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INTRODUCTION TO RADIO ASTRONOMY

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Radio astronomers measure, analyze, and interpret the radio radiation from sources outside the atmosphere of the Earth. Most of the radio sources of interest are outside our solar system.

It is important to emphasize that an astronomer can do no real experimentation. He is entirely limited to measuring the intensity (and possibly polarization) of distant radiation sources as a function of frequency, position, and time.

THE RADIO WINDOW

Radio astronomy takes advantage of the "radio window" in our atmosphere.

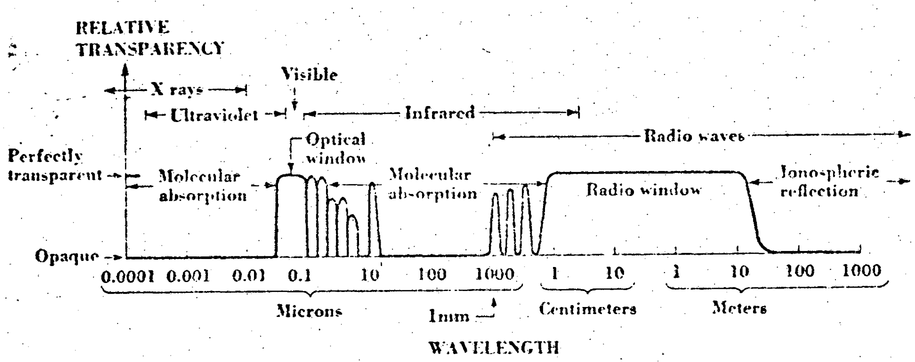
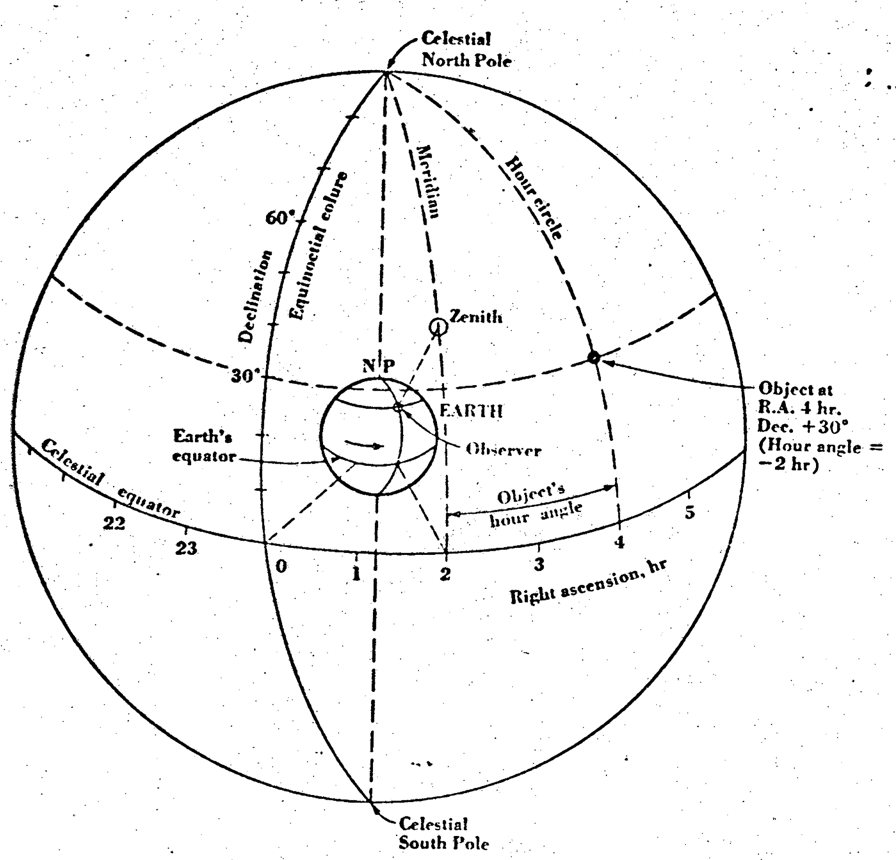


Fig. 1-1. Electromagnetic spectrum showing relative transparency of the earth's atmosphere and ionosphere.

