Summer Student Lecture: Kurt Gordon
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SPHERICAL COORDINATE SYSTEMS

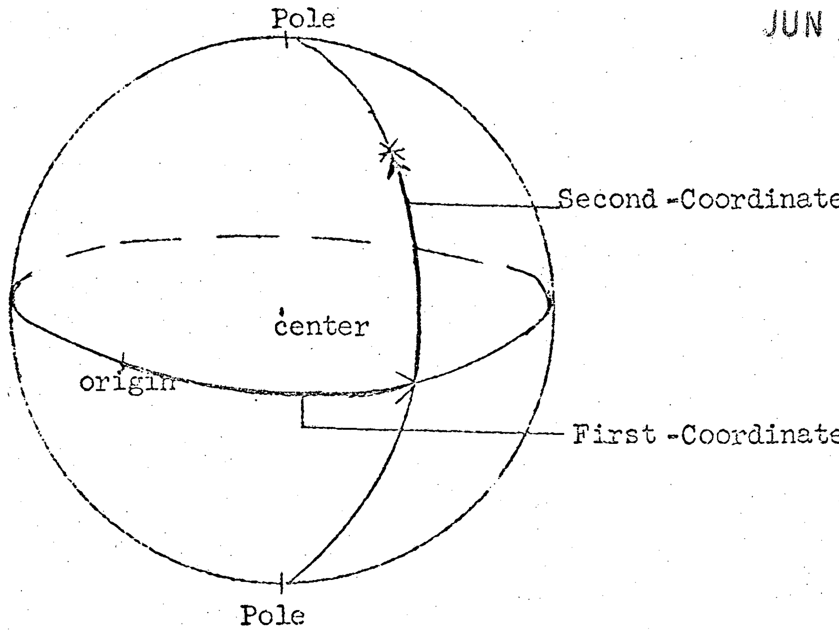


Figure 1

Great Circle: The intersection of a plane passing through the center of a sphere with the sphere.

Fundamental Circle: An arbitrary great circle of the sphere.

Poles: The two points 90 degrees from the fundamental circle.

Secondary Circles: The great circles passing through the poles and therefore perpendicular to the fundamental circle.

The Origin: An arbitrary point on the fundamental circle.

First-Coordinate: The angular distance measured from the origin along the fundamental circle, in a specific direction, to the secondary circle passing through the object. Unit of measurement.

Second-Coordinate: The angular distance measured from the fundamental circle, in a specific direction, along the secondary circle passing through the object. Unit of measurement.

(See Figure 1)

THE EQUATORIAL SYSTEM OF COORDINATES AND SIDEREAL TIME

Part I. Equatorial Coordinates:

Just as there is a system of latitude and longitude by which we can locate positions on the earth's surface, so is there a system by which positions can be located in the sky. In order to attach meaning to the terms latitude and longitude a number of definitions must first be established. We define the axis of rotation as the line about which the earth turns; the North and South poles are the points where this axis intersects the earth's surface; the equator is a circle which is everywhere 90 degrees removed from the poles; and a meridian is an arc drawn from the North to the South pole, intersecting the equator at a right angle. Longitude is measured along the equator from some arbitrary starting point. Latitude is measured north or south of the equator along a meridian. In this way, any point on the surface of the earth can be uniquely specified by a pair of numbers.

The equatorial system of coordinates is a logical extension into the sky of the system of latitude and longitude used on the earth's surface. This will become apparent from the following definitions:

Celestial Sphere: Assume the sky is represented by a very large sphere and let the earth be placed at the center of this sphere. An observer on the surface of the earth, when looking up, will see the inner surface of this globe and exactly one-half of it will be visible to him at any one time. This globe is called the celestial sphere.

North Celestial Pole: If the earth's axis of rotation is extended through the North Pole until it intersects the celestial sphere, the point of intersection is called the North Celestial Pole, abbreviated NCP. The north star is located near this point.

Celestial Equator: The celestial equator is the circle on the celestial sphere which is everywhere 90 degrees from the NCP. It can also be defined as the circle which would result if the earth's equator were expanded until it intersected the celestial sphere.

