1/14/89

NLSRT Memo No. 46

Beam Scan Properties of Nonparabolic Reflectors

P.J. NAPIER

National Radio Astronomy Observatory, Socorro, New Mexico 87801

Abstract. Nonparabolic reflector systems such as shaped dual reflector systems and spherical reflectors with Gregorian feed systems have poor beam-scan properties. The consequences of this need to be considered before selecting a shaped or spherical reflector system for future radio telescopes.

1. INTRODUCTION

Von Hoerner (1988) has pointed out the advantages of selecting shaped reflector pairs and spherical reflectors with a Gregorian feed system for future radio telescopes. One obvious disadvantage of a shaped reflector is the loss of prime focus feed performance, although this may be acceptable at longer wavelengths (James, 1983). It is the purpose of this memo to point out that such nonparabolic reflector systems also have the disadvantage of poor off axis performance when compared to a parabolic reflector system having the same effective F/D (focal ratio) at the feed. Poor off axis performance means that the beam peak gain drops and the sidelobe levels increase rapidly as the antenna beam is scanned off axis by moving the feed laterally away from the focal point in the focal plane. This disadvantage may outweigh the sensitivity advantage of the shaped system and the manufacturing advantage of the spherical reflector, so it needs to be given careful consideration before selecting a nonparabolic geometry for a future telescope. A less restrictive measure for obtaining improved sensitivity might be the use of a removable, illumination modifying lens in front of the feed (Hudson et.al., 1987). The poor off axis performance of nonparabolic reflector systems can be seen in previous reports (Hoppe and Stanton, 1987, Kildal, 1986). In this report we attempt to explain qualitativley why this seems to be a fundamental property of these reflector types.

Existing applications of shaped reflectors for radio telescopes (Cambridge 5km array, VLA and VLBA) have been for synthesis telescopes where it is not important for the antenna to have a field of view greater than a beamwidth. It is likely that future high frequency synthesis arrays, such as the NRAO Millimeter Array, will benefit from having a field of view of many beamwidths to simplify mosaic mapping of large objects and to help phase calibration. There are several reasons why a future, high frequency single aperture telescope needs to have good off axis performance:

(1) There is growing interest in equipping radio telescopes with focal plane arrays (Yngvesson,1985) to significantly increase their sensitivity and mapping speed. The goal, presumably, is eventually to do the same thing as is done on optical telescopes and obtain, in real time, an image of the whole field of view available in the focal plane. With continuing developments in integrated feeds and receivers it does not seem unreasonable to have a long term goal of imaging a region of the sky several tens

of beamwidths in diameter, at least at short wavelengths where feeds are physically small and many astronomical objects are large compared to the beamwidth.

(2) At short wavelengths fluctuations in atmospheric absorbtion significantly degrades the quality of radio images made with single dishes. This problem can be solved by beam switching between two or more feeds (Emerson et.al., 1979). It is important that the beams are well isolated from each other and have good efficiency. The useful beam throw is limited only by the requirement that the propagation paths of the two beams through the troposphere overlap significantly. (Emerson,private communication). For example, for a 100m dish at a wavelength of 1cm, it could be expected that beam separations of about 20 beamwidths would be useable.

(3) It should be possible to correct for aberrations in single dishes by collecting all the energy in the focal plane with an array and then correlating between array elements (Cornwell and Napier, 1988). This technique is simplest when the focal plane and aperture plane coherence functions are related by a Fourier transform, as is true for a parabola-hyperbola geometry with effective F/D at the feed greater than about 1. For a reflector geometry in which this Fourier transform relationship does not hold, which is equivalent to saying that the geometry has poor off-axis performance, the technique may still be applied but at the expense of increased computational complexity. The distance off axis that the focal plane array needs to extend is inversely proportional to the scale size of the aberrations in the aperture plane that have to be corrected. For example, for aberrations in the aperture plane of scale size one tenth of the aperture diameter, the focal plane array needs to extend out to at least ten beamwidths.