The figure shows the relative transparency of the Earth's atmosphere to radiation of different wavelengths. Until recently we have dealt exclusively with either the optical window or the radio window. In the last few years infra-red astronomy (0.1μ to 10μ) and mm-wave astronomy has grown tremendously. For example, only a few weeks ago molecular lines of CO and CN were found with the NRAO mm-wave antenna at Tucson. However, most of radio astronomy still deals with the wavelength range from two cm to about 20 m.

COORDINATES

Before going on to more interesting topics, let us pause and give a moments attention to the problem of specifying the position of a radio source in the sky. This means choosing a convenient coordinate system.



The most important coordinate system is the "equatorial" which is based on the properties of the Earth's rotation. The above figure defines all the details. Basically, the equator of the Earth defines a plane and all coordinates are measured with respect to this plane and the Earth's axis, which is perpendicular to this plane. The essential definition of these coordinates is based on the definition whereby every object in the sky is said to return to a particular position every 24 hours. For this reason, the celestial equator, which is the imaginary projection of the Earth's equator upon the sky, is divided into 24 hours, with 0^h corresponding to a particular point in the sky. This 0^h point could be chosen arbitrarily, but is chosen on the basis of the point (one of the two) in the sky where the celestial equator intersects the plane in which the solar system moves.

Consider any point in the sky. The line to this point will project onto the equatorial plane. Counting eastward from the 0^h point, the amount of the 24 hour scale needed to reach the projection is called the right ascension of the point in the sky. Since the point will usually not be on the celestial equator, one then measures, in degrees, the NS angle the point makes with respect to the celestial equator; this angle is the declination of the point in the sky. In the above figure an object which has a right ascension of 4^h and a declination of +30° is indicated.

Of course, while the coordinate system is fixed in the sky, the Earth rotates with respect to this coordinate system. For a particular position on the Earth at a particular time, the meridian has a right ascension which is called the sidereal time. In the above figure the observer is at 2^h sidereal time.

At any particular time, any object in the sky will have a particular hour angle, which is the difference between the right ascension of the object and the sidereal time (right ascension of the meridian).

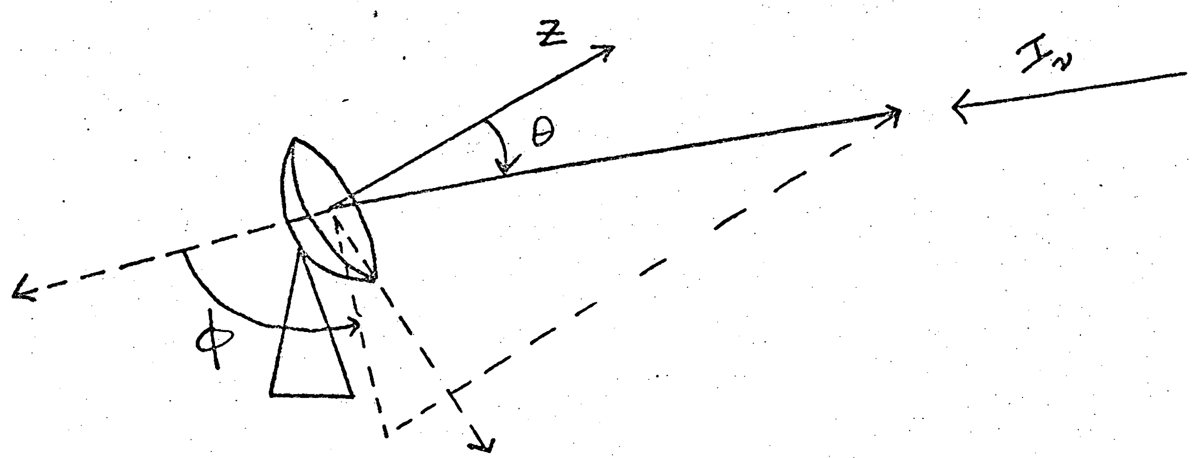
As you know, sidereal time is different from the time we normally use, which is based upon the movements of the Sun. The sidereal day differs from the solar day by about four minutes. At the autumnal equinox, in the fall, the two times agree, but thereafter they shift with respect to each other by about 4 minutes a day.

