

## Radiative Processes in Astrophysics

### Outline

I. Definitions: thermal + non, black body, Maxwellian; relations (Kirchoff's law).  $I \propto \frac{S}{\theta^2}$ .

II. Single-particle radiation processes

A. Involving bound states (QM)

B. Free-particle (include IC; just upgrade, not really "new" photons)

III. Detail on TB + SR from ensembles

A. Brems.

B. Synchrotron rad'n

## Definitions

What do you need to know to describe radiation fully?

How much power? ( $\text{en/time}$ )? From how much sky? How big the detector? What frequency? What polarization?

All of these except pol. are part of

### Specific Intensity

$$I_\nu dt dA dv d\Omega = dE = \underbrace{\text{energy received from } d\Omega}_{\text{in time } dt} \text{ through area } dA \text{ in band } dv.$$

So units  $I_\nu$ :  $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$  (or  $\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$ )

Let a source emit  $L_\nu$   $\text{erg s}^{-1} \text{Hz}^{-1}$ , - at dist.  $d$ , with radius  $R$ . (dCCR)

What is  $I_\nu$ ?  $dE(\text{em}) = L_\nu dt dv \left(\frac{d\Omega}{4\pi d^2}\right) \left(\frac{dA}{4\pi R^2}\right)$

$$dE(\text{rec.}) = I_\nu dt dv dA \left(\frac{\pi R^2}{d^2}\right)$$

so  $I_\nu = \frac{L_\nu}{\frac{dA}{4\pi R^2} \frac{(4\pi R^2)}{d^2} 4\pi d^2} = \frac{L_\nu}{4\pi^2 R^2} \text{ independent of } d!$

How can this be? What about inverse-square law?

Ans: As you get farther away, source subtends less sr.,  $\propto \frac{1}{d^2}$ .

What is your  $d\Omega$ ? Beam. Can't look at less  $d\Omega$  at a time than this.

(But can look at more.) So as long as  $\Omega_{\text{source}} > d\Omega(\text{beam})$ , measure const intensity.

If  $\Omega_{\text{source}} < d\Omega_{\text{beam}}$ , want another quantity:

Spectral Flux.  $S_\nu = \int I_\nu d\Omega$  over area of interest (e.g., a source  
units =  $\text{erg cm}^{-2} \text{s}^{-1} \text{Hz}^{-1}$  (not  $\text{sr}^{-1}$  anymore))

Defs. 2.

So  $d\Omega = \frac{\text{Area}}{d^2}$ ,  $S_v \propto \frac{1}{d^2}$  for a source. If subtends  $\theta$  rad,  
 $\theta \ll 1, d\Omega \sim \theta^2$ ;  
 $S_v \approx I_v d\Omega$

$I_v$ : indep. of dist but refers to different fraction of source at  
different freqs dists. ( $I_v = \frac{S_v}{\theta^2}$ )

$S_v$ :  $\propto \frac{1}{d^2}$  but describes whole source  $S_v \approx I_v \theta^2$

Can refine total flux (integrated flux)  $\equiv \int S_v d\Omega = \text{em/area time } (\text{W m}^{-2}$   
 $\text{erg s}^{-1} \text{cm}^{-2})$

Where does radiation come from?

Define emissivity. (How much power? In which direction? what freq.?  
How much volume?)

$$j_v dt dV dv d\Omega = dE \text{ (produced)}$$

$$\text{so } [j_v] = \text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1} \text{sr}^{-1} \\ (= \text{W m}^{-3} \text{Hz}^{-1} \text{sr}^{-1})$$

Sim. to  $I_v$  but not quite;  $[I_v] = [j_v] \times \text{length}$

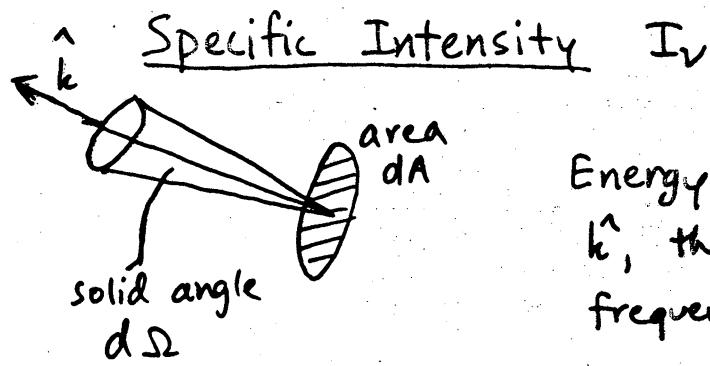
Absorption? This depends on how much intensity already there.  
( $\propto I_v \Rightarrow$  no abs!) So define

$$k_v I_v = dE \text{ (absorbed) in } dt dV dv d\Omega .$$

$$\text{so } [k_v] \times \frac{dE}{dt dV dv d\Omega} = \frac{1}{\text{length}} \text{ only. (all the other units in } I_v.)$$

# Radiative Quantities: Definitions

Describe all properties of radiation except polarization with



Energy received from  $d\Omega$  about some direction  $\hat{k}$ , through area  $dA \perp \hat{k}$ , in time  $dt$ , in frequency interval  $dv$ :

$$dE = I_v dt dA dv d\Omega$$

so  $I_v = \text{energy} / (\text{time, area, frequency, solid angle})$

units:  $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$  (cgs)

or  $\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$  (SI)

