

# Next Generation Very Large Array Memo No. 1

## Fast switching phase calibration at 3mm at the VLA site

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### Abstract

I consider requirements and conditions for fast switching phase calibration at the Very Large Array site in the context of 3mm observing with the future next generation VLA. I consider the effects of tropospheric phase errors on image dynamic range and image resolution. Using historical data from the VLA atmospheric phase monitor (Site Test Interferometer (STI)), I show that fast switching phase calibration at 3mm should be viable, day or night, for faint source studies (loss of resolution  $\leq 20\%$ ) for most of the year with a 30sec total calibration cycle, outside the monsoon months of August and September. Even a 60sec cycle is adequate at night most of the year. I review calibrator areal density at 3mm, based on numerous studies done for the MMA and ALMA. There should be a 25mJy calibrator within  $2^\circ$  in the vast majority (98%) of observations. This strength calibrator is adequate to ensure that the residual rms phase noise due to SNR on the phase calibrator is much less than that due to the troposphere, even for a 30sec cycle time with only 3sec on the calibrator each visit.

## 1 Introduction

The next generation Very Large Array entails an array on the high plains of western New Mexico, possibly extending out to northern Mexico, West Texas, and Arizona. The goal is an effective collecting area of 10 times the current Very Large Array at 40GHz, covering frequencies from about 1 GHz to 115GHz, with baselines of a few hundred kilometers, occupying, for the most part, the high desert plains of western New Mexico. This region is all above 2000m elevation, and shares the major southwest US weather pattern. Hence, we expect the statistics for weather behaviour at the VLA site will apply reasonably to most of the high plains. Further site testing in areas of West Texas or Northern Mexico would be required to extrapolate beyond the high plains of New Mexico.

We have decades of data on the quality of the VLA as a millimeter interferometric site, including extensive studies of opacity and phase stability. The VLA site was used for acceptance testing of the original ALMA antennas, including observations up to 230GHz, and the experience was that the VLA site, at 2124m elevation is a quality 90GHz site, comparable to the Plateau de Bure site in overall quality (Thomson, Moran, Swenson 2004 (TMS) table 13.3). We also have years of continuous data from the STI, with extensive analysis in the VLA Test, MMA, and LAMA, memo series by Holdaway, Butler, Owen, Foster, myself, and others.

In this memo, I consider the viability of fast switching phase calibration at 3mm at the VLA site, based on the historical data. Other methods of phase calibration are being considered, such as radiometric phase correction, and paired-antenna or a separate calibration array. These other methods need to be investigated.

## 2 Concepts in interferometric phase calibration

The model for interferometric phase fluctuations due to variations in the tropospheric water vapor is built on the concepts of Kolmogorov turbulence and the frozen screen approximation for the propagation of the turbulence over the array (Coulman 1991; Carilli & Holdaway 1999; TMS 13.1). The frozen screen approximation assumes that the turbulence is convected over the array much faster than it dissipates or changes character. This model has been shown to apply most of the time at the VLA site, except in instances of very localized weather patterns, such as thunderstorms.

In this model, the rms phase fluctuations due to the troposphere on a baseline of length  $b$  obey a power law relationship of the form,  $\phi_{rms} \propto b^\beta$ , with three regimes: (i)  $\beta = 5/6$  on baselines shorter than the width of the turbulent layer (3D turbulence), (ii)  $\beta = 1/3$  on baselines longer than the width of the turbulent layer (2D turbulence), (iii)  $\beta \sim 0$  beyond the outer scale of the turbulence. Detailed studies at the VLA have verified this basic theory, with the outer scale typically being about 6 km (Figure 1), and the turbulent layer being one to a few km.