POWER ABSORBED BY A RADIO TELESCOPE

Let us describe the radiation that comes to us from the sky in terms of an intensity, I_ν. We define

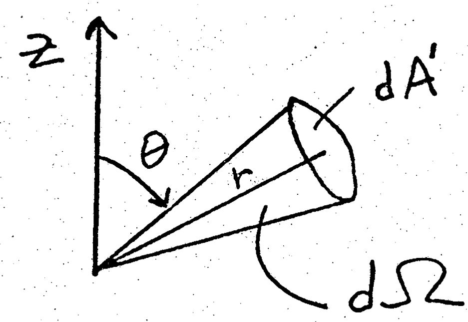
$$I_\nu d\nu\ dA\ d\Omega\ dt$$

to be the amount of energy in the frequency interval ν to $\nu+d\nu$ which passes through an area dA into a solid angle $d\Omega$ oriented about a particular direction, during the time interval dt. The units of I_ν are watts m^{-2} sterad $^{-1}Hz^{-1}$.



We adopt a coordinate system with the z - axis perpendicular to the surface of a radio telescope. In this system every point in the sky has coordinates (Θ,Φ).

Consider an area dA' which is a distance r from the telescope. The solid angle $d\Omega$



is defined by

$$d\Omega = \frac{dA'}{r^2} .$$

A source in the sky occupying a solid angle Ω_s is then said to have a flux density, S_ν, where

$$S_\nu = \int_{\Omega_s} I_\nu(\Theta,\phi)d\Omega \text{ watts } m^{-2}Hz^{-1}. \qquad (1)$$

Let us consider radiation in the frequency range ν to $\nu+d\nu$ incident upon an element of surface area, dA, of a telescope of area A. The power incident upon dA from a solid angle $d\Omega$ is given by

$$dP_{inc} = I_\nu(\Theta,\phi) \cos \Theta dA \; d\Omega \; d\nu \quad \text{watts.}$$

The total power incident on the telescope is then

$$P_{inc} = \int_{antenna} \iint_{4\pi} \int_{\nu}^{\nu+d\nu} I_\nu(\Theta,\phi) \cos\Theta \; dA \; d\Omega \; d\nu. \qquad (2)$$

If I_ν is uniform over the observing band-width, $\Delta\nu$,

$$P_{inc} = \Delta\nu\, A \int_{4\pi} I_\nu\, (\Theta,\phi)\, \cos\,\Theta d\Omega. \tag{3}$$

In the special case where I_ν is uniform over a small source and can be considered as zero elsewhere, we can use equation (1) to obtain

$$P_{inc} = \Delta\nu\, A\, \cos\,\Theta\, S_\nu\,. \tag{4}$$

For all real antennas, not all of the incident power is absorbed. One can then write

$$P_a = 1/2\, \Delta\nu \int_{4\pi} A_e\, (\Theta,\phi)\, I_\nu\, (\Theta,\phi)\, d\Omega \tag{5}$$

where the factor 1/2 indicates that typically only one polarization is absorbed and $A_e\, (\Theta,\phi)$ describes the absorption properties of the antenna in terms of an effective area.