2. Off Axis Performance of Reflector Antennas

Figure 1 shows a general two reflector antenna system. The reflectors are profiled so that all rays from the secondary focal point have equal pathlength to the aperture plane. Hence, the on axis feed produces a constant aperture phase distribution. The effect of moving the feed a small distance laterally in the focal plane is equivalent to multiplying the radiation pattern of the on axis feed by a phase pattern $\delta_f(\theta)$ where

$$\delta_f(\theta) = \frac{2\pi}{\lambda}\Delta x sin(\theta)\ (radians) \tag{1}$$

where θ and Δx are defined in figure 1 and λ is the wavelength. For simplicity we will, for the moment, consider only the one dimensional case which is adequate for understanding the basic phenomenon. For any commonly used reflector system there is a unique relationship between θ, the feed pattern angle and r, the radius in the aperture to which a ray travels. Note that $\delta_f(\theta)$ is a linear function of $sin(\theta)$. Therefore, if r is a linear function of $sin(\theta)$, a lateral feed displacement will produce a linear phase gradient across the aperture which in turn causes only a change in pointing and no beam degradation. The more non-linear the relationship between $sin(\theta)$ and r, the more non-linear will be the aperture phase error, $\delta_a(r)$, that results from a lateral feed shift. The non-linear part of $\delta_a(r)$ causes loss of gain and increase in coma sidelobes (Ruze, 1965). The main point of this memorandum is that, of the reflector sytems commonly used for radio astronomy, only high F/D parabolic systems, such as reasonably high magnification Cassegrain and Gregorian systems, have linear $r(sin(\theta))$

relationships. Small F/D parabolas, shaped reflector systems and spherical reflectors with Gregorian feeds all have highly non-linear $r(sin(\theta))$ relationships and therefore poor off axis performance.

Examples of the relationship between $sin(\theta)$ and r for these different reflector systems are shown in figure 2 in which $sin(\theta)$ and r are normalized to 1.0 at the edge of the reflectors. The curves for the three parabolic systems $F/D = .26$, .35 and 2.2 are computed using the geometric relationship

$$sin(\theta) = \frac{\frac{r}{F}}{1 + (\frac{r}{2F})^2} \tag{2}$$

F is the effective focal length of the optical system. Note that, in (2), the relationship between $sin(\theta)$ and r becomes increasingly linear as F increases. $F/D = .35$ is the best fit parabola for the VLBA primary reflector and $F/D = 2.2$ is the parent Cassegrain geometry used to generate the VLBA shaped reflector pair. The curve for the VLBA and VLA shaped geometries results from the reflector shaping procedure (Galindo, 1964). For the VLBA the shaping transforms a feed pattern that is -14dB at the subreflector edge at $\pm 13.3°$ into an aperture amplitude distribution that is uniform out to 95% of the primary reflector radius then rolls off to -15dB at the edge. For the VLA the shaping transforms a feed pattern that is -11dB at the subreflector edge at $\pm 9.0°$ into a uniform aperture amplitude distribution. The curve for the spherical/Gregorian system corresponds exactly to the example given in Figure 7 of Von Hoerner (1987)($F/D = 0.55$, subreflector edge $\pm 7.5°$). It was computed using the formulas in Holt and Bouche (1964).

Several interesting features can be seen in Figure 2. Clearly the large F/D parabola/hyperbola Cassegrain geometry has the most linear $r(sin(\theta))$ relationship. The form of the curve for the shaped VLBA is to be expected because the parent parabola/hyperbola have been shaped so as to bundle rays more closely together on the outer part of the main reflector, thereby increasing the illumination. It seems inescapable that any shaped reflector pair that is generated by modifying a parent parabola/hyperbola pair so as to significantly alter the aperture illumination of the parent pair must have a significantly non-linear $r(sin(\theta))$ relationship. Note that for both the VLBA and spherical/Gregorian systems the non-linearity is so large that even for a beam scan as small as one beamwidth, when the phase gradient will have a value of approximately $\lambda/2$ at the edge of the main reflector, the non-linear part of $\delta_a(r)$ will be large enough to cause significant gain loss. To quantify this, detailed three-dimensional raytracing calculations for the VLBA shaped reflector system predict gain losses of -0.4dB, -1.7dB and -3.6dB for one, two and three beamwidths off axis respectively. For the same amount of gain loss Ruze (1965) predicts that these beam throws are only one seventieth of the scan available with a parabola/hyperbola having the same F/D (2.2) at the feed.

As an example of a deep parabola the case $F/D = .26$ is included in Figure 2. For this case the curves in Ruze (1965) predicts gain losses of -0.2dB, -0.9dB and -2.6dB for one, two and three beamwidths off axis respectively for the case of uniform illumination. This is comparable to the shaped VLBA losses which is to be expected because the amount of non-linearity evident in Figure 2 is similar for the two cases, with the VLBA being worse because its non-linearity is larger towards the edge of the reflector where there is more area. The opposite senses of the curvature of the curves for shaped and unshaped systems in Figure 2 is expected from their different illumination properties. If rays are traced from the feed to the aperture this curvature results in reduced ray density towards the edge of the reflector for the parabolic systems, causing the reduced edge illumination usually called "space attenuation". The opposite curvature for the shaped systems causes increased ray density at the reflector edge providing increased illumination. The increased edge illumination properties of the spherical/Gregorian system are well known (Holt and Bouche, 1964). The opposite curvature of the $r(sin(\theta))$ relationship for parabolic and non-parabolic reflectors means that, with an offset feed, the sign of the non-linear part of the aperture phase error will be opposite for the two reflector types. This has the interesting consequence that the increased coma sidelobes will be on different sides of the scanned beam in the two reflector types. In a parabolic system the coma sidelobes are on the antenna axis side of the scanned beam (Ruze, 1965), so for a shaped reflector antenna they will be on the side away from the axis. This has been verified by detailed raytracing calculations of the VLBA geometry. This means that if you wish to do beam switching between an on-axis and off-axis feed in a shaped reflector antenna, and you can stand the gain loss of the off-axis beam, then isolation between the two beams should not be a problem.