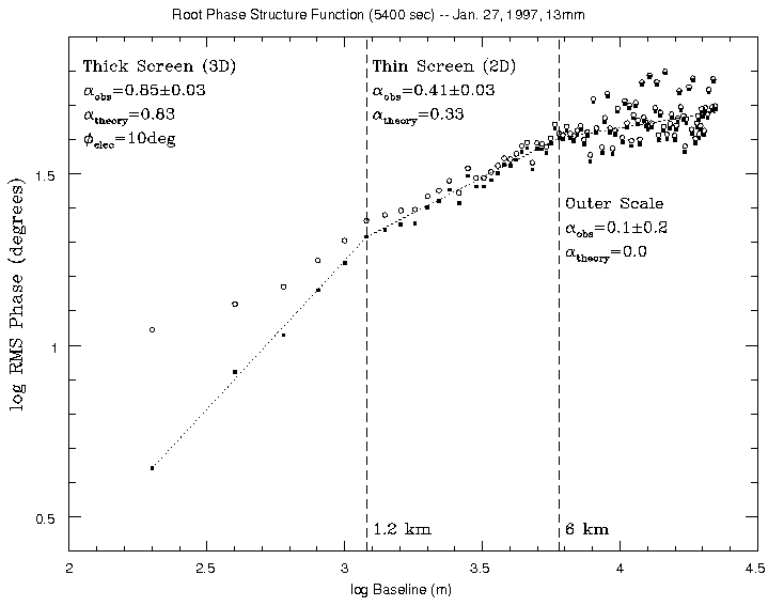


Figure 1: *The root phase structure function from observations at 22 GHz in the BnA configuration of the VLA (Carilli & Holdaway 1999). The open circles show the rms phase variations vs. baseline length as measured on the 1 Jy celestial calibrator 0748+240 over a period of 90 minutes. The filled squares show these same values with a constant noise term of  $10^\circ$  subtracted in quadrature. The three regimes of the root phase structure function as predicted by Kolomogorov turbulence theory are indicated.*

An important implication of the frozen screen approximation is that baseline length and time can be related through the characteristic velocity of the turbulent boundary layer,  $v_a$ , ie. the 'winds aloft'. The fundamental principle behind fast switching is that by cycling between a nearby phase calibrator and the target source, tropospheric phase fluctuations are 'stopped' at an effective baseline length,  $b_{eff} \sim v_a \times (t_{cyc}/2)$ , where  $t_{cyc}$  is the full cal-src-cal cycle time.

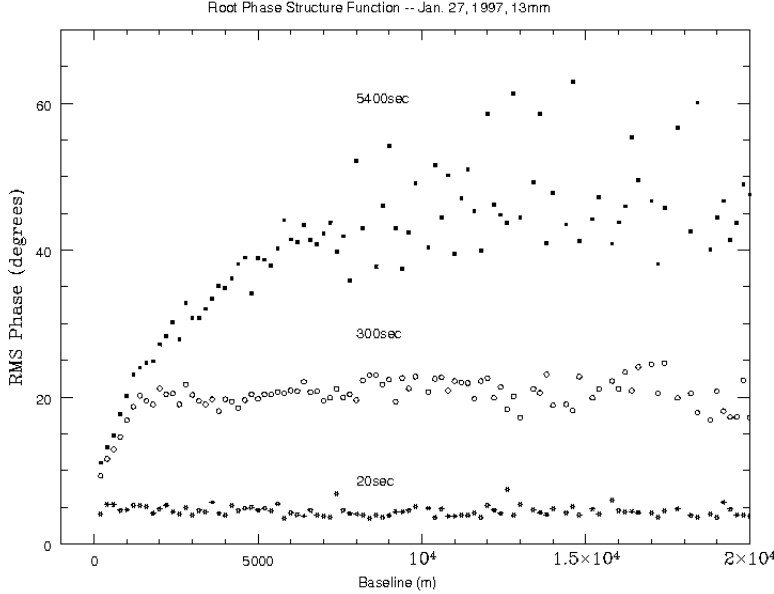


Figure 2: *The solid squares are the same as for Figure 6, but now on a linear scale (Carilli & Holdaway 1999). The open circles show the residual rms phase variations vs. baseline length after self-calibrating the data with an averaging time of 300 seconds. The stars show the residual rms phase variations vs. baseline length after calibrating with a cycle time of 20 seconds.*

Application of fast switching phase calibration is shown in Figure 2. It can be seen that application of fast switching phase calibration does indeed stop the troposphere, such that the residual tropospheric phase noise is set by the effective baseline length.

A second important 'effective baseline' is set by:  $b_h \sim \theta_{cal} \times h_{trop}$ , where  $\theta_{cal}$  is the source-calibrator angular separation in radians, and  $h_{trop}$  is the scale height of the troposphere, typically 1 km to a few km (Butler 2002). For source-calibrator separations  $\sim 2^\circ$  (see below), this implies a minimum effective baseline relevant to calibration of about 100 meters. This minimum baseline due to source-calibrator separations sets a noise floor for tropospheric phase calibration that applies to both fast switching and paired-antenna calibration. For fast switching, the practical implication is that it does not help to have  $t_{cyc} < 2(h_{trop} \times \theta_{cal})/v_a$  sec. For the VLA site, this corresponds to  $\sim 10$ sec, for source-calibrator separations  $\sim 2^\circ$ .

