US/GRBK)

Lecture Notes

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THE LITTLEST RADIO SOURCE

I. Introduction

The development of very long baseline (VLB) interferometry has enabled radio astronomers to achieve virtually unlimited spatial resolution with their telescopes. With it the question arises of just how much resolution is necessary to resolve the finest details in the structure of radio sources. This question is important in planning VLB experiments because an experiment to study source structure is of little value if, on the baseline chosen, the sources are either completely resolved or completely unresolved. On baselines separated by more than a few tens of degrees on the earth, it is not possible to get more than $\sim$ 50% foreshortening of the baseline and still have a source above the horizons of both telescopes; so tracking a source over large hour angles will not circumvent the problem of choosing the optimum baseline. Another important aspect of this question is that presently the length of baselines are limited by the size of the earth. The difficulty and expense of bypassing this limitation by placing a VLB interferometer in space or on the moon would hardly be justified if sources were completely resolved on such extraterrestrial baselines. In this case it would be worse than shooting a gnat with an elephant gun -- it would be like shooting at a gnat with an elephant gun and missing.

In this paper we consider four effects which will place a limit on how small a radio source can appear to be; they are: 1) interstellar scattering, 2) interplanetary scattering, 3) synchrotron self-absorption, and 4) the inverse Compton effect. The former two effects act on radio waves as they pass through the intervening medium between the source and observer and are independent of the nature of the source. The latter two concern only sources of incoherent synchrotron radiation and are applicable to compact extragalactic radio sources. Currently, VLB experiments are being used to study in more detail these four effects so that the numbers used in this paper will undoubtedly be changed, but from previous work I expect the changes will not be considerable and will not change the qualitative conclusions drawn here.

II. Interstellar Scattering

Interstellar scattering (ISS) has been invoked to explain the long term ($\sim$ minutes) variations of pulsar pulse heights and also their time dependent frequency structure. Recent comprehensive studies of the effects of ISS on pulsars have been made by B. J. Rickett (MNRAS, 150, 67 (1970)) and K. R. Lang (Ap. J., 164, 249 (1971)).

Summarizing the theory: a wave of length λ passing through a blob of size a with a deviant electron density Δn from the average density n undergoes a phase change

$$\Delta\phi = \Delta n \, r_e \, \lambda a \quad \text{where} \quad r_e = \frac{e^2 m}{c^2} = 2.8 \times 10^{-13} \text{ cm}.$$

Through a thickness L of blobs an r.m.s. phase change of

$$\phi_o = \left(\frac{L}{a}\right)^{1/2} \left< \Delta n^2 \right>^{1/2}$$

occurs. Finally this yields an r.m.s. scattering angle of

$$\theta_s = \left(\frac{\lambda}{2\pi a}\right) \phi_o = \frac{1}{2\rho} \left(\frac{L}{a}\right)^{1/2} \left< \Delta n^2 \right>^{1/2} r_e \lambda^2 .$$

The time delay of a ray coming from an angle θ_s with respect to one suffering no deflection is

$$\tau \approx \left(\frac{L\theta_s^2}{8c}\right)$$

(cf. Fig. 1).

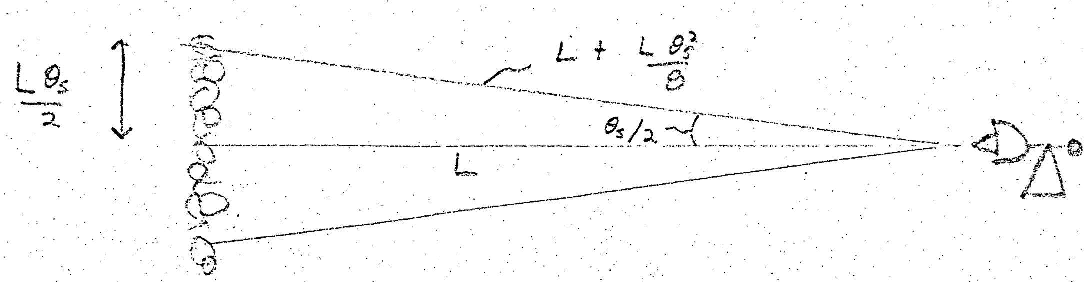


Figure 1.

