CONVERSION OF TWO-WAY TIMING DATA TO DOPPLER DATA

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ABSTRACT

In this memo we describe the how the NRAO OVLBI Earth Station two-way timing data from the two-way phase transfer with a satellite are transformed into "doppler data" to be used for orbit determination. We find that the two-way phase transfer with the VSOP and RADIOASTRON satellites inherently introduces an error in the conventional doppler count. We also discuss the effects that clock errors and errors in the determination of the phase residuals have on the conventional doppler data.

Introduction

In two-way phase transfer, the goal is to have the satellite receive a constant frequency uplink signal from a ground tracking station. The signal that is received by the satellite is downlinked back to the tracking station where it is compared to a predicted downlink signal allowing the signal that was received by the satellite to be analyzed. A constant frequency signal, however, cannot realistically be sent to the satellite due to uncertainties in the predicted orbit of the satellite. Thus the difference between the actual downlink signal and the predicted downlink signal contains information on the errors in the predicted orbit. The difference in the phase of the actual downlink signal with the predicted downlink signal is what is measured at the Earth station and is called the phase residual. The phase residuals can be converted to units of time by dividing by the nominal downlink signal of the satellite [1]. The format of the doppler data are specified in [2].

Determining the Round Trip Time

The round trip time is defined as the time that the signal takes to go from the tracking station to the satellite and back to the tracking station. The round trip time, $D(t)$, is the essential quantity that needs to be determined in order to produce doppler data from two-way timing measurements. The true round trip time of the signal is given by

$$D^{true}(t) = \tau_d(t) + \tau_u(t') + \tau_{atm}(t') + \tau_{atm}(t) + \tau_{orbit}(t) + \tau_{spacecraft}(t) + \tau_{position}(t) \quad (1)$$

where t is the time that the downlink signal is received by the tracking station, t' is the true time that the uplink signal was broadcast, τ_d is the predicted downlink time, τ_u is the predicted uplink time, t_{atm} are the delays due to the signal having to propagate through the the Earth's troposphere and ionosphere, and $\tau_{spacecraft}$ represents the hardware delay between the time that the uplink signal is received by the satellite and when the downlink signal is broadcast by the satellite. The effect of the errors between the true orbit and the predicted orbit are represented by τ_{orbit}. The effects of the error in determining the exact position of the tracking station are represented by $\tau_{position}$. The time listed for τ_{orbit},$\tau_{position}$ and $\tau_{spacecraft}$ is the downlink reception time in order to indicate which signal was effected by these delays. Also we adopt the general notation that signal paths (in seconds) are noted by a τ while times of events are given by a t. Since the tropospheric and the ionospheric delays will typically be closer to the tracking station than to the satellite we assume that these delays occur at the transmission and reception time for the uplink and downlink signal respectively. The true uplink time is determined by solving the equation

$$t' = t - \tau_d(t) - \tau_u(t') - \tau_{atm}(t') - \tau_{atm}(t) - \tau_{orbit}(t) - \tau_{spacecraft} - \tau_{position}(t) \tag{2}$$

which must be solved iteratively. We have assumed that the spacecraft hardware delay is constant. The projection of the tracking station position error along the line of sight to the spacecraft will be time varying, making $\tau_{position}$ a time varying quantity.

Clock Errors

The Earth station's clock can have an offset from true UTC and a drift of this offset with time. We will assume that this drift is linear during a single tracking pass. We will use the following notation: t is the true UTC time, $t_s(t)$ the tracking station's clock offset from true UTC and $\frac{\partial t_s}{\partial t}$ is the drift rate of the clock offset. The clock error and drift will be measured for the Earth station. For the uplink and downlink predicted ranges, the clock offset will mean that we use an incorrect predicted range to the satellite. Thus, for a round trip signal being received at time t we have actually used the predicted ranges $\tau_u(t' + t_s(t'))$ and $\tau_d(t + t_s(t))$. The predicted round trip time is thus

$$D^{pred}(t) = \tau_d(t + t_s(t)) + \tau_u(t' + t_s(t')) \tag{3}$$

We note that no attempt to compensate for the effects of the atmosphere, the ionosphere nor any other delays is made by the Earth station for the uplink and downlink signals from the satellite.