### 3 Effects of tropospheric phase noise

One effect of phase noise in interferometric imaging is dynamic range. For random phase errors on each baseline of order  $\phi_{rms}$  (in radians), the image dynamic range will be limited to:  $DNR \sim N/\phi_{rms}$ , where N is the number of antennas in an array (Perley 1999 equ. 13-6). Dynamic range is mostly relevant for bright sources, in which case, as long as a starting model can be synthesized, self-calibration can be used to fix the residual phase errors. For faint sources, dynamic range is obviously less of an issue. There will be an intermediate regime where dynamic range is relevant, but some increase in phase coherence will be required to allow for longer averaging time in self-

calibration.

The more important issue for faint sources is what might be called 'radio seeing', ie. source position wandering over time to the next due to the troposphere. For long record lengths, this can be calculated via visibility coherence vs. baseline length (Carilli & Holdaway 1999). When summing short record lengths, the dominant effect is the wandering of the source from record to record, ie. a radio 'speckle'. Very roughly speaking, the total flux density of the source might be preserved, but the source is smeared by the tropospheric-induced seeing.

TMS consider this effect in section 13.1 (eqs 13.85 - 13.93). They show that the increase in the FWHM of the source,  $\Delta\theta_t$ , due to tropospheric phase noise with rms  $\phi_{rms}$  (in degrees) on a baseline,  $b$ , obeys:

$$\Delta\theta_t \sim 2(\ln(2))^{1/2}(\phi_{rms}/360^\circ)(\lambda/b)$$

where  $\lambda$  is the observing wavelength. Considering an array like the VLA, the typical synthesized beam FWHM for robust weighting with  $R \sim 0.5$ , has a value  $\theta_{syn} \sim \lambda/b$  radians. Hence, tropospheric phase noise results in a fractional change in the resolution of the array of order:

$$\Delta\theta_t/\theta_{syn} \sim 2(\ln(2))^{1/2}(\phi_{rms}/360^\circ)$$

For example, if we want the resolution of the array to be degraded by no more than 10%, we would require residual tropospheric phase fluctuations after calibration to be  $\phi_{rms} \leq 22^\circ$ .

## 4 Site quality of the Very Large Array

The STI at the VLA site has been operating for close to 20 years (Carilli et al. 1998, VLA Test memo 213). I reanalyze the data presented in Butler & Desai (1999, VLA Test Memo. 222), in the context of 3mm observing at the VLA. The STI is a single baseline interferometer of 300m length observing a geostationary satellite beacon at 11.3GHz.

Note that tropospheric water vapor is non-dispersive, ie. changes in the index of refraction are frequency independent. The implied phase errors then increase linearly with frequency, and STI results at 11.3GHz can be extrapolated easily to other frequencies.

Butler & Desai show that the predicted rms phase fluctuations after calibration on the longer baselines can be calculated from:

$$\phi_{rms} = (1.9/\lambda_{cm})(t_{cyc}/2t_{corn})^\beta 2^{1/2} \sigma_{tropo}^{STI}$$

where  $\sigma_{tropo}^{STI}$  is the rms of phase fluctuations over long timescales (root(2) smaller than the saturation value of the phase structure function measured over long delays),  $t_{corn}$  is the 'corner time', which relates to the STI baseline length and the velocity of the troposphere over the array as  $\sim 300m/v_a$ . The parameter  $\beta$  is the measured powerlaw slope of the phase structure function vs. time up to the corner time. There is also a root dependence on air mass, which I ignore herein.

Butler & Desai have tabulated one year's worth of average values day and night for the STI for each month. I have used these values to extrapolate to residual rms phase fluctuations at 3mm after calibration. Note that both  $\beta$  and  $t_{corn}$  have large fitting errors. Moreover, the values at  $\beta$ ,  $t_{corn}$ , and  $\sigma_{tropo}^{STI}$  for 10% and 50% conditions may not correspond to each other. Hence, I adopt the values for  $\beta$  and  $t_{corn}$  obtained for the 50% measurement in all months.

