POINTING ACCURACY OF THE 45-FOOT ANTENNA

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INTRODUCTION

There are several factors which can cause pointing errors, including encoder offsets, gravitational sag, atmospheric refraction, non-perpendicularity of the beam and elevation axis, non-perpendicularity of the elevation axis and the azimuth axis, azimuth axis error, elevation axis error, and non-uniform thermal expansion of the telescope structure. Many of these factors are due to invariant structural imperfections of the telescope, and should therefore produce systematic, repeatable pointing errors that can be described accurately by a suitable pointing model. Other effects, like non-uniform thermal expansion and atmospheric refraction, are sensitive to the time of day, weather conditions, etc., and can not be expressed as simple equations of azimuth and elevation.

Previous work on characterizing the pointing of the 45-foot telescope is reported in [1]-[3]. The present effort, conducted as a summer student project, is intended to extend and refine the earlier work and in particular to estimate how well atmospheric refraction can be corrected.

The pointing model currently used at the 45-foot telescope can be represented with nine coefficients, which describe the predicted azimuth error and elevation error as functions of azimuth a and elevation e:

$$\Delta e = C_1 + C_2 \cos a + C_3 \sin a + C_4 \cos e$$

$$\Delta a \cos e = C_5 \cos e + C_6 \cos a \sin e + C_7 \sin a \sin e + C_8 \sin e + C_9.$$

C_1 and C_5 are the encoder offsets for elevation and azimuth, respectively. C_2 and C_7 represent the North-South component of tilt of the azimuth axis, and C_3 and C_6 represent the East-West component. C_4 accounts for gravitational sag of the antenna. C_8 represents the non-perpendicularity of the axes, and C_9 the non-perpendicularity of the beam and elevation axis.

The values of the coefficients are determined by measuring Δe and Δa at many different (a, e) positions in the sky, and performing a least squares solution for the coefficients. Several strong radio sources of known position are observed at various positions in the sky. By recording the output from the telescope at points surrounding the source's expected position (in both azimuth and elevation), it is possible to determine the pointing deviation $(\Delta a, \Delta e)$. The least squares solution for the coefficients can then be reduced to a system of nine linear equations which is easily solved. Unfortunately, this method treats the nine coefficients as independent parameters, while it is known that C_2 should equal $-C_7$, and C_3 should equal C_6. The coefficients are related because the elevation axis error and azimuth axis error cause related azimuth and elevation pointing errors. Imposing this constraint, however, makes the problem non-linear, and more difficult to solve.

When a suitable set of coefficients is obtained, they can be added to the on-line program. The pointing coefficients used by the online program need to be accurate enough to point the antenna to within about 200 arcseconds of the source. If the telescope is pointed too far from expected source, it will be impossible to measure the new pointing error for refining the coefficients.

The data files produced by the on-line program contain information about the pointing correction that is currently used. The off-line program calculates the offsets of the source, relative to the commanded antenna position. It then adds in the existing pointing correction to obtain the total offset. This total offset is then used in the least squares fit.

REFRACTION

The original pointing model consisted of ten coefficients, which included an extra term of the form $A\cot(e)$. This term described the effect of refraction. The refraction due to the atmosphere can be accurately approximated by this term at higher elevations, but below ten degrees it becomes unreliable. The refraction can be better estimated by including a $\cot^3(e)$ term, giving a refraction equation of the form $A\cot(e) + B\cot^3(e)$. This model is accurate at lower elevations, but the coefficients depend upon the weather conditions. Because the refraction is not strictly a function of azimuth and elevation, it was removed from the pointing model.

Using local weather data, which is collected by the on-line program, it is possible to predict the coefficients A and B. The following equations describe how the refraction coefficients are calculated from the temperature, pressure and dew point.

The on-line program provides the following data:

T, the site temperature (in degrees C);

T_d, the dew point temperature (in degrees C); and

P, the barometric pressure (in millibars).