As long as the clock offset and its rate are small then we can expand τ_u and τ_d in a Taylor series about the true UTC time t:

$$\begin{aligned} D^{pred}(t) &= \tau_d(t) + \frac{\partial \tau_d(t)}{\partial t} t_s(t) + \frac{1}{2}\frac{\partial^2 \tau_d(t)}{\partial t^2} t_s^2(t) \\ &\quad + \tau_u(t') + \frac{\partial \tau_u(t')}{\partial t} t_s(t') + \frac{1}{2}\frac{\partial^2 \tau_u(t')}{\partial t^2} t_s^2(t') + \mathcal{O}(t_s^3). \end{aligned} \tag{4}$$

The first derivative of τ_u and τ_d are typically referred to as the range rate while the second derivative is referred to as the range acceleration. We can let $\tau_{clock}(t)$ represent the error in the predicted round trip time that the clock offset and drift introduces:

$$\begin{aligned} \tau_{clock}(t) &= \frac{\partial \tau_d(t)}{\partial t} t_s(t) + \frac{1}{2}\frac{\partial^2 \tau_d(t)}{\partial t^2} t_s^2(t) \\ &\quad + \frac{\partial \tau_u(t')}{\partial t} t_s(t') + \frac{1}{2}\frac{\partial^2 \tau_u(t')}{\partial t^2} t_s^2(t') + \mathcal{O}(t_s^3) \end{aligned} \tag{5}$$

and

$$D^{pred}(t) = \tau_u(t') + \tau_d(t) + \tau_{clock}(t). \tag{6}$$

Using the expansion

$$t_s(t') = t_s(t) + \frac{\partial t_s}{\partial t}(t'-t) = t_s(t) - \frac{\partial t_s}{\partial t} D^{true}(t) \tag{7}$$

it can be shown that

$$\begin{aligned}\tau_{clock}(t) \;=\; & \frac{\partial \tau_d(t)}{\partial t} t_s(t) + \frac{1}{2}\frac{\partial^2 \tau_d(t)}{\partial t^2} t_s^2(t) + \frac{\partial \tau_u(t')}{\partial t} t_s(t) + \frac{1}{2}\frac{\partial^2 \tau_u(t')}{\partial t^2} t_s^2(t) \\ & - \frac{\partial \tau_u(t')}{\partial t}\frac{\partial t_s}{\partial t} D^{true}(t) + \frac{\partial^2 \tau_u(t')}{\partial t^2}\left[\frac{1}{2}\left(\frac{\partial t_s}{\partial t}\right)^2 \left(D^{true}(t)\right)^2 - t_s(t)\frac{\partial t_s}{\partial t} D^{true}(t)\right] \\ & + \mathcal{O}(t_s^3)\end{aligned} \tag{8}$$

Phase Residuals

The phase residuals, in units of time, measured by the NRAO OVLBI Earth Station are given by

$$\tau_\phi(t + t_s(t) + t_\phi) = D^{true}(t) - D^{pred}(t) - \delta t_o \tag{9}$$

where δt_o accounts for any measurement ambiguities. The quantity t_ϕ represents the hardware delay between the instant that the downlink signal is received by the tracking station and when it is measured. Since the phase residuals are a quantity measured by the Earth station we must also include the error due to the clock offset in the the time tag of the phase residuals. When the two-way link is established with the satellite a default value, typically $\tau_\phi(t_o + t_s(t_o) + t_\phi) = 0$, is assumed since the exact range to the satellite is unknown at this time [3]. Thus δt_o represents the error in the range to the satellite upon acquisition so that

