Next Generation VLA Memo 45

Polarization Calibration

With Linearly Polarized Feeds

I. Introduction – Calibrating Circular Polarizations

Polarization is an important diagnostic for the origin of the radio radiation from celestial sources. This is especially true of linear polarization which occurs in many classes of radio sources. Circular polarization is relatively rare, and occurs in detectable amounts in a small percentage of radio souces.

Historically, therefore, the earliest interest was in measuring linear polarization. In the early days of radioastronomy, receiver gains were not very stable, so that measuring polarization by differencing the power from orthogonal linear feeds was a fussy and failure prone procedure. If, on the other hand, the two radio receivers were oppositely circularly polarized, the polarized signal appears as the correlation between the two receivers, with the relatively rare circular polarization being determined by differencing the power received in the two receivers. This was therefore an attractive way to build radio telescopes, especially radio interferometers.

No feed is going to be perfect. Therefore there will need to be procedures to determine, and correct for, the polarization properties of the system. I will describe one such procedure for a radio interferometer with circularly polarized feeds to illustrate how this calibration may be done.

An interferometer with two, oppositely polarized, receivers, will produce two IF signals.. Let us label them L and R. Then the backend of the receiver consists of devices that produce the correlations of these IFs. All four correlations are produced: LL, RR, LR, RL. For an interferometer array, observing a point source of unit strength, we may write these correlations in terms of a complex gain for each receiver involved, and a leakage from one polarization into another.

$$C^{LL}_{i,j} = G^{L}_{i} * G^{L*}_{j}$$

$$C^{RR}_{i,j} = G^{R}_{i} * G^{R*}_{j}$$

$$C^{LR}_{i,j} = G^{L}_{i} * D^{R*}_{j} + D^{L}_{i} * G^{R*}_{j} + G^{L}_{i} * G^{R*}_{j} * P * P^{*}$$

$$C^{RL}_{i,j} = G^{R}_{i} * D^{L*}_{j} + D^{R}_{i} * G^{L*}_{j} + G^{R}_{i} * G^{L*}_{j} * P * P^{*}$$

$C^{LL}_{i,j}$ is the correlation between the L receiver of antenna i and the L receiver of antenna j, etc. G^{L}_{i} is the gain of the L receiver of antenna i, etc. D^{L}_{i} is the leakage into the L receiver from the right in antenna i, etc. The superscript asterisk indicates complex conjugation. $P * P^{*}$ is the source polarization power. Quantities of second order have been dropped, and circular polarization is ignored.

Calibrating the polarization properties of the system consists of determining the two G values and the two D values for each antenna. That is, there are four (complex) quantities for each antenna to be solved for. Since each baseline makes four (complex) measurements, and there are $n*(n-1)/2$ baselines, four antennas have six baselines giving 24 complex measurements, and there are 16 complex quantities to be determined, one should be able to solve the system. However, the system is singular. The most obvious singularity is that all instrumental quantities appear as the product of a quantity from the first antenna with the complex conjugate of a quantity from the second; therefore, adding a constant phase to all quantities leaves the measured values unchanged. This singularity is conventionally removed by designating a ‘reference antenna’, and setting its phase to zero.

One polarization calibration procedure (there are others) is to observe an unpolarized point source. The parallel hand (LL and RR) correlations above are separately solved for the G gains, usually with the same ‘reference antenna’ designated for the L and R gain solutions. Then the crossed hand correlations (LR and RL) are examined to determine the leakage (D) terms, using the known gains. Then a source of known polarization is observed. The polarization is derived from this observation using the known G and D terms. This will usually agree in magnitude with the known polarization, but will have an arbitrary angle of the E vector of the polarized radiation. Adding a constant phase to the G factors of one polarization will rotate the direction of the derived polarization, and the value which aligns the polarization with that known for the source completes the calibration. (Thus we see that designating a ‘reference antenna’ for both polarizations is a temporary step of convenience; setting only one polarization to zero logically suffices.)

There is a more subtle singularity in the linear equations above, in that polarization leakage is expressed with respect to the polarization desired, which we have taken to be the circular polarizations. But the feeds do have actually a small degree of ellipticity, and in the practical solution, the leakages are with respect to that reference. The solution of the equations is often abetted by, in this case also, designating a reference antenna, which is defined to have zero leakage. The full, non linear equations for the correlations, observed over a range of hour angles, removes this singularity, but designating a reference antenna seems, in practice, to produce leakages that suffice to make astronomically useful polarization images.

II. Polarization calibration with linearly polarized receivers

For linearly polarized receivers, it is generally difficult to calibrate system gain without measuring all four correlations, because most calibration sources have some degree of linear polarization. Although the observer may have no interest in linear polarization (often the case for thermal sources), the measurement and calibration of polarization is the most straightforward approach to an accurate flux calibration.

