandwidth of B_1 and also spacing between center frequencies of B_1, so that $B_1 = B_T/J$. The filter 30-dB bandwidth will be designated $B_{30} = \beta B_1$, where β is a filter shape factor. The selection of crossover at the 1-dB bandwidth is somewhat arbitrary, but it is approximately the bandwidth where the spectra can be determined without additional loss in statistical uncertainty due to coarse quantization (see Weinreb [3], p. 70). The 30-dB bandwidth determines the required sampling rate; $f_s = 2B_{30}$ for ≤ 0.1 percent aliasing of an out-of-band signal. The number of autocorrelator lags m required to achieve a given resolution b is given by $m = f_s/2b = \beta M/J$. For cross correlators, 2 m lags are required for either positive and negative lags, or sin and cos components. For a specified m, a higher shape factor requires either more filters or more lags.

A key parameter for evaluating the cost or complexity of a correlator is the total number of multiplications per second, R. Each sample in each of N signals must be multiplied by m previous samples of the same signal, and 2 m samples of the $N-1$ other signals. There are N autocorrelators and $N(N-1)/2$ cross correlators giving a sum of $m \cdot N^2$ products per sample. Multiplying by J filters and the f_s sampling rate gives

$$R = N^2 \cdot m \cdot J \cdot f_s. \tag{1}$$

Substituting $m = \beta M/J$ and $f_s = 2\beta B_T/J$

$$R = 2 \cdot N^2 \cdot \beta^2 \cdot B_T \cdot M/J. \tag{2}$$

Thus R is proportional to the bandwidth analyzed, B_T; the number of frequency channels, M; and is inversely proportional to the number of filters, J.

It is important to note that the required multiplication rate can be achieved by any product of multiplier elements and multiplication rate per element f_m which need not be equal to f_s. If $f_m = K f_s$ where K is an integer, then a multiplier can be time-shared to act upon K data streams stored in memories. (This technique is sometimes called "recirculation.") On the other hand, if $f_s = K f_m$, then K multipliers can be time-multiplexed to act upon one data stream. The least costly correlator is one which utilizes multiplier elements having the lowest ratio of cost to speed taking into account the cost of buffers or multiplexers.

Designating this correlator cost ratio as E in k\$ per 10^9 multiplications/seconds and the filter cost factor as F in k\$

TABLE I
SINGLE SPECTROMETER COSTS (1984$)
($N = 1$, $B_T = 2$ GHz, $M = 2000$)

CASE	β	E Corr. Cost k$/GHz	F Filter Cost k$	J Filters	C Total Cost k$	B_1 Filter Width MHz	f_s Sampling Rate MHz	m Corr. Per Filter	Comment
A	2	0.5	0.6	1	16,000	2,000	8,000	4,000	No filter bank
B	2	0.5	0.6	2,000	1,600	1	-	-	No correlator
C	2	0.5	0.6	163	197	12.2	48.8	24.5	Min. cost, β = 2
D	1.33	0.5	0.6	109	131	18.3	48.9	24.5	Min. cost, β = 1.33

TABLE II
ARRAY SPECTROMETER COST (1984$)
(ALL MINIMUM COST, $\beta = 1.33$)

CASE	N Ants.	E Corr. Cost k$/GHz	F Filter Cost k$	J Filters Per Ant.	C Total Cost M$	B_T Total BW GHz	M Total Freq. Points	B_1 Filter Width MHz	Comment
A	40	0.5	0.6	344	16.5	1	1,000	2.9	R = 11.6 x 10^{12} mults/sec; R = 5.7 x 10^{15} mults/sec if J = 1
B	40	0.2	0.6	217	10.5	1	1,000	4.6	1990 projection of lower digital cost
C	50	0.2	0.6	122	7.4	0.5	500	4.1	More antennas, less BW

per unit including samplers and delay lines, the total system cost C, excluding development cost, is given by

$$C = 2 \cdot E \cdot N^2 \cdot \beta^2 \cdot B_T \cdot M/J + F \cdot N \cdot J. \quad (3)$$