$$\delta t_o = \tau_{atm}(t_o) + \tau_{atm}(t'_o) + \tau_{orbit}(t_o) + \tau_{spacecraft} + \tau_{position}(t_o) - \tau_{clock}(t_o) \tag{10}$$

where t_o is the time that the two-way link is established. For the duration of a tracking pass δt_o will remain constant but will not be constant from one tracking pass to the next. Substituting equations (1), (6) and (10) into equation (9) we have

$$\begin{aligned}\tau_\phi(t + t_s(t) + t_\phi) \;=\; & \tau_{atm}(t) + \tau_{atm}(t') + \tau_{orbit}(t) + \tau_{position}(t) - \tau_{clock}(t) \\ & - \tau_{atm}(t_o) - \tau_{atm}(t'_o) - \tau_{orbit}(t_o) - \tau_{position}(t_o) + \tau_{clock}(t_o)\end{aligned} \tag{11}$$

showing that the phase residuals only measure the quantities not compensated for by the tracking station in the two-way phase transfer process and the effects of the clock errors.

Determining the True Uplink Time

The true round trip time can be determined once the true uplink time is known. However, we cannot precisely determine the true uplink time due to the δt_o term in the phase residuals. There are two estimates for the true uplink time that we will investigate. The first of these is found by solving

$$t'_1 = t - \tau_d(t) - \tau_u(t'_1) - \tau_\phi(t + t_s(t) + t_\phi) \tag{12}$$

which gives us

$$t'_1 - t' = \tau_u(t') - \tau_u(t'_1) - \delta t_o \approx -\frac{\partial \tau_u(t')}{\partial t}(t'_1 - t') - \delta t_o. \tag{13}$$

The second estimation is determined from solving

$$t'_2 = t - \tau_d(t) - \tau_u(t'_2) - \tau_{atm}(t) - \tau_{atm}(t'_2) \tag{14}$$

which gives us

$$t'_2 - t' = \tau_u(t') - \tau_u(t'_2) + \tau_{orbit}(t) + \tau_{position}(t) + \tau_{spacecraft}. \tag{15}$$

The determination of the uplink time using equation (14) is in error due to only the orbit error at that given time while the solution to equation (12) depends on the orbit error at the beginning of the tracking pass. We note that use of equation (12) would allow for the error in determining the true uplink time to be constant. However, there will still be a time varying error in the round trip signal time since the range rate of the satellite will be changing in time. We have assumed that τ_{orbit} is the dominant error in the above arguments. At the Earth Station we have chosen to use equation (12) for solving for the estimated uplink time. This choice was made on the basis of the computational requirements for each determination.

Estimating the Round Trip Time

We will use the general equation for the round trip time of

$$D^{est.}(t) = \tau_d(t) + \tau_u(t'_{pred}) + \tau_\phi(t + t_s(t) + t_\phi). \tag{16}$$

Before we can continue we must correct for the time tag of the phase residuals. We are able to adjust the time tag of the phase residuals to the true UTC for those phase residuals to within a time tag error of t_Δ. There will be a remaining error (t_Δ) in the time tag due to the fact that we cannot measure $t_s(t)$ and t_ϕ with infinite accuracy. It should be noted that we are assuming that we are completely removing the effects of the clock drift rate in the time tag of the phase residuals. After we have corrected the time tag for the phase residuals we are left with

$$\tau_\phi(t + t_\Delta) \sim \tau_\phi(t) + \frac{\partial \tau_\phi(t)}{\partial t} t_\Delta + \frac{\partial^2 \tau_\phi(t)}{\partial t^2}(t_\Delta)^2 + \mathcal{O}(t_\Delta^3) \tag{17}$$

By inserting equations (11) and (17) into equation (16) we have the following estimation for the round trip signal time:

$$D^{est.}(t) = D^{true}(t) - \tau_{clock}(t) + \tau_u(t'_{pred}) - \tau_u(t') - \delta t_o + \frac{\partial \tau_\phi(t)}{\partial t} t_\Delta + \frac{\partial^2 \tau_\phi(t)}{\partial t^2}(t_\Delta)^2 + \mathcal{O}(t_\Delta^3). \tag{18}$$

Thus we see that there is an intrinsic error in determining the round trip signal time. This error cannot be removed without *a priori* knowledge of the orbit error (τ_{orbit}).