The water vapor pressure and wet and dry components of the refractive index can be calculated from

$$\begin{aligned} P_w &= \exp\left[\frac{17.27 \cdot T_d}{237.3 + T_d} + 1.81\right] \\ (n_0 - 1)_{\text{dry}} &= 0.0000776 \cdot P/(T + 273.2) \\ (n_0 - 1)_{\text{wet}} &= 0.0000776 \cdot 4810.0 \cdot P_w/(T + 273.2)^2. \end{aligned}$$

Defining the variables x and y:

$$\begin{aligned} x &= \left[(n_0 - 1)_{\text{wet}} + (n_0 - 1)_{\text{dry}}\right] \\ y &= \left[(n_0 - 1)_{\text{wet}} \frac{H_{\text{wet}}}{r_0} + (n_0 - 1)_{\text{dry}} \frac{H_{\text{dry}}}{r_0}\right], \end{aligned}$$

the refraction model coefficients (in radians) can then be expressed as

$$\begin{aligned} A &= x - y \\ B &= y - \frac{1}{2}x^2. \end{aligned}$$

There are two constants which are used in the previous equations, H_{dry} and H_{wet}. These represent the dry and wet scale heights of the atmosphere. The other constant r_0 is the earth's radius.

There are a few assumptions inherent in this model. First of all, it assumes that the atmosphere is spherically symmetric. The local weather conditions can predict the atmospheric density distribution directly overhead, but at lower elevations the radiation from the source is passing through a larger cross section of the atmosphere, over which the weather conditions might vary considerably, especially if weather conditions are inclement or turbulent. A second assumption of this model is that the scale heights do not change appreciably. It is known that the wet and dry scale heights undergo seasonal variation; the values currently used are average values.

The first step in adding real-time refraction calculations to the pointing routines was to modify the on-line program so that the appropriate weather data was included in the pointing data files. After this modification was made, it was possible to calculate the refraction in off-line software. Even though the refraction correction wasn't applied in real time, and the on-line program was still using a ten coefficient model, the off-line analysis software can use the reported weather data to simulate what the output would be if only nine of the coefficients were used and the calculated refraction were factored in in real-time. The analysis software then performs a nine coefficient least squares fit to the resulting data. Later, the online program was modified to use the 9 coefficient model, and calculate refraction in real time.

Calculating Offsets

In order to measure the pointing offsets, the on-line telescope control program measures the total power at five points in the vicinity of the expected source position. The locations of these points are –4, –1, 0, 1 and 4 units from the expected source position, where the unit spacing (Δx) is chosen appropriately so that the scan should cover the entire source. Similar scans are done in elevation and azimuth, allowing separate measurements of elevation and azimuth offsets.

The +4 and −4 positions should be far from the source, and are used only to establish a baseline for the central three measurements. If the total power at each scan location be represented by f_{-4}, f_{-1}, f_0, f_1, f_4, then the baseline can be subtracted from the central three points as follows:

$$\begin{aligned} f_{-1} &= f_{-1} - \frac{1}{4}(3f_{-4} + f_4) \\ f_0 &= f_0 - \frac{1}{2}(f_{-4} + f_4) \\ f_1 &= f_1 - \frac{1}{4}(3f_4 + f_{-4}) \end{aligned}$$

Notice that the baseline interpolation assumes that the points are spaced equally. Although the points are not equally spaced in location, they *are* equally spaced in time. Subtracting out the baseline should therefore remove any linear time variation which might be present.

It is assumed that the shape of the response is a gaussian of fixed beamwidth:

$$f(x) = A \exp - \left(\frac{(x - \hat{x})}{w}\right)^2$$

If the unit spacing between the scan points is Δx, then the offset can be calculated from the total power at the −1 and +1 points:

$$\begin{aligned} f_{-1} = f(-\Delta x) &= A \exp - \left(\frac{(\Delta x + \hat{x})}{w}\right)^2 \\ f_1 = f(\Delta x) &= A \exp - \left(\frac{(\Delta x - \hat{x})}{w}\right)^2 \end{aligned}$$

Taking the quotient of these equations eliminates A and allows one to solve for $\hat{x}$:

$$\hat{x} = \frac{w^2}{4\Delta x} \ln\left(\frac{f_1}{f_{-1}}\right)$$

If either of the +1 or −1 measurements exceeds $0.3 f_0$, then the offset is computed with the same method, but using f_0 and either f_1 or f_{-1}, whichever is larger. After $\hat{x}$ is found, the peak value of the gaussian, A, is chosen so that $f_0 = A \exp(\frac{\hat{x}}{w})^2$.