Hour Circle: An arc drawn from the NCP, perpendicular to the celestial equator, and through any particular point on the celestial sphere is the hour circle of that point. Thus the term "hour circle" corresponds to the word "meridian" on the surface of the earth.

Right Ascension (α): Right ascension corresponds to longitude on the surface of the earth. It is measured eastward along the celestial equator from a point on the equator called the Vernal Equinox (Υ). Instead of being measured in degrees, right ascension is usually measured in hours, from 0 to 24. Thus

24 hours = 360 degrees
12 hours = 180 degrees
6 hours = 90 degrees
1 hour = 15 degrees

Declination (δ): Declination corresponds to latitude on the surface of the earth. It is measured from the celestial equator, along the hour circle of the object in question. Declinations north of the equator are considered positive, while those south of the equator are considered negative. Declination is measured in degrees, being 0 at the equator, +90 degrees at the NCP, and -90 degrees at the SCP.

Example: Vega is one of the brightest stars in the sky. Its right ascension is $18^h35^m.2$ and its declination is 38°44'. In order to locate Vega in the sky, start at the Vernal Equinox and go eastward along the celestial equator for 18^h35^m. Then, go toward the NCP for 38°44', and there it is.

Part II. Sidereal Time:

The celestial sphere is pictured as being stationary in space, but since the earth is constantly turning on its axis, an observer on the earth has the impression that he is standing still and that the celestial sphere is slowly rotating from east to west. The apparent axis of rotation of the celestial sphere must coincide with the axis of rotation of the earth, so that all objects on the celestial sphere appear to rotate about the NCP. This apparent motion is called diurnal rotation.

It is evident that this apparent rotation of the celestial sphere affords an easy method of measuring the actual rotation of the earth. But the rotation of the earth is the basis of our system of measuring time. Thus the phenomenon of diurnal rotation is in itself a clock by which the astronomer can tell time.

The following definitions are now necessary:

Transit: When a star crosses the meridian it is said to transit. Upper transit is a crossing of the part of the meridian measured from the NCP to the SCP which includes the observer's zenith; lower transit is a crossing of the part of the meridian which includes the observer's nadir.

Circumpolar Star: A circumpolar star is one for which both transits are either above or below the horizon.

Observer's Meridian: The hour circle of the zenith, that is, the arc drawn from the NCP through the zenith, perpendicular to the celestial equator, is called the observer's meridian.

Hour Angle (t): The hour angle of an object on the celestial sphere is the arc along the celestial equator included between the observer's meridian and the object's hour circle. If the hour circle is west of the observer's meridian, the hour angle is positive. If the hour circle is east of the observer's meridian, the hour angle is negative.

Sidereal Time (θ): The sidereal time is defined as the hour angle of the Vernal Equinox. It may also be defined as the right ascension of the observer's meridian. The latter is usually a more practical working definition.

There is a simple relation between the right ascension and hour angle of an object and the sidereal time.

right ascension + hour angle = sidereal time

or

$$\alpha + t = \theta \quad .$$

It can easily be shown that this formula is consistent with the definitions of sidereal time above. Since the right ascension of the Vernal Equinox is zero, the formula states that the hour angle of the Vernal Equinox equals θ. Also, since the hour angle of the observer's meridian is zero, the formula states that the right ascension of the meridian equals θ.

	Earth System	Horizon System	Equatorial System
Fundamental Circle	Equator	Horizon	Celestial equator
Poles	North and South Poles	Zenith and Nadir	N.C.P. and S.C.P.
Secondary Circles	Meridians	Vertical Circles	Hour Circles
Names of coordinates	(1) Longitude (2) Latitude	(1) Azimuth (2) Altitude	(1) Right ascension (2) Declination
Origin of (1)	Greenwich, England	South point	Vernal equinox
Positive direction of (1)	West	West	East (counterclockwis
Negative direction of (1)	East	None	None
Positive direction of (2)	North of equator	North of horizon	North of Celestial equator
Negative direction of (2)	South of equator	South of horizon	South of Celestial equator
Unit of (1)	h, m, s or o, ' "	o, ', "	h, m, s
Unit of (2)	o, ', "	o, ', "	o, ', "

Note the following points:

(1) The altitude of the North Celestial Pole (NCP) equals the geographic latitude of the observer.