Let us define,

$$A_e = A_e\, (\Theta=0\,,\phi=0) = \text{effective aperture,}$$

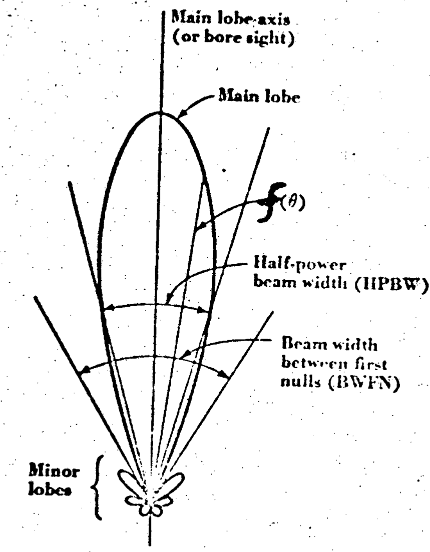
then

$$f(\Theta,\phi) = \frac{A_e\, (\Theta,\phi)}{A_e} = \text{antenna pattern.}$$

In these terms,

$$P_a = 1/2\, \Delta\nu\, A_e \int_{4\pi} I_\nu(\Theta,\phi) f(\Theta,\phi)\, d\Omega \tag{6}$$

For a typical antenna, a cross-section of the antenna pattern is shown at the right.



The following quantities describe various properties of the antenna:

$$\Omega_A = \int_{4\pi} f(\Theta,\phi)\,d\Omega = \text{beam solid angle}$$

$$\Omega_M = \int_{\substack{\text{MAIN}\\\text{LOBE}}} f(\Theta,\phi)\,d\Omega = \text{main beam solid angle}$$

$$\varepsilon_a = \frac{A_e}{A} = \text{aperture efficiency}$$

$$\varepsilon_M = \frac{\Omega_M}{\Omega_A} = \text{beam efficiency}$$

$$D = \frac{4\pi\ A_e}{\int_{4\pi} A_e(\Theta,\phi)\,d\Omega} = \frac{4\pi}{\int_{4\pi} f(\Theta,\phi)\,d\Omega} = \frac{4\pi}{\Omega_A} = \text{directivity.}$$

A further relation which may be derived is

$$A_e\ \Omega_A = \lambda^2$$

where λ is the observing wavelength. Using this we can re-write equation (6) as

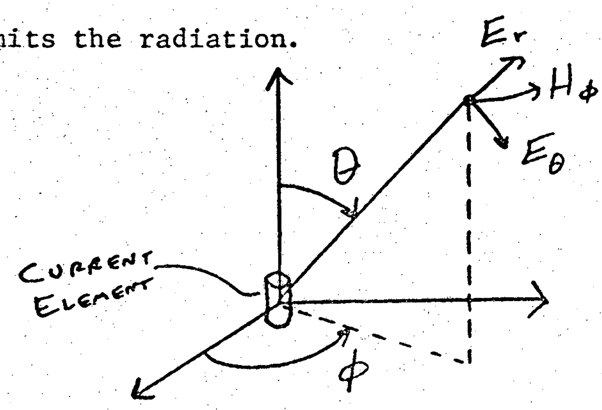
$$P_a = 1/2\ \frac{\lambda^2}{\Omega_A}\ \Delta\nu \int_{4\pi} I_\nu(\Theta,\phi)\ f\ (\Theta,\phi)\ d\Omega\ . \tag{7}$$

ABSORPTION AND EMISSION BY AN ANTENNA

What happens to the power absorbed by an antenna? We will discuss almost all of the relevant physics in terms of a very simple problem: the radiation emitted and absorbed by a current element of length Δz carrying a sinusoidal current

$$I = I_o e^{i\omega t}$$

where $\omega = 2\pi\nu$. We will first discuss the case where we supply the current and the current element emits the radiation.