The pertinent parameters in the calculation are w, which is a measure of the beamwidth of the antenna, and Δx, which is the spacing of the points. w is related to the half-power beamwidth b by $b = 2w\sqrt{\ln(2)}$. For all of the data sets we used $b = 360$ arcseconds. Until 7/11/92, the value of Δx was 240 arcseconds for all sources. On 7/11/92, Δx was changed to 200 arcseconds for strong sources and 170 for the weaker sources.

Δx is the angular spacing between the points on the celestial sphere. For elevation scans, Δx is the same as the increment of elevation between points, however when scanning in azimuth, the actual spacing between points is $\Delta x / \cos(e)$. The reason for this is that at higher elevations a given azimuth angle covers a smaller solid angle on the celestial sphere. Wherever offsets and residuals are quoted in this document, the azimuth offset/residual actually refers to the RMS value of $\Delta a \cos(e)$.

Notice that in most cases the offsets are calculated from the +1 and −1 scan positions, and do not depend on the central position. For weak sources, the total power at the +1 and −1 positions can be small, and the calculated $\hat{x}$ is quite sensitive to errors in the measurements. This is why in more recent scans, Δx was lowered for the weaker sources.

Certain characteristics of the data indicate when the measurements are unreliable, and the off-line analysis program recognizes and eliminates these cases. Whenever the signal to noise ratio, which is determined by taking the ratio of the peak value of the gaussian to the standard deviation of the measurements, is less than 3.5, the scan is rejected. The output measurements are on a scale of −10 to +10 volts. Whenever any of the five measured values is within .1 of +10 or −10, the point is rejected, because this indicates that saturation occurred. If the absolute difference between the +4 and −4 values is larger than half of the peak value, the point is filtered out. When the baseline is subtracted out, if any of the three resulting values are negative, the scan is rejected. In recent scans, this process filtered out about 15% of all the data points.

Summary of Results

The following table summarizes the results of all of all of the good data sets taken between 5/20 and 7/16. During this time interval, there were a few serious augmentations to the pointing measurement routine that should be taken into consideration. Starting with the 920601 data set, a new and different observation schedule was used, that caused the telescope to track sources down to lower elevations than before. The pointing coefficients used by the on-line program were updated before the 920612 data set. Before the 920623 data set, the on-line program was modified so that it used a nine parameter model to correct the pointing, and calculated the refraction (which was previously a tenth parameter) in real-time from current weather data.

DAYTIME AND NIGHTTIME DATA

	920520	920601	920612	920623	920628	920706	920711	920713	920716
Number of Pts	317	452	353	329	335	351	445	345	248
Accepted	225	283	264	218	291	281	367	287	211
Rejected	92	169	89	111	44	70	78	58	37
Offsets									
RMS az off	62.90	45.30	28.14	50.59	28.34	25.90	33.30	36.33	34.16
RMS el off	111.56	84.07	38.45	51.13	36.44	41.05	45.59	36.20	40.88
Residuals									
RMS az res	28.18	27.39	19.93	31.19	22.18	22.63	22.76	25.92	27.50
RMS el res	26.28	23.85	32.73	29.50	27.04	28.11	30.75	24.94	25.11

WITHOUT USING CALCULATED REFRACTION (DAYTIME AND NIGHTTIME DATA)

	920520	920601	920612	920623	920628	920706	920711	920713	920716
RMS el off	116.16	94.59	30.43	57.46	40.60	47.75	44.57	35.80	41.00
RMS el res	27.86	23.74	27.30	34.52	28.98	34.21	32.55	26.05	27.35

NIGHTTIME DATA (USING CALCULATED REFRACTION)

	920520	920601	920612	920623	920628	920706	920711	920713	920716
Number of Pts	134	207	121	125	126	128	247	147	150
Accepted	101	171	108	66	111	98	207	131	124
Rejected	33	36	13	59	15	30	40	16	26
Offsets									
RMS az off	63.71	38.77	18.69	37.75	17.59	21.97	33.51	21.35	20.67
RMS el off	100.37	79.53	27.58	45.73	26.21	30.90	41.27	31.16	38.75
Residuals									
RMS az res	20.92	22.15	14.75	25.25	16.24	18.28	19.62	16.09	15.02
RMS el res	20.05	20.13	23.25	29.96	21.16	17.90	21.47	16.06	18.46