A very similar procedure to that above may be used to calibrate polarization for linearly polarized receivers. A source known to be unpolarized may be observed to derive the G gain factors of all receivers from the parallel hand correlations, and the D leakage terms from the crossed hand correlations. Then a source of known polarization may be used to derive the relative phase between the two orthogonal linear receiver system, which is needed to derive source polarizations. There is a weakness to this procedure, however. If the source polarization happens to align with the polarization of one of the feeds, it conveys no information about the relative phases of the receivers for the two polarizations.

There are thus several impediments to using this procedure for the ngVLA. First, there is the problem of finding a source known to be unpolarized. Due to the large range of baselines of the ngVLA, one is compelled to use calibrators with very small structures to provide calibration power for the distant antennas. Such sources tend to be relatively strongly polarized, and variable in polarization. Similarly, sources with strong polarization to use for the relative phasing of the two polarizations also will tend to be variable, so maintaining a set of suitable calibrators is a substantial effort. This means that the sources known to be unpolarized are likely to be far from sources with known polarization, so that visiting both is likely to be a substantial overhead on the scheduling block. Then, when scheduling the observation, care must be taken that the parallactic angle at the time of observation does not cause the calibrator's polarization to align with one of the dipole directions. All this tends to make for complicated and inefficient array usage.

It is possible to ameliorate these effects by exploiting an additional degree of freedom, by installing the linearly polarized receivers at different angles in different antennas. Let us designate the two polarizations as X and Y. Then we may write the equivalent of the equations of the first section. Again second and higher order terms are neglected, and again, the equations apply to a point source with unit flux.

$$C^{XX}_{i,j} = G^X_i * G^{X*}_j * \cos(\varphi_j - \varphi_i) - (G^X_i * D^{X*}_j + G^{X*}_j * D^X_i) * \cos(\varphi_j + \varphi_i) +$$
$$G^X_i * G^{X*}_j * p * \cos(\varphi_i - \psi) * \cos(\varphi_j - \psi)$$

$$C^{YY}_{i,j} = G^Y_i * G^{Y*}_j * \cos(\varphi_j - \varphi_i) + (G^Y_i * D^{Y*}_j + G^{Y*}_j * D^Y_i) * \cos(\varphi_j + \varphi_i) +$$
$$G^Y_i * G^{Y*}_j * p * \sin(\varphi_i - \psi) * \sin(\varphi_j - \psi)$$

$$C^{XY}_{i,j} = G^X_i * G^{Y*}_j * \sin(\varphi_j - \varphi_i) - (G^X_i * D^{Y*}_j + G^{Y*}_j * D^X_i) * \sin(\varphi_j - \varphi_i) +$$

$$G^X_i * G^{Y*}_j * p * \cos(\varphi_i - \psi) * \sin(\varphi_j - \psi)$$

$$C^{YX}_{i,j} = G^Y_i * G^{X*}_j * \sin(\varphi_j - \varphi_i) + (G^Y_i * D^{X*}_j + G^{X*}_j * D^Y_i) * \sin(\varphi_j - \varphi_i) -$$

$$G^Y_i * G^{X*}_j * p * \sin(\varphi_i - \psi) * \cos(\varphi_j - \psi)$$

Here φ_i is the angle of the receiver X polarization on the i^{th} antenna relative to some fiducial mark common to all antennas. Polarization is represented as p, the percentage linear polarization and ψ, the polarization angle relative to the same fiducial.

I have set up a simulation using these equations, and run it for ten antennas. It verifies that this single observation of a point source results in a satisfactory recovery of the gains, leakages and source polarization put into the simulation to calculate the correlations. Since the problem is non-linear, an iterative solution was used, which converged very quickly. For purposes of the simulation, I imposed the extra condition that the sum of all the leakage terms is zero. It is not clear that this condition is necessary, but it is at least not harmful.

For the simulation, I set the φ to be evenly distributed around a semicircle. I believe the procedure would work as well if half the antennas were set at zero degrees and half at 45 degrees.

Note that the method does depend on the assumption that the feeds are very close to real linear polarizations. For the most accurate work, a subsequent correction may need to be applied to allow for the fact that they will actually be slightly elliptically polarized, and may even be slightly non-orthogonal. This correction will probably be small enough that it can be ignored for most observations seeking only linear polarization.