This has two observable effects: 1) a pulse of duration less than τ seconds will be smeared out over a duration τ, and 2) for observing bandwidths $f_\nu < \tau^{-1}$ fluctuations in the received signal will appear or, alternatively, the spectra will have features of width f_ν. It is from these effects that the parameters for ISS are measured.

If there is relative motion between the earth-medium-pulsar, the features will appear to drift and also cause the long time variations in the pulse strengths through the relation

$$\tau = \frac{\lambda}{v\theta_s}$$

where v is the relative velocity through the medium perpendicular to a line joining the earth and pulsar.

All this is caused by the diffraction pattern on the ground due to the grainy medium. The scale of this diffraction pattern is λ/θ_s (cf. Fig. 2)

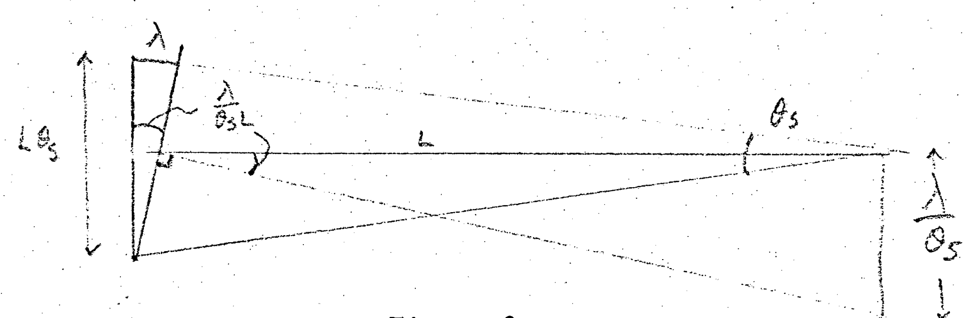


Figure 2.

Sources smaller than $\psi = \frac{\lambda}{\theta_s L}$ will give scintillations - for sources larger than this, the patterns from different parts of the source will interfere and smear the pattern away. If the cone defined by θ_s does not contain more than one blob, then the beams from various blobs will not interfere, and if the phase change is not strong, the scintillations will become weak. These conditions are $\frac{2a}{L} < \theta_s$ and $\phi_o > 1$.

If one assumes that the distances to the pulsars are related to the dispersion measure, one finds observationally that

$$\langle \Delta n^2 \rangle^{1/2} \approx 3 \times 10^{-5} \text{ cm}^{-3}$$

$$a \approx 10^{11} \text{ cm}$$

so that the size for scintillations becomes

$$\psi \approx 4 \times 10^{-7} \frac{\nu}{408} \cdot (L)^{-3/2} \text{ arc sec}$$

where ν is the observing frequency in MHz, and L is the pulsar distance in kpc. Thus only pulsars are small enough to scintillate via ISS. The validity of this theory at higher frequencies (2388 MHz) has been confirmed by the observations of Downs and Riechley (Ap. J., 163, L11 (1971)) and at distances deep into the galactic plane by Lang (Ap. L. 7, 175 (1971)).