3. Need For Further Investigations

The discussion above has been mainly qualitative with a goal of explaining why the scan properties of parabolic and non-parabolic reflector systems are necessarily different. The investigation can easily be made quantitative by following the analysis used by Ruze (1965). Gain loss, coma sidelobe level and beamwidth as a function of scan can be produced for a range of shaping parameters. Since a high F/D parabolic system has a linear $r(sin(\theta))$ relationship, and since the illumination properties can only be altered by altering this relationship, it seems inescapable that a shaped Cassegrain will have a degraded beam scan capability. The question of whether there are other reflector pairs, not based on the parabola/hyperbola, that have both good scan and illumination performance is open, although it seems likely that if such a system exists it will have poor prime focus performance. The discussion on reflector systems with improved scan performance contained in Hansen (1964) may be a good starting point for this study.

REFERENCES

Cornwell, T.J. and Napier, P.J. (1988), *The focal plane coherence function of an imaging antenna and its use in measuring and correcting aberrations*, Radio Science **23**, 739-748.

Emerson, D.T., Klein, U. and Haslam, C.G.T. (1979), *A multiple beam technique for overcoming atmospheric limitations to single-dish observations of extended radio sources*, Astron. Astrophys. **76**, 92-105.

Galindo,V., (1964), *Design of Dual-Reflector Antennas with Arbitrary Phase and Amplitude Distributions*, IEEE Trans. Antennas Propagat. **AP-12**, 403-408.

Hansen, R.C., (1964), Microwave Scanning Antennas, Vol 1, Academic Press pub., 141-143..

Holt, F.S., and Bouche, E.L., (1964), *A Gregorian Corrector for Spherical Reflectors*, IEEE Trans. Antennas Propagat. **AP-12**, 44-47.

Hoppe, D. and Stanton, P. (1987), *Calculated 70-Meter Antenna Performance for Offset L-Band and C-Band Feeds*, JPL TDA Progress Report **42-90, April-June**, 12-20.

Hudson, J., Planbeck, R. and Welch, W.J. (1987), *Aperture Efficiency Enhancement In A Cassegrain System By Means Of A Dielectric Lens*, Radio Science **22**, 1091-1101.

James, G.L., (1983), *Primary-Focus Operation of Shaped Dual-Reflector Antennas*, IEEE Trans. Antennas Propagat. **AP-31**, 537-538.

Kildal, P-S., (1986), *Arecibo-Antenna Upgrading: Diffraction Analysis of Two Dual-Reflector Feeds With 8ft Distance From Paraxial Focus*, NAIC Arecibo Upgrade memo series..

Ruze, J., (1965), *Lateral-feed displacement in a paraboloid*, IEEE Trans. Antennas Propagat. **AP-13**, 660-665.

Von Hoerner, S. (1987), *What Next? Suggestions for Future Radio Telescopes*, NRAO, NLSRT Memo No. 2.

Yngvesson, K.S, (1985), *Imaging front-end systems for millimeter waves and submillimeter waves*, Proc SPIE **598**, 104-113.