III. Polarization within the beam

The optics specified for the antenna should result in a very well behaved instrumental polarization (Srikanth, GBT Memo 102 for example), with cross polarized sidelobes of order one percent. There will also probably be a pattern due to slight differences in the E- and H-plane beams of the particular feed. Having different orientations for the linearly polarized feeds will result in a cross polarized pattern differing with the orientations of the feeds in the antennas of the baseline. Spreading the angles of the feeds over a large arc will tend to cause the cross polarized sidelobes to cancel. The expectation would be that for the purpose of most astronomical observations, this cancellation might be good enough that no correction need be applied for polarization within the beam. On the other hand, if a correction is required, it would simplify the calculation and application of the correction if half the antennas were at zero degrees and half at 45 degrees. Then there are only three baseline dependent systematic polarization patterns to deal with.

IV. Correcting for leakage and conversion to Stokes Parameters

After the leakage terms and gains have been determined on a point calibrator, they can be applied to correct the correlations.

$$C^{XX'}_{ij} = C^{XX}_{ij} + (G^{X}_{i}*D^{X*}_{j} + G^{X*}_{j}*D^{X}_{i}) * C^{XY}_{ij}$$

$$C^{YY'}_{ij} = C^{YY}_{ij} + (G^{Y}_{i}*D^{Y*}_{j} + G^{Y*}_{j}*D^{Y}_{i}) * C^{YX}_{ij}$$

$$C^{XY'}_{ij} = C^{XY}_{ij} + (G^{X}_{i}*D^{Y*}_{j} + G^{X*}_{j}*D^{Y}_{i}) * C^{XX}_{ij}$$

$$C^{YX'}_{ij} = C^{YX}_{ij} + (G^{Y}_{i}*D^{Y*}_{j} + G^{X*}_{j}*D^{Y}_{i}) * C^{XY}_{ij}$$

These corrected correlations can then be converted to the usual Fourier transforms of the image stokes parameters.

$$C^{I}_{ij} = (C_{ij}^{XX'}/(G^{X}_{i}*G^{X*}_{j}) + C_{ij}^{YY'}/(G^{Y}_{i}*G^{Y*}_{j})) * \cos(\varphi_j - \varphi_i) + (C_{ij}^{XY'}/(G^{X}_{i}*G^{Y*}_{j}) + C_{ij}^{YX'}/(G^{Y}_{i}*G^{X*}_{j})) * \cos(\varphi_j - \varphi_i)$$

$$C^{q}_{ij} = C_{ij}^{XX'}/(G^{X}_{i}*G^{X*}_{j})*\cos(\varphi_i)*\cos(\varphi_j) + C_{ij}^{YY'}/(G^{Y}_{i}*G^{Y*}_{j}) *\sin(\varphi_j)*\sin(\varphi_i) + C_{ij}^{XY'}/(G^{X}_{i}*G^{Y*}_{j}) *\sin(\varphi_j)*\cos(\varphi_i) + C_{ij}^{YX'}/(G^{Y}_{i}*G^{X*}_{j})*\sin(\varphi_i)*\cos(\varphi_j)$$

$$C^{u}_{ij} = C_{ij}^{XX'}/(G^{X}_{i}*G^{X*}_{j})*\sin(\varphi_i)*\sin(\varphi_j) + C_{ij}^{YY'}/(G^{Y}_{i}*G^{Y*}_{j})*\cos(\varphi_j)*\cos(\varphi_i) + C_{ij}^{XY'}/(G^{X}_{i}*G^{Y*}_{j}) *\cos(\varphi_j)*\sin(\varphi_i) - C_{ij}^{YX'}/(G^{Y}_{i}*G^{X*}_{j})*\cos(\varphi_i)*\sin(\varphi_j)$$

V. Calibrating Circular Polarization

In the case of parallel linear polarized feeds, the calibration of circular polarization is primarily that of determining the relative phases of the two polarizations, with circular polarization, for a point source, appearing in phase quadrature to the I stokes parameter. With receiver angles depending on the antenna, as above, circular polarization depends on the relative phase and on the accurate subtraction of the amplitudes. For the extragalactic sources, levels of circular polarization tend to be small, so accurate calibration is necessary and special techniques will probably be needed. For the sun and for pulsars, high levels of circular polarization are observed, so the calibration above will suffice. For these cases, the formal solution may suffice:

$$C^{v}_{ij} = (C_{ij}^{XX'}/(G^{X}_{i}*G^{X*}_{j}) - C_{ij}^{YY'}/(G^{Y}_{i}*G^{Y*}_{j}) + i*C_{ij}^{XY'}/(G^{X}_{i}*G^{Y*}_{j}) - i*C_{ij}^{YX'}/(G^{Y}_{i}*G^{X*}_{j})) *\exp(i*(\varphi_i+\varphi_j))$$

VI. Conclusions

The ability to calibrate both gain and polarization on a single point source is a significant operational advantage, with a consequent saving in observational overhead. There seem to be no disadvantages to the method other than the possible mechanical complexities of having the various antennas differing in the orientations of the receivers.