For such a current element one can solve Maxwell's equations for the E and H vectors:

$$E = (E_r, E_\theta, E_\phi), \quad H = (H_r, H_\theta, H_\phi)$$

obtaining

$$E_r = \frac{I_o \Delta z \cos\Theta}{2\pi \varepsilon_o} \left(\frac{1}{cr^2} + \frac{1}{i\omega r^3} \right) e^{i\omega(t - \frac{r}{c})}$$

$$E_\Theta = \frac{I_o \Delta z \sin\Theta}{4\pi \varepsilon_o} \left(\frac{i\omega}{c^2 r} + \frac{1}{cr^2} - \frac{i}{\omega r^3} \right) e^{i\omega(t - \frac{r}{c})}$$

$$H_\phi = \frac{I_o \Delta z \sin\Theta}{4\pi} \left(\frac{i\omega}{cr} + \frac{1}{r^2} \right) e^{i\omega(t - \frac{r}{c})}$$

$$E_\phi = H_r = H_\Theta = 0.$$

Examining these solutions, we see terms depending upon r^{-1}, r^{-2}, and r^{-3}. Obviously, as r becomes large, terms involving r^{-1} will dominate the solutions. These terms are called components of the "far field":

$$E_\Theta = \frac{i\omega\, I_o\, \Delta z \sin\Theta}{4\pi\, \varepsilon_o\, c^2 r}\, e^{i\omega(t - \frac{r}{c})}$$

$$H_\phi = \frac{i\omega\, I_o\, \Delta z \sin\Theta}{4\pi\, cr}\, e^{i\omega(t - \frac{r}{c})}$$

and all other components are part of the "near field". For obvious reasons, all astronomical objects at large distances will be in the far field.

Note that $\frac{E_\Theta}{H_\phi} = \frac{1}{\varepsilon_o c} = \sqrt{\varepsilon_o \mu_o}\ \frac{1}{\varepsilon_o} = \sqrt{\frac{\mu_o}{\varepsilon_o}} = z_o = 377$ ohm

which is a general result for the far field.

Now let

$\overline{S}$ = time average energy flux (watts m^{-2} Hz^{-1})

$$\overline{S} = \frac{1}{2} R_e\, (E_\Theta H_\phi{}^* - E_\phi H_\Theta{}^*) = \frac{1}{2}\, E_\Theta H_\phi{}^*$$

$$\overline{S} = \frac{(I_o\, \Delta z \sin\Theta)^2\, \omega^2}{32\pi^2 \varepsilon_o\, c^3\, r^2}$$

The power passing through an area dA is

$$dP = \overline{S}dA = \frac{I_o \ \Delta z \ \sin \ \Theta)^2 \ \omega^2 dA}{32\pi^2 \ \varepsilon_o \ c^3 \ r^2}$$

$$dP = \frac{(I_o \ \Delta z \ \sin \Theta)^2 \quad \omega^2 d\Omega}{32\pi \ \varepsilon_o \ c^3}$$

$$\frac{dP}{d\Omega} = \frac{15\pi}{\lambda^2} (I_o \ \Delta z \ \sin \Theta)^2.$$

Now $$f\ (\Theta,\phi) = \frac{dP/d\Omega}{(dP/d\Omega)_{max}} = \sin^2 \Theta$$

hence $$\Omega_A = \int_{4\pi} f(\Theta,\phi)\ d\Omega = \frac{8\pi}{3}$$

$$D = \frac{4\pi}{\Omega_A} = \frac{3}{2}$$

$$\sin^2\Theta = \frac{1}{2} \quad \text{at } \Theta = 45°$$

hence HPBW = 90°.

The total power emitted by the current element, P_e, is

$$P_e = \int_{4\pi} \left(\frac{dP}{d\Omega}\right) d\Omega = \frac{15\pi}{\lambda^2} (I_o \ \Delta z)^2 \int_{4\pi} \sin^2\Theta \, d\Omega$$

$$= \frac{15\pi}{\lambda^2} (I_o \ \Delta z)^2 \ \Omega_A$$

$$P_e = \frac{40\pi^2}{\lambda^2} (I_o \ \Delta z)^2.$$

Now all of this has been very simple. We are basically saying that electrons in an alternating current (with frequency ν) element radiate into space at this frequency with a radiation pattern for which we have calculated the properties.