I calculate the expected  $\phi_{rms}$  after fast switching phase calibration assuming a 30sec and 60sec cycle time, and under 10% and 50% conditions. Figure 3 and 4 shows the results as a function of month and night/day. Clearly, the monsoon months, principally August and September, are somewhat problematic for high frequency observing. The results can be summarized as follows (ignoring the monsoon months):

- For 60sec total calibration cycle time, night time observing provides reasonable conditions at least 50% of the time ( $\phi_{rms} < 40^\circ$ ) at 3mm. Note that  $\phi_{rms} < 40^\circ$  would degrade the spatial resolution of the array by no more than 20%, according to the equations in TMS.
- For 30sec cycle time and 50% conditions, reasonable calibration results ( $\phi_{rms} < 40^\circ$ ) will be obtained day or night, over most of the year. If we demand  $\phi_{rms} < 20^\circ$ , or resolution loss  $< 10\%$ , night time is still adequate most of the year.

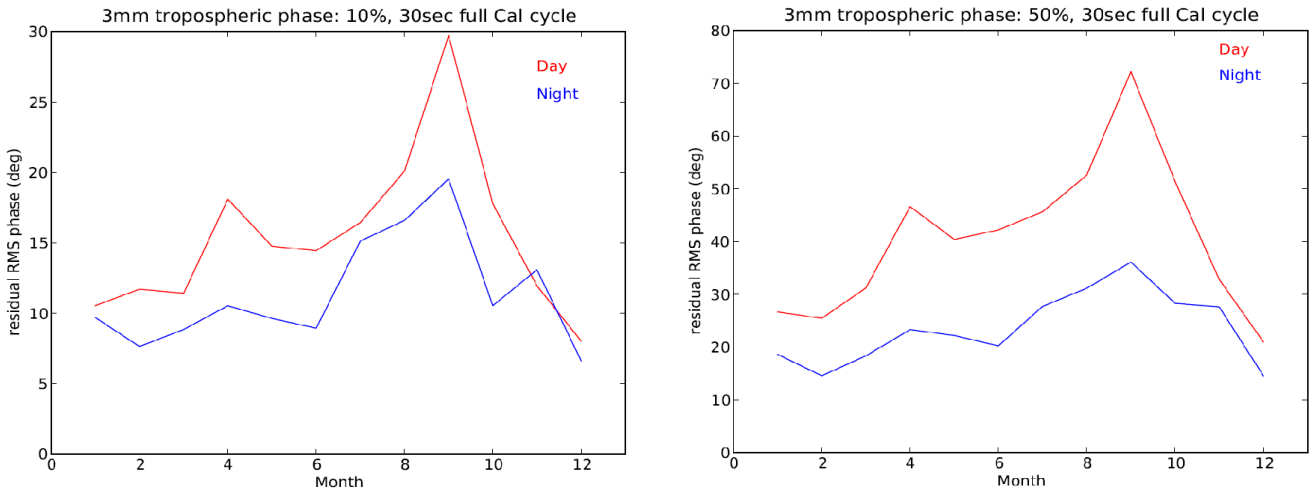


Figure 3: *Predicted rms phase noise after fast switching phase calibration as a function of month and day/night (red/blue; derived from STI data presented in Butler & Desai 1999). A 30sec total calibration cycle was used, and 10% and 50% conditions are plotted.*

## 5 Number density of phase calibrators

There have been many studies of the number density of 3mm phase calibrators, with the results published (mostly) in the MMA memo series by Holdaway, Owen, Foster, Fomalont, Rupen...). I review the results from Foster (1994), although all the studies agree within a factor two or so.

As a rule of thumb, we expect about 20,000 sources over the whole sky stronger than 25mJy at 3mm. Foster performed a monte carlo simulation, asking what fraction of the time one expects a calibrator of a given strength at 3mm within a given distance of a target source? He shows that 70% of the positions in the sky will have a 100mJy (or greater) calibrator within  $2^\circ$ . Lowering this to 25mJy at 3mm leads to 98% of positions having a calibrator within  $2^\circ$ .