Harris, Zeissig and Lovelace (Astron. and Ap., 8, 98 (1970)), using Salpeter's theory and Rickett's measurement, have derived the following angular size for sources due to ISS

$$\theta = 25'' \, L^{1/2} \, \nu^{-2} \, . \qquad \nu \text{ in MHz and } L \text{ in pc.}$$

Rickett, using Uscinski's theory, and his own measurements gets

$$\theta = 5''.3 \, L^{1/2} \, \nu^{-2} \, .$$

Taking the ∿ geometric mean of these, I get

$$\theta_{ISS} = 10'' \, L^{1/2} \, \nu^{-2} \, .$$

The result of this is that sources smaller than the scattering size should appear as large as the scattering size and not the true size. The size depends on the inverse square of the frequency and the $(\text{distance})^{1/2}$ through the galactic medium. For extragalactic sources, this would become a dependence on galactic latitude and for pulsars in the galaxy, the dispersion measure. The effects of ISS have already been made manifest in the turnover of the scintillation index for interplanetary scintillation, which was pointed out by Harris, Zeissig, and Lovelace.

ISS has been invoked as a possible explanation for the resolution of the OH line sources (Burke et al. A. J., 73, S168 (1968)); however, I think that this is not correct because the observations would imply values for $\frac{\langle \Delta n^2 \rangle^{1/2}}{a^{1/2}}$

of 10^{-6} - 10^{-7} $cm^{-3.5}$, whereas even the most optimistic value from the pulsar data would be 10^{-9} and more likely 10^{-10} $cm^{-3.5}$.

III. Interplanetary Scattering (IPS)

The interplanetary medium will also cause scattering in a way analogous to the interstellar medium. This scattering manifests itself through interplanetary scintillations and has been used in studying the sizes of compact radio sources (see for example, M. H. Cohen, Ann. Rev. Astron. and Ap., 7, 619, (1969)). The data on which the following discussion is based comes from direct measurements of the angular sizes of sources at low frequencies which was compiled by Erickson (Ap. J., 139, 1290 (1964)).

Erickson (Radio Science, 1, 109 (1966)) finds the following empirical relation for, θ, the scattering angle, from data taken between 11 and 1.7 meters and solar elongations between 8 and 90 solar radii:

$$\theta = 50' \frac{\lambda^2}{(R)^2}$$

where λ is the observing wavelength in meters, R is the impact parameter in solar radii. At opposition this becomes

$$\theta = 12''.5 \frac{\lambda^2}{(215)^2}$$

or

$$\theta_{IPS} \doteq 1450'' \nu^{-2} \qquad \nu \text{ in MHz}$$

Thus for IPS the scattering angle scales also as λ^2 but depends on the solar elongation of the source. The degree of solar activity will have an effect and perhaps the azimuth angle of the sources with respect to the sun will have an effect on the scattering angle as well.

It is immediately obvious that except for enormous values of L in the ISS formula that IPS should dominate. The empirical evidence at low frequencies near the sun is hard to refute; however, it is open to question how far this relation can be extrapolated in frequency and solar elongation. Assuming that typical blob sizes which cause the scattering is 100 km at 100 $R_{\odot}$ and Erickson's relation is valid, and that the effective screen distance is 1 a.u., then using the relation for the scattering angle being greater than the angle subtended by the blob when viewed from 1 a.u.,

$$\frac{2a}{L} < \theta_{IPS}$$

we obtain

$$\nu \lesssim \frac{300 \text{ MHz}}{\left(\frac{R}{100\ R_{\odot}}\right)^{3/2}} \qquad (1)$$

At frequencies higher than this, the source would appear smaller than θ_{IPS} but would wander about with rms excursions of magnitude θ_{IPS}. Observations by the Canadian observers (reported by J. Locke at the Rumford Symposium, Boston, April 1971) at 408 MHz seem to confirm this. The source was at 30° solar elongation and the observations were made on a 3074 km baseline. The fringes were strong but the phases were severely and abruptly changed frequently during the observation. The existence of fringes on a baseline this long at this frequency indicate structure on the order of 10^{-2} arc sec, while IPS scattering size is $\sim 10^{-1}$ arc sec for this frequency. For this measurement formula (1) is invalid and the lack of stability is exactly what would happen if the source were being smeared out sequentially rather than simultaneously. If these observations were synthesized into a map, the result would appear to be a source of size $\theta_{IPS} \gtrsim 10^{-1}$ arc sec.