The number of filters J_0, which minimizes the cost and the minimum cost C_0, is then

$$J_0 = \beta \sqrt{2NB_T \cdot M \cdot E/F} \quad (4)$$

$$C_0 = 2 \cdot F \cdot N \cdot J_0 = 2\beta N \sqrt{2NB_TMEF}. \quad (5)$$

Some typical values for costs of a single spectrometer and a 40- or 27-element array are shown in Tables I and II. A correlator cost factor $E = 0.5\text{k\$}/10^9$ multiplications/s is used for most of the computations. The Very Large Array (VLA) correlator [4] built in 1977 has 11 000 multipliers at 10^8 multiplications/s and has a total correlator cost of ~ $800k; thus $E = 800/(11\,000 \times 10^8) = 0.73\text{k\$}/10^9$ multiplications/s for this case. A filter cost factor F of 0.3k$ per filter has been used. The filter cost includes a phase-locked local oscillator for conversion to baseband and a sampler. Both E and F may be substantially reduced by clever design, use of gate arrays for correlation, and use of new IC's for LO synthesis.

III. FILTER AND FREQUENCY CONVERSION METHODS

A. Quadrature-Phase Sideband Selection

A common method for down conversion and filtering is shown in Fig. 2. The filters in this system are usually low-pass filters and the accepted input spectrum is then as shown at the top of Fig. 3. A gap where the two sidebands are not separated then exists in the converted spectrum near the local oscillator frequency due to the inability of the phase shift networks to maintain 90° phase shift close to zero frequency. The gap can be made smaller with more complex phase shift networks; a 10-pole network could give 30-dB sideband rejection over a 10^4 frequency range (i.e., 5 kHz to 50 MHz). Another remedy is to analyze the spectra in the gap region with alternate converters offset by $f_s/4$ in frequency; this forms a system with the filter shape factor β defined in Section II equal to 2.

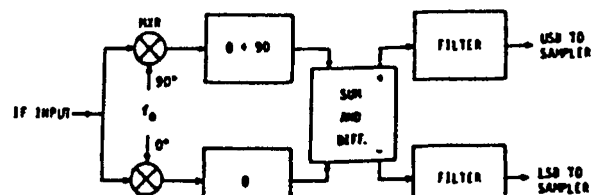

Fig. 2. Quadrature phase image-rejecting mixer system.

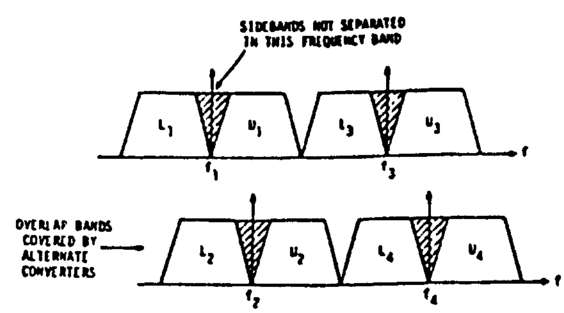

Fig. 3. Input spectra converted as upper sideband (U) or lower sideband (L) by system of Fig. 2. *Note:* The sideband rejection gap near the LO frequency can be made small with many-pole, phase shift networks, or left large but analyzed by an alternate frequency converter as illustrated in the lower half of the figure.

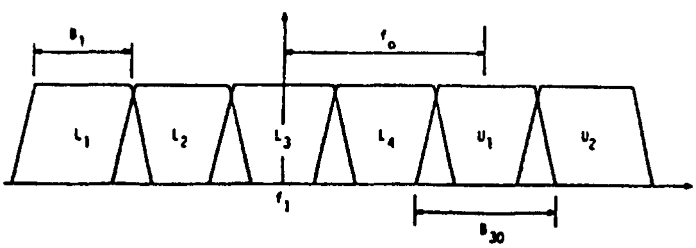

Fig. 4. Interlaced bandpass filters. *Note:* L_1 and U_1 are lower- and upper-sideband spectra accepted by one quadrature-phase mixer with LO frequency, f_1. Other mixers with LO frequencies spaced above f_1 in steps of B_1 fill in the gaps between lower and upper sidebands. By proper selection of the center frequency of each filter, the output may be sampled with $f_s = 2 \times B_{30}$ and aliasing error < 0.1 percent