Determining the Doppler Count

The doppler count, $N(t)$, is the quantity which is produced for the doppler data that is used for orbit determination [2]. The conventional doppler count is equal to the change in the range to the satellite, in cycles of the downlink frequency, from the time when the measurements began, plus the number of cycles of a bias signal passing through the receiver system. In the conventional doppler method a constant frequency signal is sent to the satellite unlike the two-way phase transfer case. For the two-way timing measurements the doppler count is given by

$$N(t) = \left[D^{est.}(t) - D^{est.}(t_o)\right] \times f_d + [t - t_o] \times f_b \tag{19}$$

where t_o is the time that the two-way timing link was established, f_d is the nominal downlink frequency and $f_b = 5 \times 10^6$ Hz is a bias frequency to keep the doppler count positive. The time that the two-way timing link was established (t_o) is taken to be the *time of reception* of the first valid two-way phase transfer signal at the tracking station.

Inserting equation (18) into equation (19) we have for the conventional doppler count measured via two-way phase transfer that

$$\begin{aligned} N(t) &= \left[D^{true}(t) - D^{true}(t_o)\right] \times f_d + [t - t_o] \times f_b \\ &+ \Bigg[\tau_u(t'_{pred}) - \tau_u(t') - \tau_u(t'_{o\ pred}) + \tau_u(t'_o) - \tau_{clock}(t) + \tau_{clock}(t_o) \\ &+ \left(\frac{\partial\tau_\phi(t)}{\partial t} - \frac{\partial\tau_\phi(t_o)}{\partial t}\right) t_\Delta + \left(\frac{\partial^2\tau_\phi(t)}{\partial t^2} - \frac{\partial^2\tau_\phi(t_o)}{\partial t^2}\right)(t_\Delta^2) + \mathcal{O}(t_\Delta^3)\Bigg] \times f_d \\ &= N_{true}(t) + N_{error}(t). \end{aligned} \tag{20}$$

This introduces and error of

$$\begin{aligned} N_{error}(t) &= \Bigg[\tau_u(t'_{pred}) - \tau_u(t') - \tau_u(t'_{o\ pred}) + \tau_u(t'_o) - \tau_{clock}(t) + \tau_{clock}(t_o) \\ &+ \left(\frac{\partial\tau_\phi(t)}{\partial t} - \frac{\partial\tau_\phi(t_o)}{\partial t}\right) t_\Delta + \left(\frac{\partial^2\tau_\phi(t)}{\partial t^2} - \frac{\partial^2\tau_\phi(t_o)}{\partial t^2}\right)(t_\Delta^2) + \mathcal{O}(t_\Delta^3)\Bigg] \times f_d \end{aligned} \tag{21}$$

into the doppler count. This error will be time varying since the error in determining the true uplink time is time varying. Also, even if the tracking station clock was perfect, the doppler count error could not be removed from the doppler data unless δt_o is known, which will not be the case for the VSOP and RADIOASTRON spacecraft. *Therefore there will always be an inherent error in the conventional doppler data determined via two-way phase transfer without knowledge of* δt_o.