At 111.7 MHz unpublished VLB observations by the author and others indicated that three sources observed during the night had sizes $\sim 10^{-1}$ arc sec, although at other frequencies these sources are known to be much smaller. For this frequency $\theta_{IPS} = 0\overset{''}{.}12$ and $\theta_{ISS} = 0\overset{''}{.}014$, so that evidently IPS is dominating.

Thus it appears that IPS is a more principal cause of source broadening than ISS - a fact which seems to have escaped the pulsar observers who recommend VLB observations to confirm the ISS theories. However, still much work remains in this area of low frequency VLB, both in confirming the λ^2 dependence of scattering size and determining at which frequencies and solar elongations IPS or ISS dominates. Further observations of this sort are in progress.

IV. Synchrotron Self-Absorption (SSA) and Inverse Compton Effect (ICE).

The effect of the self absorption on the size of a synchrotron source is well known and has the following relation (cf. Kellermann and Pauliny-Toth, Ap. J., 155, L71, (1969))

$$\theta_{SSA} = 82'' \, S_m^{1/2} \, B^{1/4} \, \nu_n^{-5/4} \tag{2}$$

where B is the magnetic field in gauss, S_m the peak flux in flux units, and ν_m the frequency of maximum flux in MHz.

In the same paper Kellermann and Pauliny-Toth also considered the competition between synchrotron and inverse Compton losses. If the source luminosity is large enough, synchrotron photons can compton scatter off the electrons. The relative energy loss for the two effects is

$$\frac{L_c}{L_s} = \frac{\gamma^2 \, \sigma \, U_{rad}}{\int 2\gamma^2 \, \sigma \, \sin^2 \theta \, d\theta \, U_{mag}} = \frac{3L_T/4\pi r^2 c}{B^2/8\pi} = \frac{6L_T}{r^2 B^2 c} \tag{3}$$

$L_T = L_c + L_s$; L_s and L_c are the synchrotron and compton losses respectively, γ is the Lorentz factor, σ the Thompson cross section for the electron U_{rad} and U_{mag} the energy densities in the radiation and magnetic fields respectively, c the velocity of light, B the magnetic field and r the radius of the source. If d is the distance to the source

$$L_s = 4\pi d^2 \int_{\nu_m}^{f_c} s d\nu = 4\pi d^2 S_{max} \, f_c \tag{4}$$

where it has been assumed that S, the flux density, has constant value of S_{max} (nearly true for flat spectra sources) up to a upper cutoff frequency of $f_c >> \nu_m$, the lower cutoff frequency. Taking r/d as the angular size of the source and plugging (4) (3) and (2) together we get, keeping second order terms,

$$\frac{L_c}{L_c} = \frac{1}{2}\left(\frac{T_{max}}{10^{12}}\right)^5 f_c \left[1 + \frac{1}{2}\left(\frac{T_{max}}{10^{12}}\right)^5 f_c\right]$$

where $f_c \sim 10^{5\pm1}$ MHz.

Above 10^{12}°K, $L_c >> L_s$ and Compton losses overwhelm the synchrotron losses As an example, Kellermann and Pauliny-Toth take a source with cutoff near 1 GHz and find a half life of 10^4 years for $T = 10^{11}$°K when $T = 10^{12}$°K it is 1 day!

So a source brighter than 10^{12}°K will quickly radiate itself away via ICE and cool to a temperature of $\sim 10^{12}$°K. They also show observational evidence for this limit.

This limit of brightness temperature also yields a limit to the angular size of the source.