The gap problem can be avoided with lower values of β by utilizing interlaced bandpass filters and bandpass sampling as shown in Fig. 4. To avoid aliasing the filter, 30 dB points must be selected to occur at $kf_s/2$ and $(k + 1) f_s/2$ where k is an integer. If p filter channels are selected to fall between upper and lower sidebands, the value of β is given by

$$\beta = \frac{p}{2k + 1} \geq 1. \tag{6}$$

Some possible values are given in the table below where values of B_{30} and filter center frequency f_0 for a $B_1 = 12$ are also given as follows:

k	p	β	B_{30}	f_0
0	2	2	24	12
1	4	1.33	16	24
1	5	1.67	20	30
2	6	1.20	14.4	36
2	7	1.40	16.8	42

The $\beta = 1.33$ solution appears to be a reasonable compromise. An 8-pole no-zero filter could achieve the required shape factor and a 2-pole, phase shift network could give 30-dB unwanted sideband rejection.

B. VHF Bandpass Filter-Mixer

The simplest approach to the filter subsystem is a bandpass filter followed by a mixer with local oscillator frequency on one 30-dB point of the filter response. Center frequencies of 100–200 MHz are appropriate for bandwidths of ~ 10 MHz. (Lower center frequencies make image rejection difficult in the preceding frequency conversion.) Mass-produced surface acoustic-wave (SAW) filter banks are available from one manufacturer (Sawtek) at a unit filter cost of ~\$50 for filters of 12.5 MHz, 3-dB bandwidth and 15 MHz, and 20-dB bandwidth. However, this method appears less desirable than the image-rejecting mixers discussed previously for the following reasons:

1) the filters are at different center frequencies and hence require many different designs;

2) the nonrecurring cost is high;

3) the stability of high Q-bandpass filters is poor. In addition, SAW filters have large time delay (1–5 μs) which is temperature dependent for low-loss (~ 25 dB) materials. The phase stability in a cross correlation system would be poor.

IV. Desirable Design Factors

A. Continuity of Frequency Point Spacing

It is obviously desirable to have the frequency spacing b of measured points on the power spectrum continuous in going from one filter to the next. This requires that $b = B_1/m'$ where m' is an integer. It is also desirable that the filter window function have nulls at dc and the sampling frequency f_s to prevent sampler imperfections from causing ripples in the spectrum; this forces $b = B_{30}/m'$ or $\beta = m/m'$. In addition, for the quadrature image rejection method, β must satisfy (6) $\beta = p/(2k + 1)$, p and k integer, to avoid aliasing in the sampled bandpass function. Fortunately, all of these constraints can be satisfied; for example, $m = 16$, $m' = 12$, $p = 4$, $k = 1$.

B. Filter Overlap

The bandpass of adjacent filters in the system will overlap to some degree. For example, filters with response crossing at 1-dB points will have an overlap region of perhaps 0.05 B_1 between 3-dB points where the spectrum is measured through two different paths. The two spectra, after correction for bandpass shape, should agree at these points; to first order even the noise fluctuations on the measured points should be the same. For coarse quantization the noise will not be exactly identical because some of the noise, particularly at band edge, is due to noise from other frequencies within the filter passband and this will be uncorrelated between the two filters.

Thus a comparison of spectral points in the overlap region is a sensitive indicator of filter instability or a gross failure in a portion of the system. The overlap region can be chosen to be close to 100 pecent by making the factor $\beta = 2$. However, this is costly and a compromise value of $\beta = 1.33$ with overlap checks between 3 dB points appears to be prudent.