Expanding the predicted uplink delays in terms of the true uplink times in equation (21) we find that

$$\begin{aligned} \frac{N_{error}(t)}{f_d} &\sim \frac{\partial\tau_u(t')}{\partial t}\left(t'_{pred} - t'\right) + \frac{1}{2}\frac{\partial^2\tau_u(t')}{\partial t^2}\left(t'_{pred} - t'\right)^2 \\ &- \frac{\partial\tau_u(t'_o)}{\partial t}\left(t'_{o\ pred} - t'\right) - \frac{1}{2}\frac{\partial^2\tau_u(t'_o)}{\partial t^2}\left(t'_{o\ pred} - t'_o\right)^2 \\ &+ \left(\frac{\partial\tau_\phi(t)}{\partial t} - \frac{\partial\tau_\phi(t_o)}{\partial t}\right) t_\Delta + \left(\frac{\partial^2\tau_\phi(t)}{\partial t^2} - \frac{\partial^2\tau_\phi(t_o)}{\partial t^2}\right)(t_\Delta^2) \\ &- \tau_{clock}(t) + \tau_{clock}(t_o) \end{aligned} \tag{22}$$

Phase Residual Measurement Errors

The measurement of the phase residuals at the Earth station appear to have an RMS error of ~ 0.001 cycles when the SNR is large. At an downlink frequency of 8 GHz this corresponds to an error of 125 femtoseconds and at 15 GHz this corresponds to 66.7 femtoseconds. The effects of the atmosphere will need to be removed from the doppler data in order to determine the orbit error. The fluctuations in the atmospheric path length which cannot be removed by any model are typically a few picoseconds. Thus we can ignore the errors in measuring the phase residuals. It should be noted that the effects of the atmosphere are only to be removed by the orbit determination group at JPL (Mark Ryne, JPL, private communication).

Estimation of the Size of the Doppler Count Error

In this section we will attempt to estimate the error in the doppler count for various realistic cases. We will neglect the range acceleration of the spacecraft in this section since the acceleration is three orders of magnitude smaller than the range rate. First we note that we can expand $\tau_u(t_1')$ in equation (13) under the assumption that the orbit error term is the dominant term in δt_o , resulting in

$$t_2' - t' = -\frac{\tau_{orbit}(t_o)}{1 + \frac{\partial \tau_u(t')}{\partial t}}. \tag{23}$$

If the range rate is much less than one then we can expand equation (23) so that

$$t_2' - t' = -\tau_{orbit}(t_o)\left[1 - \frac{\partial \tau_u(t')}{\partial t}\right]. \tag{24}$$

Inserting equations (24) and (8) into equation (22) gives us

$$\begin{aligned}
\frac{N_{error}(t)}{f_d} \sim & \left(\left[\frac{\partial \tau_u(t')}{\partial t}\right]^2 - \frac{\partial \tau_u(t')}{\partial t} - \left[\frac{\partial \tau_u(t_o')}{\partial t}\right]^2 + \frac{\partial \tau_u(t_o')}{\partial t}\right)\tau_{orbit}(t_o) \\
& + \left(\frac{\partial \tau_\phi(t)}{\partial t} - \frac{\partial \tau_\phi(t_o)}{\partial t}\right)t_\Delta + \left(\frac{\partial^2 \tau_\phi(t)}{\partial t^2} - \frac{\partial^2 \tau_\phi(t_o)}{\partial t^2}\right)(t_\Delta^2) \\
& + \frac{\partial \tau_d(t)}{\partial t}t_s(t) + \frac{\partial \tau_u(t')}{\partial t}t_s(t) - \frac{\partial \tau_u(t')}{\partial t}\frac{\partial t_s}{\partial t}D^{true}(t) \\
& - \frac{\partial \tau_d(t_o)}{\partial t}t_s(t_o) - \frac{\partial \tau_u(t_o')}{\partial t}t_s(t_o) + \frac{\partial \tau_u(t_o')}{\partial t}\frac{\partial t_s}{\partial t}D^{true}(t_o).
\end{aligned} \tag{25}$$