Using the Rayleigh-Jeans formula,

$$S = \frac{2kT\Omega}{\lambda^2}, \quad \Omega = \frac{\pi\theta^2}{4}$$

and using $T = 10^{12}$°K we get

$$\theta_{ICE} = 1''.3\ S_m^{1/2}\ \nu_m^{-1}.$$

These two effects, SSA and ICE, limit the size of an incoherent synchrotron source in a way which depends on the source strength, the cutoff frequency, and in the case of SSA on the magnetic field. It should be noted that a given source has the same size at all frequencies for the models chosen here and we re-emphasize that ν_m is the cutoff frequency, not the observing frequency. However, if a source has a varying brightness distribution, there may be frequency dependent size effects as various parts of the source become optically thick at different frequencies.

V. Discussion

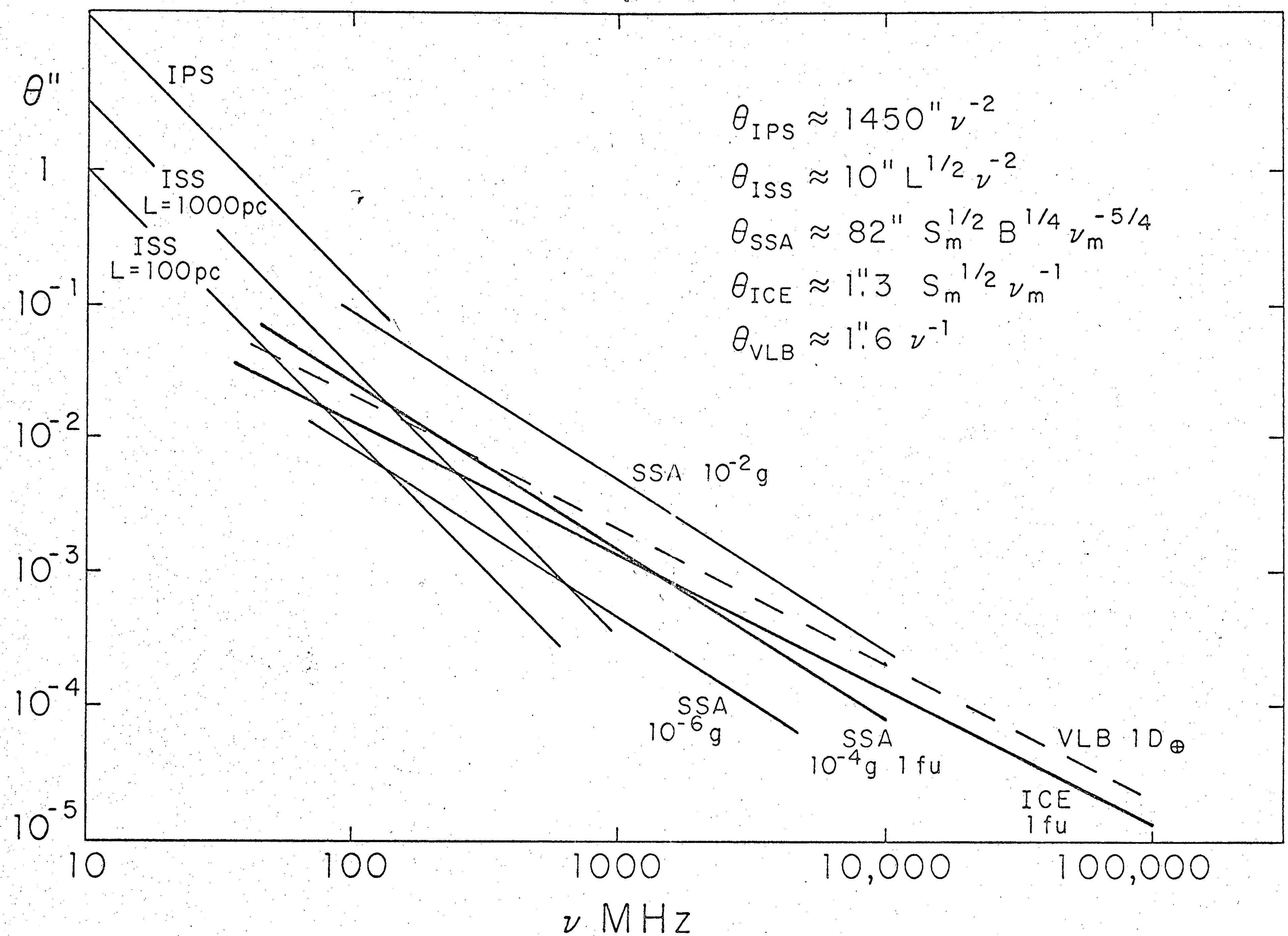
The various size limiting formulae are drawn on the accompanying figure. It is seen that at low frequencies IPS and ISS dominate, at moderate frequencies SSA takes over, and except for sources of very high magnetic fields ICE dominates at high frequencies. For SSA and ICE, the lines are for 1 flux unit maximum sources. For these, the graph is read by deciding which is the frequency at which the maximum occurs. The ordinate of this point is then the angular size for that source. For IPS and ISS the size of the source will follow their respective curves.