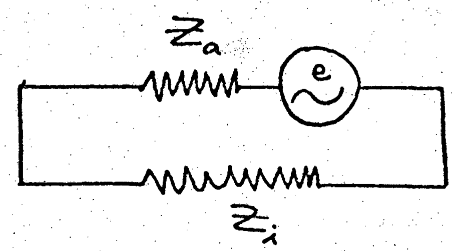
The reciprocal statement is also true. Radiation from space with a frequency ν will induce in a current element an alternating current with frequency ν. The receiving antenna pattern $(f(\Theta, \phi))$ and its parameters are the same as for a radiating current element.

This is what happens to the power absorbed by an antenna; if P_a is the power put into the electron currents induced in the system

$$P_a = P_e .$$

All useful radio telescopes utilize the same principles. They generally consist of a dipole current element which is placed at the focal point of a reflecting surface. A dipole is simply a current element which is optimized for maximum absorption properties at a particular wavelength.

We can discuss many of the important properties of radio telescope systems in terms of an electrical circuit in which the power observed by the antenna appears as a source of emf.



The antenna has an impedance $Z_a = R_a + iX_a$, and is matched with a receiver with impedance Z_i (including all transmission lines). The receiver has a function which will be clear in a moment, though we will not discuss many of the details.

In the above circuit the current I is given by

$$I = \frac{e}{Z_i + Z_a}$$

and

$$P_a = |I|^2 R_a = \frac{\overline{e^2}}{|Z_i + Z_a|^2} R_a .$$

In a properly matched system, the impedances are chosen so that $|Z_i + Z_a|^2$ is a minimum, so the absorbed power is maximized; this occurs when

$$R_i = R_a$$

$$X_i = -X_a .$$

hence

$$P_a = \frac{\overline{e^2}}{4R_a} .$$

Clearly knowing R_a and measuring $\overline{e^2}$ will give a measure of P_a.

We now invoke Nyquist's law:

In any resistor, the r.m.s. voltage is related to the temperature of the resistor by

$$\overline{e^2} = 4kT\Delta\nu R$$

where $\Delta\nu$ is the band-width of frequencies propogating in the resistor.

Hence

$$P_a = kT_a \, \Delta\nu \qquad (8)$$

where T_a is called the antenna temperature. This is the basic reason for the preoccupation of astronomers with measuring everything in terms of temperatures.

Let

P_i = internal noise power when telescope is not absorbing radiation.

and

$T_i = \frac{P_i}{k\Delta\nu}$ = corresponding internal temperature.

When observing

$$P = P_a + P_i \quad , \quad T = T_a + T_i,$$

but usually

$$P_a << P_i \quad \text{and} \quad T_a << T_i.$$

Hence internal noise dominates the power in the system. This means that one must be very careful when separating a signal from the noise fluctuations of the system.

Now fluctuations of radio noise in the system are random events. According to the statistics of random events, n measurements of a randonly fluctuating quantity will typically deviate from the mean by a fraction of the order of $n^{-1/2}$.

The use of a pass-band $\Delta\nu$ corresponds to taking $\Delta\nu$ separate measurements per second. Thus, in a time τ, $\Delta\nu\tau$ independent measurements are taken and averaged together. Thus the deviation from the mean (ΔP, ΔT) will be given by

$$\frac{\Delta P}{P} = \frac{\Delta T}{T} \cong (\Delta\nu\tau)^{-1/2} .$$

The time τ is the integration time of the receiver.

Without the telescope absorbing power

$$P \cong \Delta P \cong \frac{P_i}{\sqrt{\Delta\nu\tau}} .$$

With the telescope absorbing power, the signal mixes with the noise. Clearly, unless

$$P_a > \frac{P_i}{\sqrt{\Delta\nu\tau}}$$

the noise fluctuations will dominate over any signal. It is usual to say that for reliable detection of a signal one must have

$$P_a \geq \frac{5\ P_i}{\sqrt{\Delta\nu\tau}}$$

or

$$T_a \geq \frac{5\ T_i}{\sqrt{\Delta\nu\tau}} .$$

In principle, one would like to make $\Delta\nu$ and τ as large as possible, but of course, there are limitations which we will not go into.