C. Channel Diversity

In some applications it would be advantageous to have the capability of rapidly reassigning the frequency of filters in the system. This could be accomplished by using an image-rejecting mixer system with programmable synthesizer local oscillators. The power at one frequency would first be measured through one filter and subsequent correlator for 250 ms (for example) and through another in the next 250 ms. After 1000 ms the four measurements would be compared and any measurement differing by a specified threshold would be discarded from the average. The rms deviation of the quartet would be available to the observer and time variations due to equipment malfunction, antenna pointing, or the signal source could be detected. A failure of one filter would then not cause a portion of the spectrum to be missed.

This diversity would complicate the processing of data and is probably not worthwhile for interferometer or array

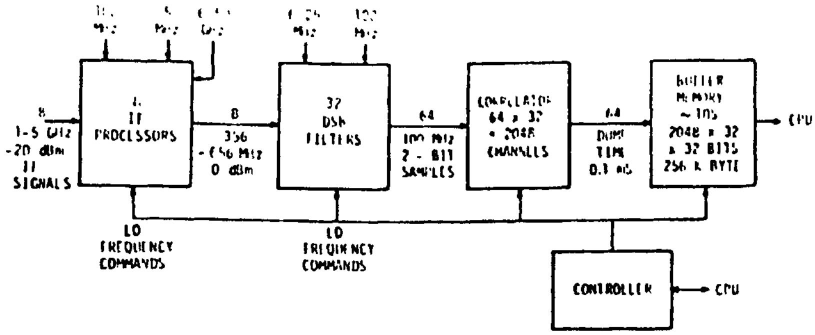

Fig. 5. Overall block diagram of the prototype system. *Note:* Eight I.F. processors shown in Fig. 6 allow input of up to 8 I.F. signals, each with 300-MHz bandwidth in the 1-5 GHz range. Each I.F. processor produces a 356-656 MHz output which drives 10 dual-sideband filter systems shown in Fig. 7. Each filter produces 3-level samples at a 100-MHz rate. The samples are then processed by a correlator operating at a 100-MHz clock rate. The correlator would be organized as 64 32-lag modules. The output accumulator-buffer memory has 32 bins for each of 2048 effective channels to allow for frequency diversity during an integration cycle.

use. However, it may be useful for single signal observations and would have little impact upon the cost. It would require programmability on a moderate speed basis of the second and probably third local oscillators in the system; these oscillators would be synthesized in any case to assure frequency stability.

V. A Sample System

A hybrid spectrometer suitable for use on the NRAO 12-m radio telescope on Kitt Peak, Arizona, will be described. As a starting point, we will assume approximately 2000 points at 2-GHz total bandwidth are desired with the option of dividing this total band among 2 (polarizations), 4, or 8 signals (multiple beams). In order to satisfy integer relations and for ease of synthesis of local oscillators, looking ahead, we have found that $M = 1536$ points and $B_T = 2.40$ GHz are convenient. The system allows B_T to be reduced, keeping the total number of channels constant, with the constraint that B_T is an integer times 37.5 MHz as shown in Table III.

The system of internal parameters will be selected to be close to the minimum cost relations of Table I as follows:

Number of Filters J		$= 64$
Filter Spacing and 1-dB Bandwidth	B_1	$= 37.5$ MHz
Filter 30-dB Bandwidth B_{30}		$= 50$ MHz
Sampling Rate f_s		$= 100$ MHz
Overlap Factor β		$= \frac{4}{3}$
Correlator Channels per Filter m		$= 32$.

