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CORRELATING ORBITING RADIOTELESCOPE DATA WITH TIME CORRECTIONS

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1.0 Real Time Corrections Within the Correlator

Let the correlator have a clock whose reading t reproduces UTC during the observation at the reference location. (The reference location is typically at the center of the earth.) For each t, the correlator determines the corresponding time of arrival of the signal of interest at each telescope; it does this by computing the propogation delay from the telescope to the reference location and subtracting:

$$t_{\rm tel} = t - D \tag{1}$$

where $t_{\rm tel}$ is the time at the telescope and D is the propogation delay. Each data sample from a telescope is labeled with a time which we will call "tape time." Nominally this is equal to the UTC at which that sample was taken, but in general the label is in error by a small amount. For ground radio telescopes this error is simply the station clock error, which is usually stable and well behaved; for orbiting radio telescopes, it is expected to be more rapidly varying. In any case, if the tape time error is known during correlation, then the correlator should use it in determining the desired tape time:

$$t_{\rm tape} = t - D - \Delta t \tag{2}$$

where Δt is the tape time error.

Synchronization of data at the correlator consists of telling the tape playback unit to produce the data sample whose time label is $t_{\rm tape}$. This is done simultaneously for all playback units of all telescopes, and the resulting data streams are all sent to the correlator proper. Each playback unit may supply the required sample by mechanically adjusting the tape position or by reading from the proper place in a previously-filled buffer, or some combination of these; such implementation details are of no concern here, except that they are all regarded as part of the playback unit. The correlator proper can then, in principle, perform all of its processing without further concern about any errors in the tape time.

However, care must be taken with fringe rotation. The need for fringe rotation arises because the astronomical signal was not recorded directly on the tape, but instead it was first mixed with a local oscillator signal to convert it to baseband. When the combined signal is delayed by $D_{\rm tel}$ for synchronization, the LO portion of the signal suffers an undesired phase shift which must be reversed by the fringe rotator. If, as is usual, the LO signal at the telescope (space or ground) is based on the same reference as the tape time, then the fringe rotator must take account of any timing error. The actual LO signal can be written

$$\exp[j2\pi f_L(t - D - \Delta t)], \tag{3}$$

where f_L is the nominal LO frequency, so that the required fringe rotator phase is

$$2\pi f_L(D + \Delta t). \tag{4}$$

One difficulty remains. Whereas the data from the playback unit is not continuous in time, but rather consists of discrete samples, the request for the sample whose tape time is $t_{\rm tape}$ usually cannot be fulfilled exactly. Instead, the playback unit supplies the sample nearest in time to that requested. A "fractional sample time" error occurs, but the value of this error is known for each sample, and it can be corrected if it is not varying too rapidly. In an FX correlator, a correction can be applied at the output of each FFT based on the average error of all samples used in that FFT. In an XF correlator, the correction must be made to the cross-correlation function or its spectrum. Thus, the maximum rate of applying this correction is the FFT rate or the dump rate, respectively. In practice the correction rate may be limited by data flow within the correlator's software/hardware structure.

The fractional sample time error and methods for its correction are well known in ground VLBI. The only difference in orbiting VLBI is that the error may vary more rapidly via the Δt term. One effect is that it may not be practical for the tape unit to follow the variations very closely, so that the "fractional sample" error might be allowed to have peak-to-peak excursions of several sample times.

Notice from (2) and (4) that the effect of the timing error Δt on correlator operation may be simply described as follows: wherever the geometrical delay D would be used in the absence of timing errors, use $D + \Delta t$ instead. Otherwise, correlator operation is unchanged.

2.0 Interpolation of Time Corrections

In present practice, measurements of Δt for the orbiting telescope are made in real time, and a time series of these is available during correlation. The independent variable for these measurements is generally UTC at the earth station where they were made, whereas for use at the correlator they must be referred to correlator time t. Also, the measurements are provided at a sampling interval appropriate to the bandwidth of the data, whereas they are needed at those discrete correlator times $t = t_k$ when the correlator software updates its geometrical model to determine D. An interpolation process is therefore needed to convert from the one time scale to the other.

Let the time series of measurements be $\{t_{\text{gnd}}(i), \Delta t(i)\}$, where $t_{\text{gnd}}(i)$ is the time "tag" of the ith measurement. Then

$$t_{\text{gnd}}(i) = t_{\text{tape}} + \Delta t(i) + D_{\text{link}}(i) \tag{5}$$

where t_{tape} is the tape time of the sample received at the earth station simultaneously with the measurement of $\Delta t(i)$, and $D_{\text{link}}(i)$ is the space-to-ground propogation delay affecting that same sample. This assumes that the downlink signal used to measure Δt suffers the same link delay as the data sample, even though they may be on physically separate communication channels. Note that the value of t_{tape} in (5) is continuous and therefore it usually does not correspond precisely to any data sample's label, since the labels are discrete time. From (2), we see that the data labeled by t_{tape} is needed in the correlator at

$$t = t_{\text{tape}} + \Delta t(i) + D. \tag{6}$$

so that

$$t = t_{\text{gnd}}(i) - D_{\text{link}}(i) + D. \tag{7}$$

Therefore, the time tags associated with the time correction values can be converted to correlator time if the corresponding downlink delays $D_{\text{link}}(i)$ are known. The original time series then becomes $\{t, \Delta t(i)\}$, where t is given by (7). This series must still be interpolated to the discrete times $\{t_k\}$ at which the correlator updates the geometry.