FITTED COEFFICIENTS
(USING DAYTIME AND NIGHTTIME DATA, CALCULATED REFRACTION)

	920520	920601	920612	920623	920628	920706	920711	920713	920716
C(1)	9.49	10.02	9.42	9.68	9.21	9.33	9.19	9.08	9.36
C(2)	-5.88	-5.91	-5.77	-5.95	-6.06	-5.79	-5.81	-5.99	-6.12
C(3)	9.39	8.69	8.62	9.74	8.82	8.44	8.22	8.55	8.58
C(4)	1.59	.62	2.32	1.34	1.72	1.52	1.80	1.92	1.45
C(5)	14.08	14.09	14.48	12.32	13.32	13.41	13.92	13.76	12.45
C(6)	10.07	9.50	9.05	9.96	9.08	9.12	8.88	8.74	8.77
C(7)	6.25	5.83	6.33	6.42	6.34	5.98	5.99	6.33	6.14
C(8)	-.23	-.93	-.33	-1.16	-1.26	-1.25	-1.17	-.81	-2.18
C(9)	-10.47	-9.88	-10.38	-8.94	-9.11	-9.02	-9.40	-9.49	-7.74

Three different least squares fits were calculated for each of the data sets. The first fit used daytime and nighttime data, and used refraction calculations based on weather data. The second fit used daytime and nighttime data, but used a fixed refraction term instead of calculating refraction. The third fit is the same as the first, but includes only data taken at night.

For all of the data sets before 920623, the on-line program was using a ten parameter model to apply pointing corrections. No real-time refraction calculations were done, but the necessary weather data was reported in the output files. In order to do the first fit, the off-line software used the provided weather data to calculate refraction. Then it factored out the $\cot(e)$ term from the reported pointing correction, and adjusted the offsets to what they would have been if the calculated refraction were used instead of the fixed $\cot(e)$ term. The analysis software then used a nine coefficient fit on the resulting data.

For all of the data sets after (and including) 920623, the on-line program was using a nine coefficient model to apply pointing corrections. The refraction was calculated and applied in real-time from the tabulated weather data, and the weather data was still included in the output data file. In order to perform the second fit, the off-line software used the provided weather data to calculate refraction, and adjusted the computed offset to what it would have been if a fixed $\cot(e)$ term had been used in place of the real-time refraction estimate. For more details about how this was done, refer to the documentation for `convert.c`.

The rows labeled `RMS az off` and `RMS el off` tabulate the azimuth and elevation RMS offsets for each data set. Recall that the offset is the position of the source relative to the commanded position (which is calculated using a previous set of pointing coefficients). The rows labeled `RMS az res` and `RMS el res` report the azimuth and elevation residuals after the nine coefficient least squares fit. As mentioned previously, the azimuth offset/residual is actually $\Delta a \cos(e)$.

In most cases, calculating refraction from current weather data yields a slightly smaller elevation offset. The 920612 data set is a notable exception. For the more recent data sets, the resulting offsets are about the same in both cases. Also, the resulting RMS residuals are slightly smaller when calculated refraction is used.

The dramatic decrease in the measured offsets between the 920601 data set and the 920612 data set can be attributed to the installation of new and more accurate set of coefficients in in the on-line code.

For data taken at night and during the day, in all but a few cases the least squares fit yielded RMS residuals of less than 30 arcseconds, and only in rare cases is the RMS residual less than 20 arcseconds. For the nighttime data, the offsets and residuals are noticeably smaller: during the night it is possible to achieve residuals of 20 arcseconds or less in both azimuth and elevation. One potential reason for this difference is the differential heating of the telescope structure during the daytime when the sun is shining more on one side of the telescope. Another possible explanation is that the atmosphere and weather conditions are more stable during the night.