A description of the system is given in Figs. 5–7. The correlator requires 2048 channels operating at a 100-MHz clock rate. This can be accomplished with 1024 VLA-1 chips and 2048 VLA-2 custom ECL IC's [5] which are

TABLE III
PROTOTYPE SYSTEM FREQUENCY RESOLUTION AND TOTAL BANDWIDTH

Resolution	1 Signal	2 Signals	4 Signals	8 Signals
b KHz	B_T MHz	B_T MHz	B_T MHz	B_T MHz
1,000	1,920	960	480	240
500	960	480	240	120
250	480	240	120	60
125	240	120	60	24
62	120	60	24	12
31	60	24	12	NA
12	24	12	NA	NA

TABLE IV
SPECTROMETER COST

8 - I.F. Processors, $3.5k each	$ 28k
80 - Dual SB I.F. Filters, $0.6K each	48k
Buffer Memories	5k
Correlator cards, sockets	10k
Cabinets, bins, power supplies	10k
Controller	10k
Cables, Miscellaneous	5k
Total	$116k

now on hand at NRAO. A cost estimate, excluding these IC's, is given in Table IV.

If VLA custom chips were not available for use in this machine, semi-custom gate-array IC's are an attractive alternative. A single Motorola MCA1200 ECL Macrocell, priced at $45 at a quantity of 1000, could contain the high-speed processing for 12 lags (i.e., dual-shift register, 3-level multipliers, and 4-bit counter) at a clock rate which may be as high as 160 MHz; the development charge is ~$23k. Other lower cost CMOS gate arrays could be used for low-speed accumulation.