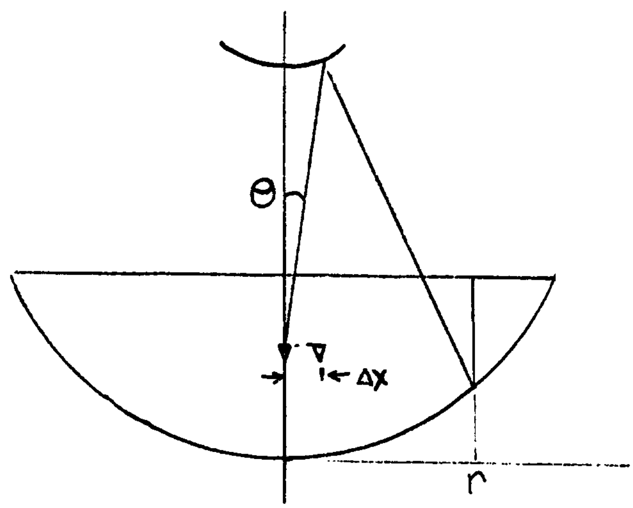

Figure 1

Reflector coordinates

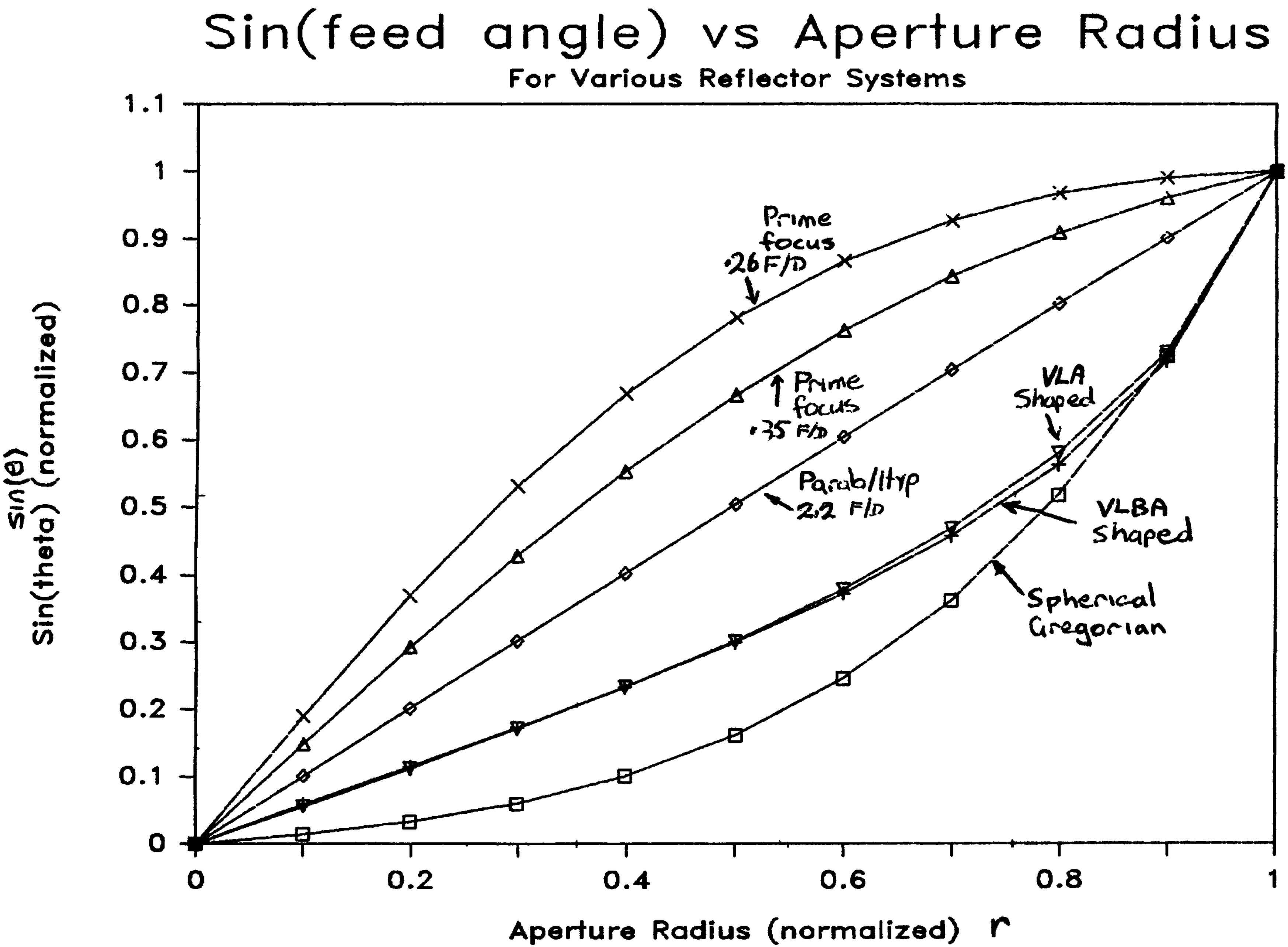
Sin(feed angle) vs Aperture Radius
For Various Reflector Systems
Sin(theta) (normalized)
sin(θ)
Aperture Radius (normalized) r
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
Prime focus .26 F/D
Prime focus .35 F/D
Parab/Hyp 2.2 F/D
VLA Shaped
VLBA Shaped
Spherical Gregorian

Figure 2