(2) Hour angle is the angular distance measured from the observer's meridian along the celestial equator to the hour circle passing through the object, measured in h, m, s of time. If the object is west of the meridian, its hour angle is positive; if east, negative.

The hour angle of an object can therefore also be thought of as the length of time elapsed since the object was on the observer's meridian. A negative number in this context means the amount of time until the next time the object will cross the meridian.

(3) Sidereal time equals (a) the right ascension of the observer's meridian, (b) the hour angle of the vernal equinox, (c) right ascension plus hour angle for any object (S.T. = R.A. + H.A.). It is important to remember that the sidereal time is exactly the same for all objects on a particular globe diagram.

(See Figure 2)

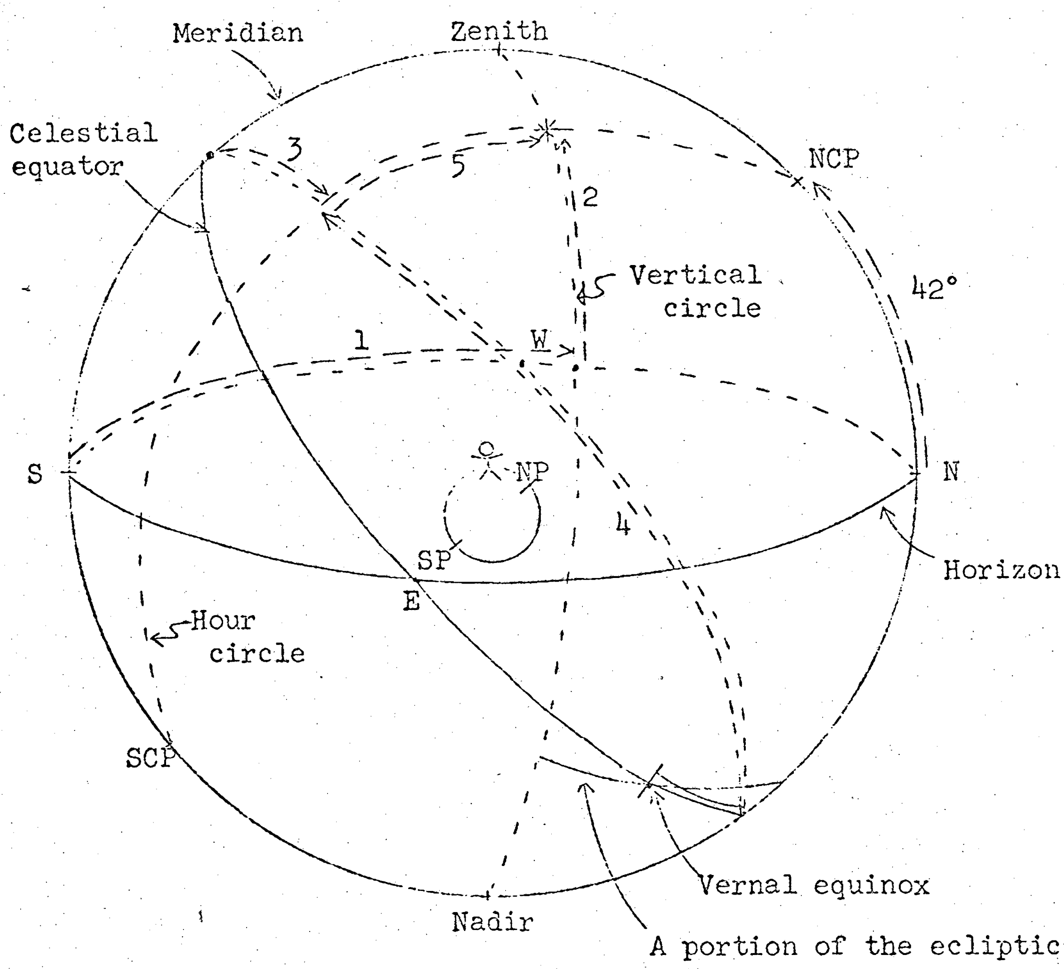


Figure 2

Celestial Sphere for Observer at Ann Arbor
for a Sidereal Time of About $13^h\ 30^m$