Problems and Unresolved Questions

The beamwidth of the antenna is estimated to be about 360 arcseconds. This value is used by the off-line software in calculating the offsets of the source. Re-measuring the beamwidth might yield better calculations of the offsets. Unfortunately, measuring the beamwidth requires a relatively strong point source, of which there are few. For extended sources, the measured response can no longer be expected to have a half-power beamwidth of 360 arcseconds. It might be worthwhile to use different HPBW's for different sources, and to find out the accepted angular size of the various strong sources that are being observed.

There are a few cases where the online program seems to give inconsistent results. For example, successive measurements sometimes give a monotonically increasing or decreasing sequence of calculated offsets, when the expected result is that successive measurements should yield very similar results. These peculiar results make it seem like the telescope isn't tracking the source accurately, and therefore the relative offset is increasing or decreasing.

It might also be worthwhile to update the coefficients in the online code once again. The least squares fits seem to yield fairly consistent results for the pointing coefficients, especially for the last fiew data sets.

It is difficult to determine the accuracy of the real-time refraction correction. We should really have data over a wide range of atmospheric conditions, including both winter and summer. The dry and wet scale heights which are used to calculate refraction are only typical values for this region. The exact values for Green Bank aren't known. One possibility is to try to find them by analyzing pointing data. As I mentioned previously, it is known that the wet and dry scale heights undergo seasonal variations. This effect might also be taken into account in the refraction calculation procedures.

Analyzing the nighttime data yields smaller RMS offsets and residuals, as previously noted. It would be useful to try to determine what causes this difference. When the telescope is pointing east or west, the differential heating effect should be most noticeable, whereas when the telescope is pointing north or south the effect should be less. Separately considering points which fall into these two categories is one way to study the effects of thermal expansion.

Computing the offset and peak from the three measurements of total power, involves solving for two quantities, A and $\hat{x}$ in a system of three equations. In general, it is impossible to find a gaussian which fits all three points exactly. If the beamwidth b is considered to be another parameter (instead of holding it constant), it is possible to obtain an exact closed form solution for the three parameters. The beamwidth, however, isn't arbitrary; it is a property of the antenna which shouldn't change. One possibility is to write some type of minimization routine that fits a gaussian of given beamwidth to the three data points. Such a solution involves solving a non-linear minimization problem, and might not be worth the trouble.

It might be worth considering whether there are systematic effects not accounted for by the nine-parameter model, so that additional terms in the model would be worthwhile. To investigate this, the offsets and residuals were plotted against elevation and azimuth; the results from the 7/16 run are given in the attached figures. Within the accuracy of these measurements, no such systematic effects are apparent.

Conclusions

The rms residuals are now consistently < 20 arcsec in each axis for nighttime data, and < 30 arcsec for day and night, compared with 30 and 50 arcsec previously [3]. This is no doubt due to improvements in the measuring technique implemented in the on-line software, and occurs in spite of the fact that recent data sets include more data at low elevations where errors are largest. Some of the improvement may be due to the real-time refraction correction. It is still true that a significant fraction of the apparent offsets and residuals is due to measurement errors, rather than actual pointing errors of the antenna; thus, these results represent an upper limit on the true pointing errors. In summary, it is quite clear that the 45 arcsec rms required by our link budget [4] will be met without the need for further improvements.