The downlink delay is most accurately estimated by a separate calculation involving the positions of the spacecraft and earth station. The correlator's software must already be tracking the satellite position, using the best available reconstructed orbit; tracking of the earth station's position is easier, and is similar to tracking the position of a ground radio telescope. As shown in the next section, the effects of the medium (ionosphere and troposphere) may be neglected.

An alternative, but less straightforward, method of converting the time tags to correlator time is possible. If the earth station can produce a separate time series of the form $\{t_{\text{tape}}(j), t_{\text{gnd}}(j)\}$, then interpolation of this series can produce values of t_{tape} at the same times as the Δt measurements. That is, the original series is converted from $\{t_{\text{gnd}}(i), \Delta t(i)\}$ to $\{t_{\text{tape}}(i), \Delta t(i)\}$. The corresponding correlator times are then given by (6), and a second interpolation is needed to convert them to the correlator's discrete time set $\{t_k\}$. This method assumes that the auxilliary time series can be supplied by the earth station with sufficient accuracy. At the correlator, additional computing is needed to do the second interpolation, possibly affecting accuracy; on the other hand, calculation of D_{link} is avoided, and this eliminates the need to know the earth station's location.

3.0 Quantitative Specifications

3.1 Time Correction (Δt) Values

The precision required in the time corrections is dominated by the need to correct for the LO phase error. An rms error of 0.1 radian will result in about a 1% loss of coherence, and at 22 GHz this corresponds to a timing error of 0.72 psec. The complete system of providing time corrections should strive for an accuracy of this order over as wide a range of time as possible, but certainly for periods up to the longest coherent integration time that might be limited by other effects. The *precision* (if not the accuracy) of the system should be considerably smaller (say 0.1 psec), and the resolution of the transmitted data should be smaller still.

3.2 Time Tags of the Correction Values

The accuracy of the time tags determines the accuracy with which the correction data can be interpolated to the times of interest. The required time precision depends on how much the corrections might change between tabulated values.

We assume that the time correction function is fully sampled so that aliasing can be neglected. (In present systems the sampling rate is 10 Hz, in which case we are assuming that the underlying time error spectrum contains no energy at frequencies above 5 Hz.) Then the sampling theorem guarantees that the correction for any time can be determined from the samples, but only if the samples are accurate. Although the bandwidth is limited, the amplitude of signal components within that bandwidth is not, so the function can change by large amounts between samples. In fact, for our signals a very large component is expected because of the error in the predicted orbit, but this component should be slowly varying; in addition, smaller but faster components are expected from fluctuations in the propogation medium. All components must be accurately represented.

Let the worst-case residual Doppler be $d_{\max} = 1000$ Hz. Then the time correction is changing at the rate of

$$d\Delta t/dt = d_{\max}/f_{\text{link}} = (1000\,\text{Hz})/(15\,\text{GHz}) = 7 \times 10^{-4}.$$

Therefore an error of ϵ in the time stamp of the Δt value (or equivalently an error in the sampling time of that value at the earth station) corresponds to an error in Δt of

$$E = \epsilon(d\Delta t/dt).$$

If the correlator is to interpolate the time corrections properly, and obtain an accuracy of, say, $E < 0.5$ psec, then we need

$$\epsilon < E/(dT/dt) = Ef_{\text{link}}/d_{\max} = 7.5\,\text{microseconds}.$$

This requirement for accuracy of a few microseconds applies not only to the station's master clock, but to the accuracy with which those clock readings make their way into the time stamps of the data transmitted to the correlator, and also to the accuracy with which they are used within the correlator's interpolation procedure.

The accuracy requirement is sufficiently loose that the effects of the troposphere and ionosphere on D_{link} can be neglected. However, it is unusually tight as a fraction of the sampling interval.

It should be apparent that the interpolation algorithm requires some sophistication; simple linear interpolation will not be adequate. Ideally, sinc function interpolation should be used, but this cannot be implemented perfectly. A reasonable approximation should be used, with a range of support of many samples, but derivation of exact requirements is beyond the scope of this note.

The above discussion applies mostly to *fluctuations* in the correction values; these must be transmitted accurately to avoid loss of coherence. The absolute accuracy with which the telescope's time is tied to true UTC is also an issue. This affects the amount of "fringe searching" that is needed during or after correlation, and depends mainly on the accuracy with which the UTC clock at the telescope is maintained. For orbiting telescopes, this clock is at the earth station, and it gets into the correlator via its reading at the special epoch when the tape clock is set; this determines a portion of Δt that stays fixed during the tracking pass (unless a dropout occurs). Relatively inexpensive GPS receivers allow determining UTC to about 100 nsec, even with Selective Availability turned on, so 200 nsec seems to be a feasible specification for the clock accuracy. Notice that good absolute accuracy of the station clock need not be maintained in real time, provided that the clock error is known during the offline processing when the time correction data are calculated.