The numbered arrows measure:

1. Azimuth of star (about 100°)
2. Altitude of star (about +60°)
3. Hour angle of star (about $+2^h$)
4. Right ascension of star (about $11^h 30^m$)
5. Declination of star (about +45°)

Conversion from hours, minutes and seconds of time to degrees, minutes and seconds of arc.

Degrees, minutes of arc and seconds of arc describe angular measurements. The degree is defined as 1/360 of a circle. A minute of arc is equal to 1/60 of a degree. A second of arc is equal to 1/60 of a minute of arc.

The following abbreviations are commonly used and should be memorized.

degree	°	hour	h
min. of arc	'	min. of time	m
sec. of arc	"	sec. of time	s

The most important system of astronomical coordinates is the equatorial system; its fundamental circle is the celestial equator. The two coordinates are right ascension which is analogous to longitude on the earth's surface and declination which is measured north and south of the celestial equator. This and other astronomical coordinate systems will be discussed more fully in a few weeks. Declination is measured in degrees while right ascension is usually counted in units of time. The conversion from arc to time may be easily accomplished by remembering that the earth makes one complete rotation of 360° on its axis every 24 hours. Hence the following relationships hold:

$$1^h = \frac{360^\circ}{24^h} = 15^\circ \qquad 15^\circ = 60^m = 1^h$$

$$60^m = (15 \times 60)' \; ; \; 1^m = 15' \qquad 60^m = 15^\circ \; ; \; \text{hence } 1^\circ = 4^m$$

$$60^s = (15 \times 60)'' \; ; \; 1^s = 15'' \qquad 60^s = 15' \; ; \; \text{hence } 1' = 4^s$$

The Ecliptic System of Coordinates - The Seasons:

The sun, moon, and planets, even though they are apparently carried around by the diurnal rotation of the celestial sphere, are not fixed in their positions with respect to the stars. In the course of a year the earth revolves around the sun, and, as a result, the sun appears to move eastward among the stars. This apparent path of the sun around the celestial sphere is known as the ecliptic. The plane of the ecliptic makes an angle of 23-1/2° with the celestial equator. Also, the north ecliptic pole and the south ecliptic pole (NEP and SEP) will be 23-1/2° from the NCP and the SCP, respectively. The NEP and the SEP are the two points on the celestial sphere which are 90° away from the ecliptic.

The celestial equator and the ecliptic cross each other at two points. These are called the equinoxes. The point in the sky where the sun crosses the celestial equator in its journey from the south to the north is called the vernal equinox. The other point, the autumnal equinox, occurs where the sun crosses the celestial equator as it moves from the northern hemisphere of the sky to the southern. It cannot be emphasized too strongly that this motion of the sun is an apparent motion, caused by the revolution of the earth about the sun. The two points on the ecliptic half-way between the equinoxes are called the solstices. The one north of the celestial equator is the summer solstice; the one south of the equator the winter solstice.

Celestial longitude (λ) is the arc on the ecliptic between the vernal equinox and the foot of a great circle drawn from the pole of the ecliptic through the body perpendicular to the ecliptic. Celestial longitude is measured eastward from the vernal equinox just as is right ascension, but it is measured in degrees instead of hours.

Celestial latitude (β) is the arc of the great circle, also in degrees, from the ecliptic to the body. It is positive northward and negative southward.

The Seasons. The earth's axis of rotation is not perpendicular to the plane of the earth's orbit about the sun, but makes an angle of 23-1/2° with this perpendicular. The axis of rotation always remains pointed in the same direction in space. Consequently, during part of the year the earth's north pole is tipped toward the sun and the northern hemisphere experiences summer; while during the other part of the year the earth's south pole is tipped toward the sun. Thus the season at some particular position on the earth depends on the position of the earth in its orbit. The fact that the earth is slightly closer to the sun about January 1 and that therefore the earth as a whole receives just a little more heat near this time has very little effect on the seasons.