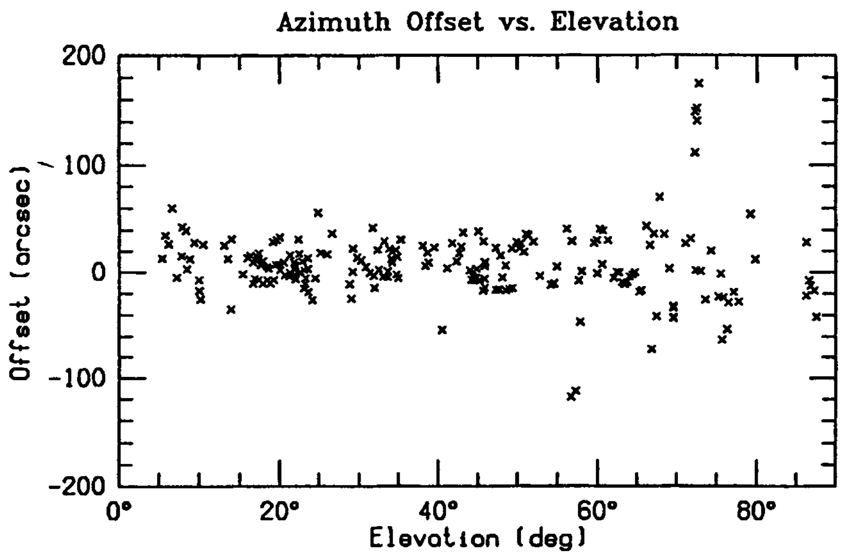
REFERENCES

[1] E. B. Fomalont, "45-Foot Pointing and Efficiency," NRAO internal memorandum, March 8, 1973.

[2] F. Ghigo, "Pointing calibration of the ESSCO 45-ft antenna at Green Bank," NRAO Electronics Division Internal Report No. 288, June 1990.

[3] L. D'Addario, "Performance measurements of the Green Bank 45-ft (13.7-m) antenna." OVLBI-ES Memo No. 14, June 13, 1991.

[4] L. D'Addario, B. Shillue and D. Varney, *The Green Bank OVLBI Earth Station: Preliminary Design.* NRAO report, July 2, 1991.


