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Interferometry and the VLA: Second day

Notes added to Summer Student Lecture 1978

Yesterday I dropped on you a fat set of notes and a very large amount of technical detail. I encourage you to reread those notes including pages 8-14 of Part II. However, today I want to try to clarify what is meant by aperture synthesis and to show you its usefullness and its culmination in the Very Large Array (VLA) now under construction in New Mexico.

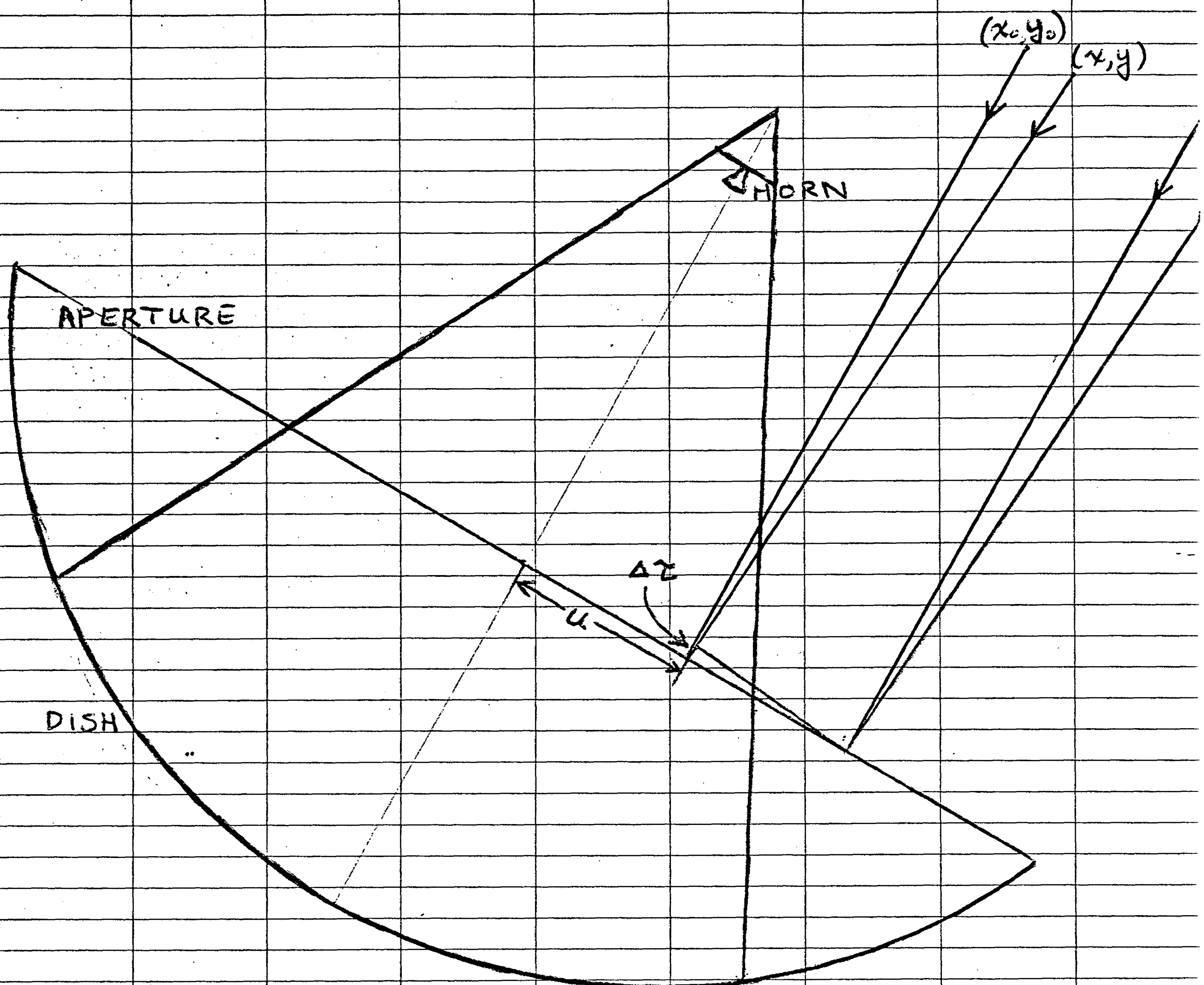
SINGLE DISH

Let us start by considering how a filled aperture - a single dish telescope - actually works. On the next page I've drawn a crude picture of a single dish telescope. The ideal single dish has a perfect parabolic surface. This surface shape has the property of maintaining, for an incoming plane wave, the phase relationships which pertain at the aperture despite the additional travel distances to the reflecting surface and then to the horn. Thus we need analyze only the relationships at the aperture. The variables we'll need are also defined on the figure on page 2.

A ray striking at a point u, v on the aperture has a phase delay

$$\omega \tau = 2\pi \left[u(x - x_0) + v(y - y_0) \right]$$

where we assume that all angles are small and linearize everything. The voltage at the horn due to an infinitesimal



Variables:

u, v	coordinates on aperture in wavelengths
x, y	sky coordinates
x_0, y_0	sky coordinates - where dish is pointed
$W(u,v)$	weighting due to reception pattern of horn
$V(x,y)$	voltage due to source
$\xi(x,y,t)$	random function
ω	angular observing frequency