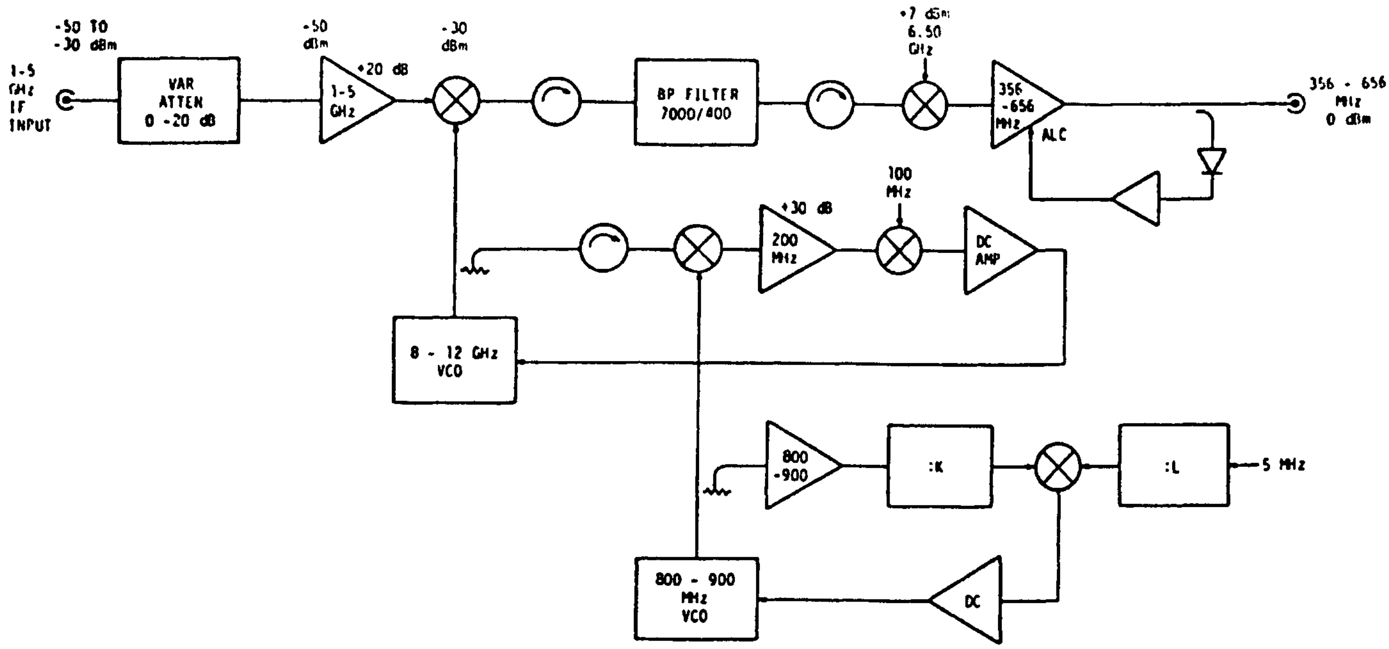

Fig. 6. Block diagram of 1 of 8 I.F. processors. *Note:* The unit allows an I.F. input with 300-MHz bandwidth anywhere in the 1-5 GHz range to be translated to the spectrometer input. The input is first up-converted to a 7-GHz center frequency by heterodyning with a synthesized 8-12 GHz local oscillator programmable in 1-MHz steps. At 7 GHz the signal is filtered and down-converted to a 506-MHz center frequency appropriate for input to the filter system.

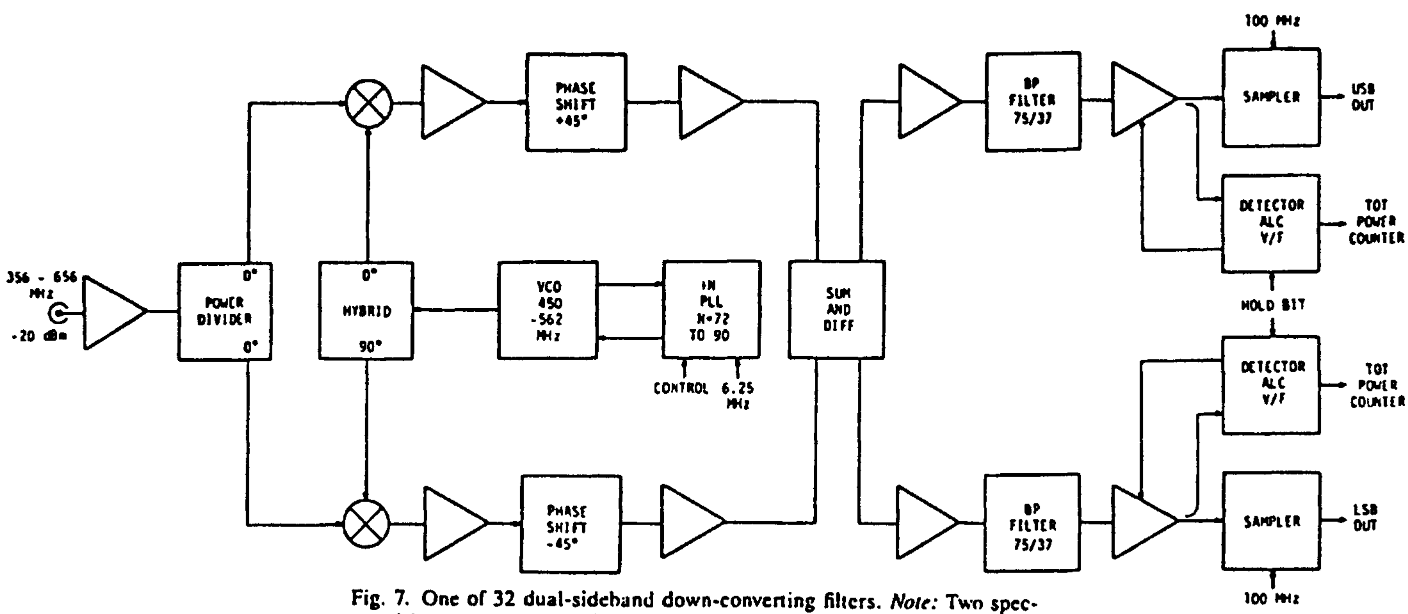

Fig. 7. One of 32 dual-sideband down-converting filters. *Note:* Two spectral bands centered at $f_0 \pm 75$ MHz with $f_0 = 450$–562 MHz in 37.5-MHz steps are down-converted to separate baseband signals by a quadrature-phase image-rejecting mixer method. The signals are then passed through filters with 75-MHz center frequency, 37.5-MHz 1-dB bandwidth, and 50-MHz 30-dB bandwidth. The amplifiers in the system are primarily for isolation purposes and are inexpensive IC units.

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