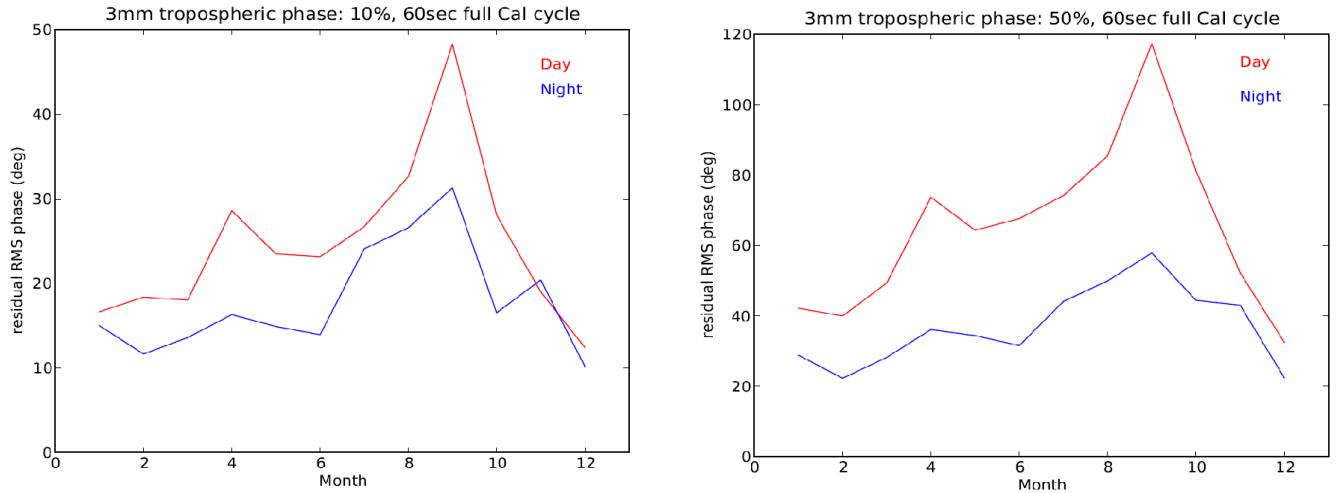


Figure 4: *Predicted rms phase noise after fast switching phase calibration as a function of month and day/night (red/blue; derived from STI data presented in Butler & Desai 1999). A 60sec total calibration cycle was considered, and 10% and 50% conditions are plotted.*

As an example, let us assume 3sec integration on the calibrator with an 8GHz bandwidth, 50% efficiency at 3mm, an 18m antenna with  $T_{sys} \sim 80K$ . This leads to an rms noise per baseline of 8mJy, or a signal-to-noise ratio (SNR) on a 25mJy calibrator:  $SNR \sim 3$ .

Walker and Wroble (1999) consider the noise distribution of phases on an interferometric baseline, given the SNR on the source. For  $SNR = 3$ , the FWHM of the distribution is  $90^\circ$ , or per baseline per calibrator scan:  $\sigma_{SNR} \sim 54^\circ$ . Assuming an antenna-based calibration of  $N$  antennas and  $N^2$  visibilities, with phase noise  $\sigma_{SNR}$  per visibility, the noise on the final antenna-based phase calibration solutions reduces by a factor  $\sqrt{N}$  (I think). For about 200 antennas, the expected rms phase noise after calibration due to SNR on a 25mJy calibrator is then  $\sim 4^\circ$ . This is much less than the residual phase rms due to the troposphere after fast switching, as per Figures 3 and 4, and hence is not a limiting factor (there may be an additional factor  $\sqrt{2}$  as well, but that does not change the conclusion).

## 6 Conclusions

- Fast switching phase calibration at 3mm should be viable, day or night, for faint source studies (loss of resolution  $\leq 20\%$ ) for most of the year with a 30sec total calibration cycle. Even a 60sec cycle is adequate at night most of the year.
- The number density of calibrators at 3mm implies that almost every position in the sky will have a phase calibrator within  $2^\circ$  brighter than 25mJy. This flux density is adequate to ensure that phase noise due to SNR on the calibrator is not a limiting factor for a 3sec integration on the calibrator.

Butler & Desai 1999, VLA Test Memo. 222

Butler 2002, VLA Test Memo. 232

Carilli & Holdaway 1999, *Radio Science*, 34, 817  
Carilli, Holdaway & Roy 1998, VLA Test Memo 213  
Coulman 1991, *A& A*, 251, 743  
Foster 1994, MMA memo 124  
Perley 1999, in *Synthesis Imaging in Radio Astronomy II* (ASP Conf. Series 180)  
Thomson, Moran, Swenson 2004, *Interferometry and Synthesis in Radio Astronomy* (Wiley)  
Wrobel & Walker 1999, in *Synthesis Imaging in Radio Astronomy II* (ASP Conf. Series 180)