part of the source and an infinitesimal part of the aperture is

$$V(u,v,x,y,t) = V(x,y)\,W(u,v)\,e^{i[\omega t + \xi(x,y,t)]}\,e^{2\pi i[u(x-x_0)+v(y-y_0)]}.$$

The receiver responds to the time averaged power:

$$R \propto \iint du'dv' \iint du\,dv \iint dx\,dy \iint dx'dy' \left\langle V(u,v,x,y,t)\cdot V^*(u',v',x',y',t)\right\rangle$$

$$= \iiiiiiiint V(x,y)\,V(x',y')\,W(u,v)\,W(u',v') \left\langle e^{i[\xi(x,y,t)-\xi(x',y',t)]}\right\rangle \cdot e^{2\pi i[u(x-x_0)-u'(x'-x_0)+v(y-y_0)-v'(y'-y_0)]}$$

where $\langle\ \rangle$ denotes a time average. Since ξ is a random, uncorrelated function – i.e. the source's radiation is not coherent – the term in $\langle\ \rangle$ is just $^2\delta(x-x', y-y')$.

Thus,

$$R \propto \iint du\,dv \iint du'dv'\, e^{-2\pi i[(u-u')x_0+(v-v')y_0]}\, W(u,v)\,W(u',v') \cdot \iint dx\,dy\; T(x,y)\, e^{2\pi i[(u-u')x+(v-v')y]} \qquad (III-1)$$

There are now 2 ways to proceed. As a sidelight, I will first show the standard approach:

(1) Rewrite equation (III-1) as

$$R \propto \iint dx\,dy\; T(x,y) \iint du'dv'\; W(u',v')\, e^{-2\pi i[(x-x_0)u'+(y-y_0)v']} \cdot \iint du\,dv\; W(u,v)\, e^{+2\pi i[(x-x_0)u+(y-y_0)v]}.$$

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Now lets define a useful function:

$$S(x,y) \equiv \iint du\,dv\; W e^{2\pi i(ux+vy)}$$

which is the Fourier transform of the horn response. Then

$$R \propto \iint dx\,dy\; T(x,y)\; S(x-x_0, y-y_0)\; S^*(x-x_0, y-y_0), \text{ or}$$

$$R \propto \iint dx\,dy\; T(x,y)\; B(x-x_0, y-y_0)$$

where $B \equiv |S|^2$ is the single dish beam pattern. In other words the single dish just adds up the sky brightness weighted by its beam pattern.