RELATION BETWEEN OBSERVATIONS AND THE OBSERVED

The starting point for the analysis of any observations is equation (7):

$$P_a = \frac{1}{2} \Delta\nu \frac{\lambda^2}{\Omega_A} \int_{4\pi} I_\nu(\Theta,\phi) f(\Theta,\phi) d\Omega \tag{7}$$

where $I_\nu(\Theta,\phi)$ represents all the accessible information about the observable universe and P_a is the directly measured quantity. However, before proceeding any further, let us transform to the temperature domain used in radio astronomy.

Since (from equation 8)

$$P_a = kT_a \Delta\nu, \tag{8}$$

we can take T_a as the primary observable, then

$$T_a = \frac{\lambda^2}{2k} \frac{1}{\Omega_A} \int_{4\pi} I_\nu(\Theta,\phi) f(\Theta,\phi) d\Omega. \tag{9}$$

If we then formally transform the intensity to a temperature by defining

$$T_b = \text{brightness temperature} = \frac{\lambda^2}{2k} I_\nu ,$$

then

$$T_b = \frac{1}{\Omega_A} \int_{4\pi} T_b (\Theta, \phi) f(\Theta, \phi) d\Omega . \tag{10}$$

Because the convention is so well established, one must be accustomed to thinking of intensity in terms of a temperature in the sky.

In principle the observational problem is to obtain maps of T_a in the sky by measuring all points in the sky with the radio telescope. Then one solves for $T_b(\Theta, \phi)$ using equation (10). In practice things can be very complicated, mainly because (1) all radiation over angles smaller than the HPBW is smeared together and (2) radiation from the whole sky reaches the antenna.

Simple cases:

1) Observations of a small source with uniform T_b over Ω_s, with T_b negligible elsewhere:

$$T_a = \frac{\Omega_s}{\Omega_A} T_b \quad , \quad T_b = \frac{\Omega_A}{\Omega_s} T_a$$

2) Observations of a small uniform source for which $\Omega_s >> \Omega_M$, then

$$T_a = \frac{T_b}{\Omega_A} \int_{\substack{\text{MAIN}\\\text{LOBE}}} f\ (\Theta,\phi)\ d\Omega$$

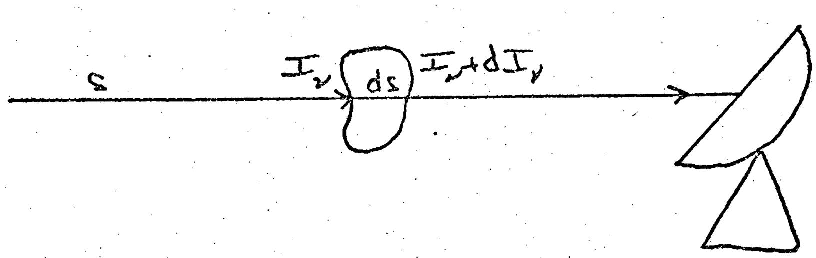
$$= \frac{\Omega_M}{\Omega_A} T_b$$

$$T_b = \frac{T_a}{\varepsilon_B}$$

RELATION BETWEEN I_ν (or T_b) AND THE UNIVERSE

Can we say anything general about how I_ν (or T_b) relates to the universe? In general terms the answer is yes, because we can relate I_ν to functions that depend only on the physical conditions at particular points in space.