The amount of heat received in 24 hours at a given point on the earth depends upon the intensity of the sunlight and upon the duration of daylight. Both vary during the year. The sunlight is more intense the higher the altitude of the sun, for when the sunlight strikes the ground obliquely, a given amount of radiation spreads over more surface and thus is less effective in heating it than when the sun is at a greater altitude. When the sun is north of the equator, it is summer in the northern hemisphere, for the sun shines more nearly vertically down and the days are also longer.

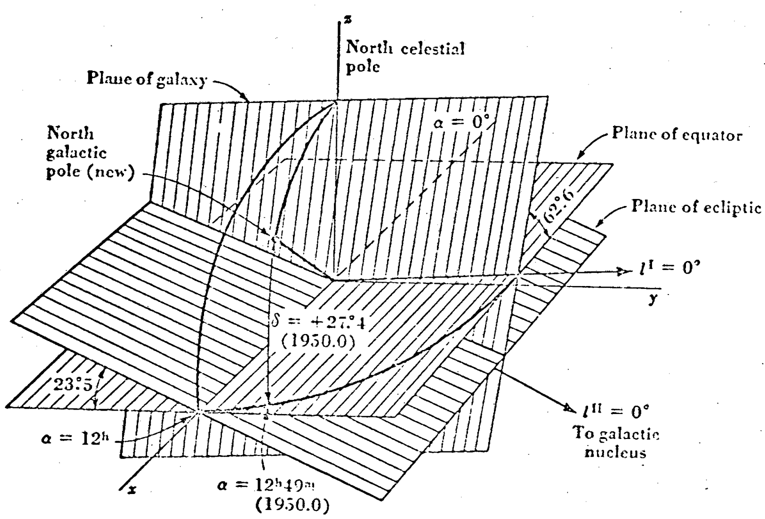
GALACTIC COORDINATES

Just as the majority of members of the solar system concentrate toward the plane of the ecliptic, most of the stars, dust, and gas in our galaxy concentrate toward the galactic plane to produce the luminous band of the Milky Way. Thus a coordinate system with a fundamental circle defined by the Milky Way is a natural coordinate system for stellar investigations, as the ecliptic system is a natural system for planetary studies, and the equatorial system is a natural system for telescope mountings and for studies of time. The usefulness of a system of galactic coordinates was recognized by Sir William Herschel as early as 1785, and several attempts have been made to set up such a system. The difficulty in setting up a galactic coordinate system stems from the fact that the Milky Way is several degrees wide with irregular outlines, making the determination of the fundamental circle from optical observations quite uncertain. The most recent galactic coordinate system based entirely on optical measurements is the Ohlsson system, in use from ca. 1930-1960. Since the gas in our galaxy has a much flatter distribution than do the stars, radio-astronomical observations gave an improved determination of the location of the galactic plane. A revised system of galactic coordinates, set up at the request of the IAU, was described in a series of five papers in the Monthly Notices of the Royal Astronomical Society, vol. 121 (1960), and has gained worldwide acceptance. Tables for conversion between 1950 equatorial and IAU galactic coordinates and between IAU and Ohlsson galactic coordinates have been published in volumes 15-17 of the Lund Observatory Annals. The fundamental circles of the two systems of galactic coordinates intersect at an angle of $1^{\circ}.5$.

Galactic longitude (ℓ) is the arc on the galactic circle between the galactic center and the foot of the great circle drawn from the galactic pole through the body perpendicular to the galactic circle. Galactic longitude is measured eastward, in degrees.

Galactic latitude (b) is the arc of the great circle, also in degrees, measured from the galactic circle to the object. It is positive toward the NGP (the galactic pole north of the celestial equator) and negative toward the SGP.

Since the galactic coordinate system is based upon concrete celestial objects rather than the motions of the earth, coordinates of objects in the galactic system are not affected by precession. Both the equatorial and ecliptic systems, with their origin at the vernal equinox, are affected by precession.