We will consider a satellite in an orbit similar to that of Surfsat's orbit. The height of the orbit is taken to be 1000 km above the Earth's surface. Thus the range to the satellite is about 3000 km near the horizon. Near the horizon the range rate is typically of the order of 10^{-5} *sec/sec*. We will consider a tracking pass which goes nearly directly through the zenith of the tracking station. The satellite will have a range rate of zero at some point near the time when it reaches maximum elevation. We will also assume the following: the clock offset is 50 nanoseconds; the clock drift rate is 25 nanoseconds per day; and the orbit error is 100 meters near the horizon and 10 meters at the zenith. At an downlink frequency of 8 GHz this then gives a doppler count error of $|N_{error}| \sim 0.03$ cycles if $t_\Delta = 0$. It should be noted that the effects of the clock drift are about 7 orders of magnitude less than the errors due to the clock offset so that we can ignore the effects of the clock drift (this is assuming that the clock drift has been taken into account for the phase residual time tags). The orbit errors determined at JPL are generally referred to in units of Hz. The above error can be put into these units by noting that the time it takes Surfsat to go from the horizon to the zenith is of the order of 10 minutes. This then gives the error as being of the order of 50 μHz.

The Earth station measures the phase residuals every tenth of a second. The hardware delay t_ϕ is a quantity which has not been directly measured. However, there is an estimation for this value [4] of $t_\phi = 2.6812183922 \pm 0.000001$ seconds. If no interpolation of the phase residuals was performed and the nearest (in time) measured phase residual value was used instead, then we would have a time tag error of $t_\Delta = 0.018781608$ seconds. By not interpolating we have effectively created a clock offset in the data since the range to the satellite changes very little in a time t_Δ and we can thus set $t_s(t) \sim t_\Delta$. For Surfsat, the phase residuals are nearly constant near the zenith and are of the order of 5 cycles/second near the horizon. This then gives a doppler error of $|N_{error}| \sim 3000$ cycles which corresponds to an error in determining the orbit of ~ 5 Hz. The reason that such a large error can occur is that the specification of the doppler file [2] only allows for even integer seconds in the time tags.

The previous error tells us that we have to compute the time tag for the phase residuals with much greater accuracy. The smallest that we can make t_Δ is set by how well we know t_ϕ. Since the smallest value of $t_\Delta = 10^{-6}$ seconds, we can ignore any clock errors smaller than this. The doppler error for $t_\Delta = 10^{-6}$ seconds is $|N_{error}| \sim 0.16$ cycles which corresponds to an error in determining the orbit of 270 μHz. This value represents the current fundamental limit to how accurately the orbit can be determined by the doppler data from the two-way timing data from the Earth station. This is one or two orders of magnitude smaller than the desired accuracy (Mark Ryne, JPL, private communication).

Comments on Using Conventional Doppler for Two-Way Phase Transfer

In order to reach the desired accuracy of orbit determination, the Earth station will have to interpolate the phase residual data due to the even second time tag requirement in [2] or the time tags reported in the doppler file will have to be allowed to be non-integer seconds. We believe that allowing for non-integer time tags in the doppler data would provide better orbit determination parameters since this method will not introduce and error due to the interpolation of the phase residuals.

We also note that there is a much better way of determining the true orbit than using the conventional doppler count when two-way phase transfer is used. This method is to directly use the phase residuals and the predicted ranges to the satellite. Using these values, the true uplink time and δt_o become predicted quantities of the reconstructed orbit. This then allows for the true orbit, the true uplink times and δt_o to be unambiguously and simultaneously determined. Furthermore, artificial errors are not introduced into this orbit determination scheme for two-way phase transfer unlike the conventional doppler method (see equation 21).

References

[1] "Reduction of Phase Residuals to Time Units", by L.R. D'Addario, OVLBI-ES Memo #56

[2] "TRK-2-30, DSN Tracking System; DSN Tracking Data Interface", K.M. Liewer

[3] "Post-Pass Processing of Two-Way Timing Measurements", by L.R. D'Addario, OVLBI-ES Memo #64

[4] "Details of the Post-Pass Processing of Time Correction Data", by L.R. D'Addario