Consider an element of matter in space,



and let s be the line of sight coordinate which passes through the element of matter in a distance ds. We can say that the change in I_ν produced by the element of matter is

$$dI_\nu = \text{emission} - \text{absorption.}$$

Now the emission and absorption properties of matter are well known and can be described by a mass emission coefficient, j_ν, and a mass absorption coefficient, κ_ν, then, if ρ is the density in the element of matter,

$$dI_\nu = j_\nu \rho ds - \kappa_\nu \rho I_\nu ds$$

hence

$$\frac{dI_\nu}{ds} = j_\nu \rho - \kappa_\nu \rho I_\nu \quad .$$

If we define

$$d\tau_\nu = \kappa_\nu \rho ds,$$

where τ_ν is the so-called optical depth, then

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\kappa_\nu} - I_\nu \quad .$$

Now

$$e^{\tau_\nu} \frac{dI_\nu}{d\tau_\nu} + e^{\tau_\nu} I_\nu = \frac{j_\nu}{\kappa_\nu} e^{\tau_\nu}$$

$$\frac{d}{d\tau_\nu} (e^{\tau_\nu} I_\nu) = \frac{j_\nu}{\kappa_\nu} e^{\tau_\nu},$$

hence integrating from $\tau = \tau_\nu^{(1)}$ where $I_\nu = I_\nu^{(1)}$ to $\tau_\nu = \tau_\nu^{(2)}$ where $I_\nu = I_\nu^{(2)}$, we get

$$e^{\tau_\nu^{(1)}} I_\nu^{(1)} - e^{\tau_\nu^{(2)}} I_\nu^{(2)} = \int_{\tau_\nu^{(2)}}^{\tau_\nu^{(1)}} \frac{j_\nu}{\kappa_\nu} e^{\tau_\nu} d\tau_\nu$$

$$I_\nu^{(2)} = I_\nu^{(1)} e^{-[\tau_\nu^{(2)} - \tau_\nu^{(1)}]} + \int_{\tau_\nu^{(1)}}^{\tau_\nu^{(2)}} \frac{j_\nu}{\kappa_\nu} e^{-[\tau_\nu^{(2)} - \tau_\nu]} d\tau_\nu \quad (11)$$

where $$\tau_\nu = \int_0^s \kappa_\nu \rho ds .$$

If we let

$$I_\nu = \frac{2kT_b}{\lambda^2}$$

and

$$\frac{j_\nu}{\kappa_\nu} = \frac{2kT_s}{\lambda^2}$$

and

$\tau_\nu^{(1)} = 0$, then

$$T_b^{(2)} = T_b^{(1)}\, e^{-\tau_\nu^{(2)}} + \int_0^{\tau_\nu^{(2)}} T_s\, e^{-[\tau_\nu^{(2)} - \tau_\nu]}\, d\tau_\nu \quad . \qquad (12)$$

If the source is uniform

$$T_b^{(2)} = T_b^{(1)}\, e^{-\tau_\nu^{(2)}} + T_s \left[1 - e^{-\tau_\nu^{(2)}}\right] \qquad (13)$$

and if $\tau_\nu^{(2)} < < 1$ $\quad T_b^{(2)} = T_b^{(1)} + T_s \ \tau_\nu^{(2)}$

while if $\tau_\nu^{(2)} >> 1$ $\quad T_b^{(2)} = T_s$.

All that we want to know about the nature of the universe is tied up in either

(1) κ_ν, j_ν, ρ or (2) τ_ν , T_s.

In general

κ_ν, j_ν = f (temperature, density, radiation fields, collision processes, position, time, etc.)

and it is this list of parameters that affect κ_ν and j_ν that we would like to determine.

Taking equation (11) in a form where S = 0 refers to the "far side of the Universe" it can be combined with equation (10) to give

$$T_a = \frac{1}{\Omega_A} \int_{4\pi} \int_{0}^{\mathrm{EARTH}} \frac{\lambda^2}{2k} \, j_\nu \rho \, \exp\left[- \int_0^s \kappa_\nu \rho ds \right] ds \, f\,(\Theta,\phi) \, d\Omega \quad . \tag{14}$$

This is the basic equation which relates all that we can measure to what we would like to determine.