Azimuth Offset vs. Elevation
Offset (arcsec)
Elevation (deg)
200
100
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-200
0°
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40°
60°
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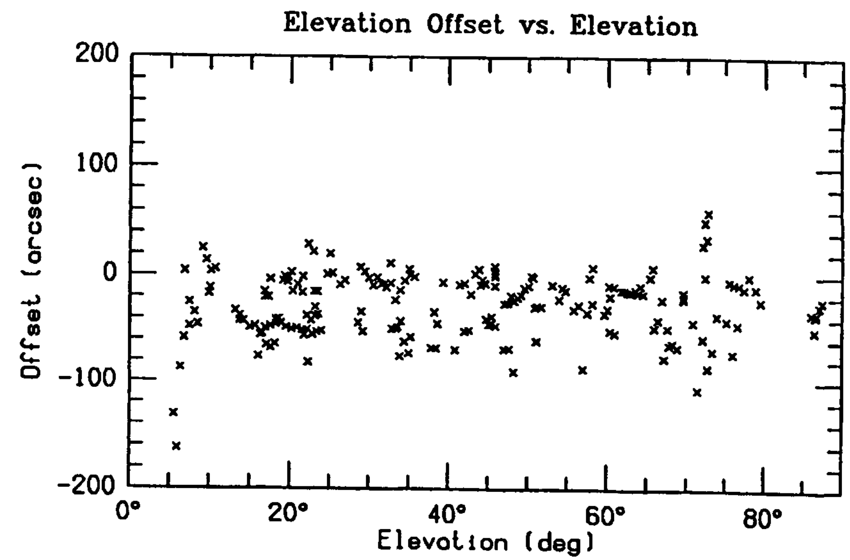
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Offset (arcsec)
Elevation (deg)
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-100
-200
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60°
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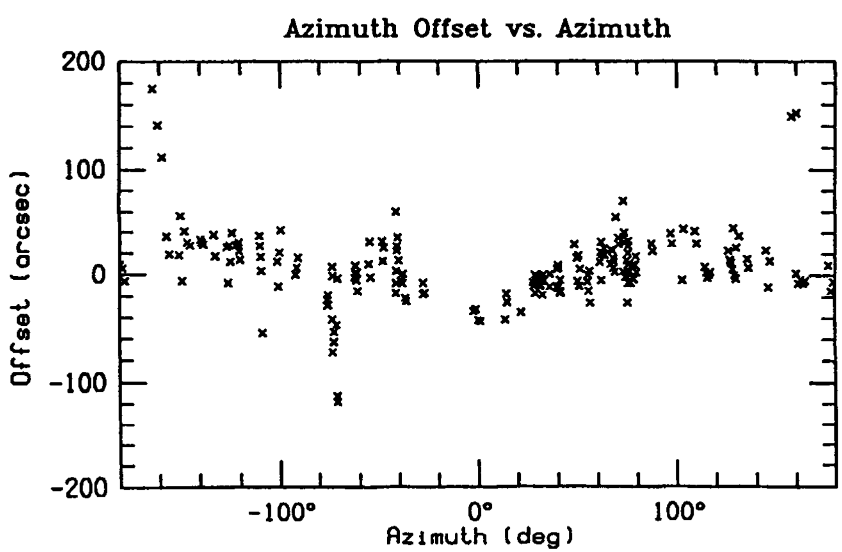
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Offset (arcsec)