Relation of plane of the galaxy to the plane of the earth's orbit (ecliptic) and plane of the earth's equator ($\delta = 0$).

SUMMARY: COORDINATES

I. Horizon System

Vertical circle - the great circle passing through the zenith, the nadir, perpendicular to the horizon, and passing through the body.

Altitude or elevation (h) - measured from the horizon to the body along the vertical circle, in degrees, positive toward the zenith.

Azimuth (A) - measured from the South point to the vertical circle along the horizon, in degrees, Westward (S point → W point → N → E). Actually, there is no universal agreement on the conventions for measuring azimuth, and one may find azimuths measured either eastward or westward from either the N or S points of the horizon.

Latitude of the observer = altitude of the celestial pole, in degrees N or S depending on which pole is visible.

II. Equator System

Hour circle - the great circle passing through the N.C.P. and the S.C.P., perpendicular to the equator, and passing through the body.

Declination (δ) - measured from the equator to the body along the hour circle, in degrees, positive Northward.

Right ascension (α) - measured from the Vernal Equinox to the hour circle along the equator, in hours, Eastward (opposite to the direction in which the celestial sphere rotates).

Hour angle (t) - measured from the meridian to the hour circle along the equator, in hours, positive Westward. Usually taken as 12 hours or less, positive or negative. (Note: it is the only coordinate in the equatorial, ecliptic, and galactic systems measured in the direction of the sky's rotation.)

Sidereal time (θ) - defined as the hour angle of the Vernal Equinox. It may also be defined as the right ascension of the meridian. Right ascension + hour angle = sidereal time ($\alpha + t = \theta$).

III. Ecliptic System

Celestial latitude (β) - measured from the ecliptic to the body along the secondary ecliptic circle, in degrees, positive Northward.

Celestial longitude (λ) - measured from the vernal equinox to the secondary ecliptic circle, along the ecliptic, in degrees, Eastward.

IV. Galactic System: IAU or "new" system

Galactic latitude (b) - measured from the galactic circle (plane) to the body along the circle perpendicular to the galactic circle and through the 2 poles and body; in degrees, positive Northward.

Galactic longitude (ℓ) - measured from the galactic center along the galactic circle to the body, in degrees, Eastward.

POSITIONS OF THE SUN

DATE	EQUATORIAL SYSTEM		ECLIPTIC SYSTEM	
	R.A.	Dec.	Longitude	Latitude
March 21	0^h	0°	0°	0°
June 21	6^h	+ 23-1/2°	90°	0°
September 22	12^h	0°	180°	0°
December 21	18^h	- 23-1/2°	270°	0°
	These positions are in the equatorial system and are therefore measured with respect to the celestial equator and the N and SCP		These positions are in the ecliptic system and are therefore measured with respect to the ecliptic and the N and SEP.	

COORDINATES OF FUNDAMENTAL POINTS IN VARIOUS SYSTEMS

SYSTEM	HORIZON		EQUATORIAL		ECLIPTIC		GALACTIC	
	A	h	α	δ	λ	β	ℓ	b
NCP	180°	+38-1/2°*	-	+90°	90°	+66-1/2°	123°	+27°.4
SCP	0°	-38-1/2°*	-	-90°	270°	-66-1/2°	303°	-27°.4
NEP	V	V	18^h	+66-1/2°	-	+90°	96°.3	+29°.8
SEP	V	V	6^h	-66-1/2°	-	-90°	276°.3	-29°.8
Zenith	-	+90°	V	+38-1/2°*	V	V	V	V
Nadir	-	-90°	V	-38-1/2°*	V	V	V	V
NGP	V	V	12^h49^m	+27°.4			-	+90°
SGP	V	V	0^h49^m	-27°.4			-	-90°
Vernal equinox	V	V	0^h	0°	0°	0°	97-3/4°	-60°.2
Galactic center	V	V	17^h42^m	-28°.9			0°	0°

Note: * = this value for an observer at Green Bank
- = the value of this coordinate is indeterminate
V = this value varies with the time of day