② A second, instuctive approach to equation (III-1) is to perform a coordinate change: $u'' = u - u'$, $v'' = v - v'$ to get

$$R \propto \iint du''dv'' \iint du'dv'\, e^{-2\pi i[u''x_0 + v''y_0]}\, W(u''+u', v''+v')\, W(u',v$$

$$\cdot \iint T(x,y)\, e^{2\pi i[u''x + v''y]}\, dx\,dy.$$

We note that the u', v' integral is a convolution and the x, y integral is a Fourier transform of the sky brightness. Defining a visibility function

$$V(u'', v'') \equiv \iint dx\,dy\; T(x,y)\, e^{2\pi i[u''x + v''y]}$$

and a new dish weighting function

$$A(u'', v'') \equiv \iint du'dv'\; W(u''+u', v''+v')\, W(u', v')$$

$$= W * W$$

we obtain, dropping the double primes,

$$R \propto \iint du\,dv\; A(u,v)\,F(u,v)\,e^{-2\pi i(ux_0+vy_0)}.$$

This is just the inverse Fourier transform of a weighted fringe visibility function evaluated solely at the dish pointing direction. In other words, the single dish performs the same operations performed by the interferometers we discussed yesterday.

To sum up the differences between single dishes and interferometers:

SUBJECT	SINGLE DISH	INTERFEROMETER
size	limited by weight	limited by Earth
area covered	full dish	fraction of aperture - traced by dishes in u-v plane as time advances
map	evaluated at 1 point only	evaluated at many poin
processing	evaluated immediately	evaluated after observat are complete
noise (°K)	receiver	receiver times area of synthesized aperture divided by area actuall covered.
resolution	set by dish diameter, but degraded by need for horn response to be zero outside the dish area	equivalent to dish of radius = maximum baseline with no horn degradation.
sidelobes	normally quite low	substantial and complicated, usually

The VLA

The traditional method of doing interferometry is to take 2 telescopes, 1 of which is movable, and gradually acquire enough baselines for some sort of aperture synthesis. Basically: one observes 1 day, moves & sets up the telescopes the next day, observes the third day, etc. ad nauseum.

One fine day a long time ago some people sat down and made a list of problems present in current interferometers and designed a better mousetrap. Their lists of problems probably ran like the following:

1. Too slow (& hence expensive) to get enough baselines
2. Regularly spaced antenna stations along a single line give bad synthesized beam patterns even at high declination.
3. Maximum baselines are too short for good spatial resolution.
4. Minimum baselines are too long to be sensitive to larger sources
5. Noise is too high because of small number of observations and insensitive receivers
6. Frequency coverage inadequate.

The VLA was their answer:

1. 27 antennas give 351 simultaneous baselines – enough to synthesize strong sources in just a few minutes.
2. A non-linear spacing of antenna stations along the 3 arms of a "Y" gives good beams.
3. The maximum baseline at the VLA will be 36 km
4. Many of the baselines will be short – except for shadowing the VLA will get full coverage.
5. Build good receivers (~50 °K) and have 351 baselines.

6. The VLA has 4 frequency ~~band~~ bands:

L	1.35 - 1.72	GHz
C	4.5 - 5.0	GHz
U	14.4 - 15.14	GHz
K	22 - 24	GHz

Further refinements:

1. The VLA has 4 scaled configurations:

A	maximum baseline -	36 km
B	"	11 km
C	"	3.4 km
D	"	1.0 km

2. 4 IF systems to simultaneously observe right and left circular polarizations at 2 frequencies. The correlator outputs 8 different fringes per baseline:

RR
LL
RL
LR
} at each frequency

3. Spectral line capability will also be provided. The number of spectral channels available varies from 8 at 50-MHz total bandwidth to 256 at 1.56 MHz bandwidth. The number of channels is fixed at 256 for smaller total bandwidths.

Current status: under construction, but extensive observations already underway

(i) Spectral correlator will begin operation soon but will be limited to continuum for the moment

(ii) We've had fringes from the 14th antenna, but the current correlator can handle only 12 antennas (& 2 IF's)

(iii) Track and stations are complete for the C and D arrays and out to AW6 (10.4 km on the southwest arm)

(iv) Waveguide installation is behind the track construction - but stations on the southwest & southeast arm are usable.