Also shown on the plot is the size which can be resolved by a VLB interferometer of baseline equal to 1 earth diameter. This size is $\theta_{VLB} = 1''.6\ \nu^{-1}$, where ν is the observing frequency in MHz. This was obtained by assuming that a source of size

$$\theta = \frac{\lambda}{3D}$$

can be resolved by the interferometer. D is the baseline, λ the observing wavelength. For sources stronger than 1 f.u., it is seen that the earth diameter baseline would almost resolve any source which radiates via incoherent synchrotron emission. To be able to completely resolve it, one could either increase the baseline or increase the observing frequency, thereby increasing the resolution. Compact sources generally have flat spectra so not much would be lost in signal strength by doubling the frequency of observation. The cost of increasing the baseline beyond an earth diameter dwarfs the cost of increasing the observing frequency, so it appears that this latter option is more reasonable. Pulsars and interstellar masers of course don't radiate via the incoherent synchrotron mechanism, and so the earth baseline may not be adequate for their resolution. However, IPS and ISS are still effective size limters for these sources, and pulsars radiate strongly only at low frequencies where these effects dominate. IPS has probably become ineffective for multiple scattering of the OH line radiation at 1660 MHz; however, ISS should still be effective. Thus, we can calculate the baseline needed to resolve an OH source 10 kpc distant size limited by ISS. This turns out to be 3.4×10^4 km or about a tenth of the earth-moon distance. So putting a VLB terminal on the moon would not be very useful for OH interferometry unless it was practical to get a great deal of foreshortening. However, the OH sources seem to be resolved by earth based interferometry anyway. The H_2O sources at 1.35 cm will hardly be effected by interstellar scattering, but at this frequency an earth baseline can resolve a source of 7×10^{-5} arc sec or 1/4 a.u. at the distance to W3.

In conclusion it seems that we have the happy circumstance that we finally have technique VLB interferometry, which is capable, without the expense of building extraterrestrial telescopes, of resolving all the radio sources. However, we must remember that similar statements were made about other instruments being as large as needed, which were later proved wrong.



θ''
1
10^-1
10^-2
10^-3
10^-4
10^-5
10
100
1000
10,000
100,000
ν MHz
IPS
ISS L=1000pc
ISS L=100pc
SSA 10^-2 g
SSA 10^-6 g
SSA 10^-4 g 1 fu
VLB 1D⊕
ICE 1 fu
$\theta_{IPS} \approx 1450'' \nu^{-2}$
$\theta_{ISS} \approx 10'' L^{1/2} \nu^{-2}$
$\theta_{SSA} \approx 82'' S_m^{1/2} B^{1/4} \nu_m^{-5/4}$
$\theta_{ICE} \approx 1''.3 S_m^{1/2} \nu_m^{-1}$
$\theta_{VLB} \approx 1''.6 \nu^{-1}$