Azimuth (deg)
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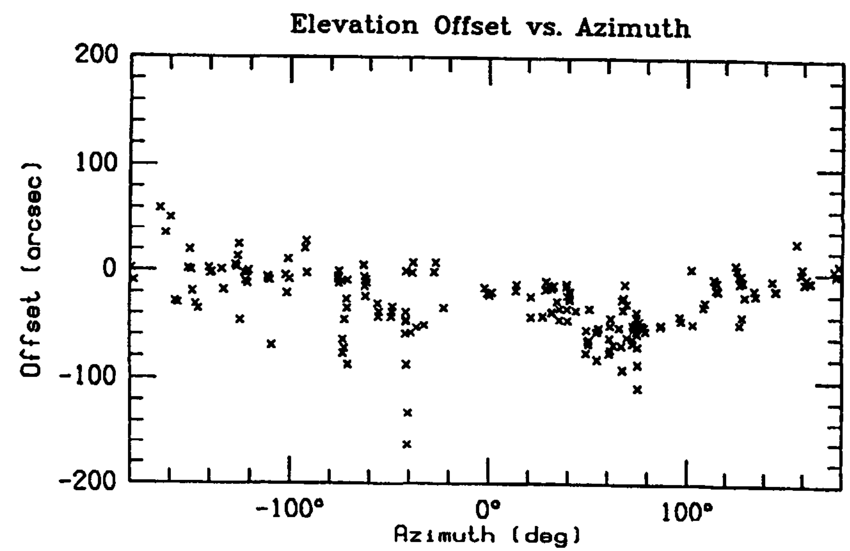
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Offset (arcsec)
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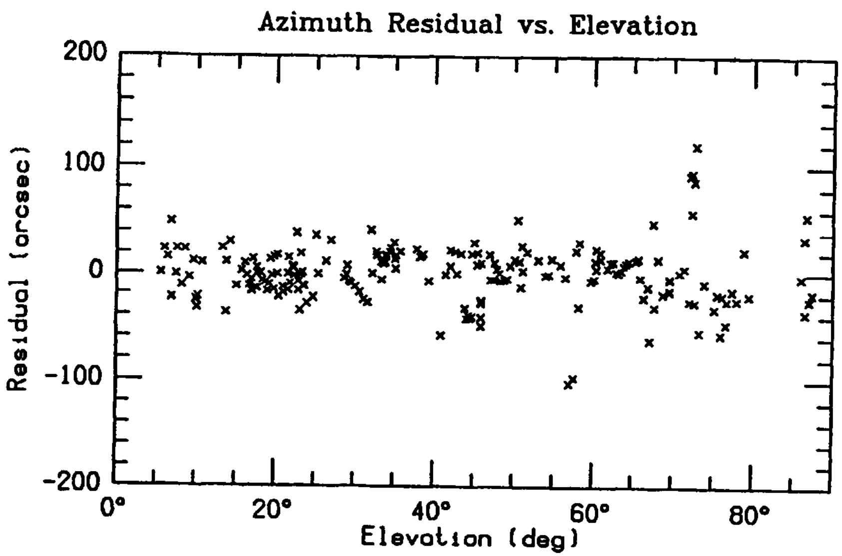
Azimuth Residual vs. Elevation
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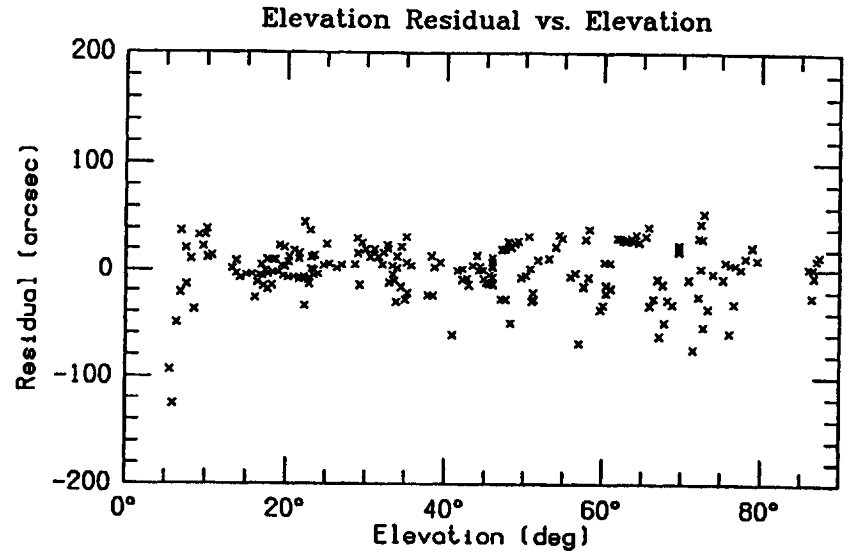
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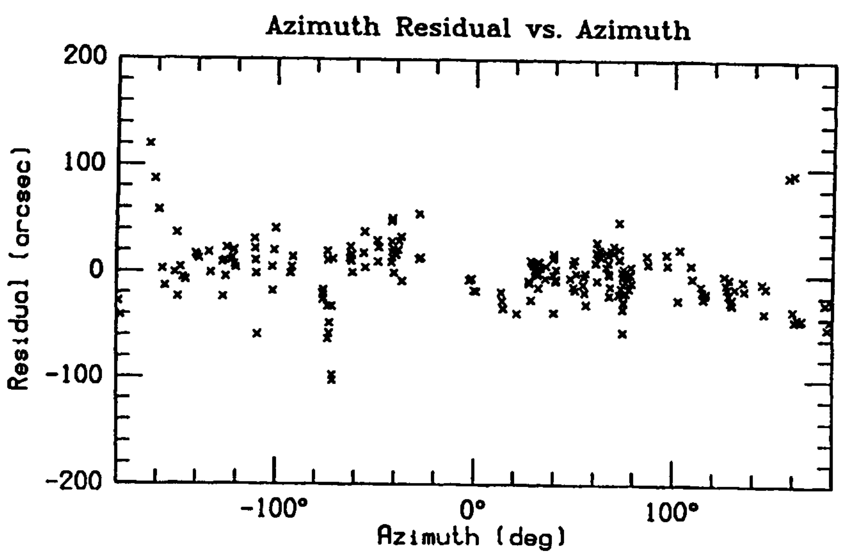
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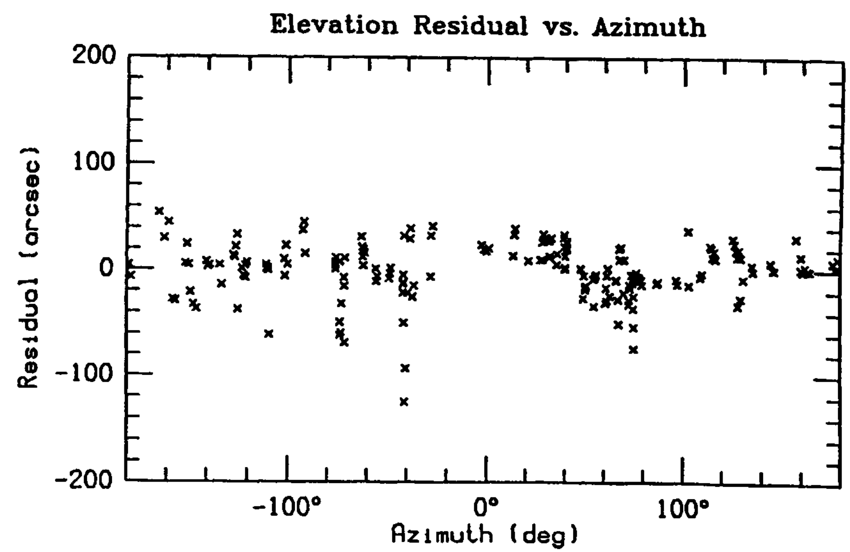
Azimuth (deg)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