OVLBI-ES MEMO NO. 56

REDUCTION OF PHASE RESIDUALS TO TIME UNITS

Larry R. D'Addario
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The two-way timing system measures residual error in terms of the phase of the downlink signal, but this data must be used to determine the timing error on the satellite. This memo addresses the narrow issue of converting the phase measurements to their time equivalents. Further details of the measurement system and associated data processing are given in OVLBI ES Memos 19, 20, 22, 25, and 38.

Consider the timing signal that is transponded at time t, was transmitted on the uplink at time $t - T_u$, and will be received on the downlink at time $t + T_d$. (Delay through the transponder is neglected.) At the time of uplink transmission, the predicted uplink delay was $\widehat{T}_u$, and at the time of downlink reception the predicted downlink delay will be $\widehat{T}_d$. Then we can write the signals at uplink transmission, uplink reception, downlink transmission, and downlink reception, respectively, as:

$$\begin{aligned} u_g(t) &= \sin[\omega_u(t + \widehat{T}_u)] && (1) \\ u_s(t) &= \sin[\omega_u(t + \widehat{T}_u - T_u)] && (2) \\ d_s(t) &= \sin[\omega_d(t + \widehat{T}_u - T_u)] && (3) \\ d_g(t) &= \sin[\omega_d(t + \widehat{T}_u - T_u - T_d)]. && (4) \end{aligned}$$

where ω_u and ω_d are the *nominal* uplink and downlink transmission frequencies, respectively, so that $\omega_d = r\omega_u$ for transponding ratio r. For simplicity, and without loss of generality, all amplitudes are taken to be unity.

In the NASA DSN/JPL station design, the received downlink signal $d_g(t)$ is phase-shifted by $\omega_d \widehat{T}_d$ during downconversion, and then is compared in a phase detector with a constant-phase reference at the nominal intermediate frequency. In the NRAO design, the signal is compared in a phase detector with a predicted downlink signal. The methods are equivalent, but this analysis follows the NRAO design.

The received downlink signal is downconverted to IF using a set of fixed-phase local oscillators whose total frequency is ω_L, yielding

$$d_{IF}(t) = \sin[(\omega_d - \omega_L)t + \omega_d(\widehat{T}_u - T_u - T_d)].$$

This signal is measured in a phase detector whose reference input has been programmed to be

$$r(t) = \sin[(\omega_d - \omega_L)t - \omega_d \widehat{T}_d)],$$

so the measured phase (signal minus reference) is:

$$\phi_{\text{meas}} = \omega_d(\widehat{T}_d - T_d + \widehat{T}_u - T_u) = \omega_d \tau$$

and

$$\tau = \phi_{\text{meas}}/\omega_d$$

where $\tau \equiv \widehat{T}_d - T_d + \widehat{T}_u - T_u$ is the two-way timing residual.

The last equation gives the conversion from measured residual phase to measured residual time. Note that ω_d is the nominal downlink frequency, not the instantaneous frequency, so the conversion merely involves dividing by a constant.