## Emissivity $j_v$

Energy emitted from volume  $dV$  in time  $dt$ , frequency interval  $dv$ , solid angle  $d\Omega$ :

$$dE = j_v dt dV dv d\Omega$$

$j_v = \text{energy} / (\text{time, volume, frequency, solid angle})$

units:  $\text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1} \text{sr}^{-1}$

$\text{W m}^{-3} \text{Hz}^{-1} \text{sr}^{-1}$

## Absorption Coefficient $\kappa_v$

$\kappa_v I_v$  is energy absorbed in vol  $dV$ , time  $dt$ , freq.  $dv$ , solid angle  $d\Omega$ .

$$dE = (\kappa_v I_v) dt dV dv d\Omega$$

so  $\kappa_v$  has units of length ( $\text{cm}^{-1}$ )

## Up from Disorder

### 1) Thermal Equilibrium.

All detailed information is lost, swallowed in 1-parameter  $B_\nu(T)$ .

True for matter + radiation coupled + in equil. <sup>strongly</sup>

Particles: Maxwellian. Photons:  $B_\nu(T)$ .

Example: Stars. (photospheres)

### 2) Thermal Particle Dist.

Collisions equilibrate particles among themselves, but photon-particle interactions not suff. to get particle-photon equil. (Commonly tr

Particles: Maxwellian. Photons: optically thin something:  
lines, brems., ...

Example: HII region radio em, stellar chromospheres, X-ray SNR

### 3) Nonthermal Particle Dist.

Some process produces non-Maxwellian particle dist, + not enough interaction to bring particles to Maxwellian.

Example: Synchrotron rad, masers.

Particles: SR: Power-law  $N \propto E^{-s}$  Photons: SR continuum power-law  
masers. Inverted pops. lines, but amplified

## Radiation Processes

Refs: Rybicki & Lightman  
[Pacholczyk]  
Longair

Radiation: =  
Production of photons

### Continuum

#### Thermal

Bremsstrahlung (radio, <sup>IR</sup>, optical, X-ray)

Collective (e.g. plasma osc.) (radio)

#### Nonthermal

Synchrotron radiation (usually radio, rarely opt, X-ray)

Inverse-Compton (usually opt  $\rightarrow$  X-ray  $\rightarrow$   $\gamma$ -ray)

### Line

Recombination (f-b) (el. trans) (radio  $\rightarrow$  X-ray)

~~Molecular~~ hyperfine (b-b) el. trans radio

Molecular rot. (b-b) radio

vibr radio  $\rightarrow$  IR

## Radiation Processes

2.

Detail: How do individual particles radiate energy?

Recall: a) Accelerated charges radiate!

$$b) P = \frac{2}{3} \frac{e^2}{c^3} a^2 \quad (\text{cgs})$$

Forms of accel:

a) QM: Discrete change in en. levels, or into (or out of) quantized level to from (to) continuum. Non-classical.

b) Free electrons:

a) Collisions. Deflected.

b) E-M fields. Electric (why not important?)

Magnetic: gyrate in field.

a: b-b, b-f, f-b. Lines or "edges". Not further discussed.

b: a) Bremsstrahlung: braking rad.

b) (Magn) Synchrotron rad. if extreme rel.

Cyclotron rad. if NR.

Note: In principle, either brems or SR can be thermal or nonthermal depending on ensemble of particles thusly radiating.

What particles radiate?

Classically:  $P \propto a^2$ . For given  $F$ ,  $a \propto \frac{1}{m}$ . Lightest  $\Rightarrow \underline{\underline{e^-}}$ .

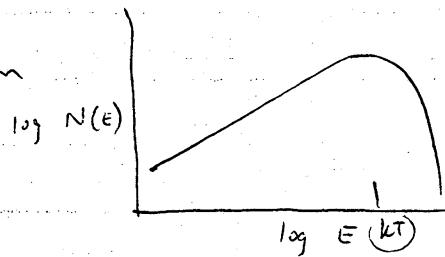
Brems:  $e^- + p^+$  collide;  $F$  is equal;  $P_{de} \sim \underline{\underline{4 \times 10^5}} \times P_{\text{proton}}$ .

## Radiation Processes

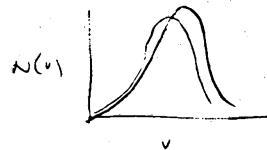
3.

What is "thermal"? - ~~Stemming~~ Having to do with a thermal distribution of particle energies.

Maxwellian



$$N(E) \propto E^{\frac{1}{2}} e^{-\frac{E}{kT}} T^{-\frac{1}{2}}$$



If  $kT \gg m_e c^2$ , can have relativistic Maxwellian.

$$N(E) \propto E^2 e^{-\frac{E}{kT}} T^{-3}$$

Very common distr., char. only by T. Get by colls  
⇒ "thermaliz."

To lowest order: All particles have  $E \sim kT$ .

Examples:		Dense
Mol.	cloud	$T \sim 10^6$
HI	cloud	$T \sim 50^6$
HII	region	$T \sim 10^6$
Young SNR		$T \sim 10^7$

So: "Nonthermal" ⇒ coming from any other distr.  $N(E)$ !

In practice, only other inferred  $N(E)$  is power law,

$$N(E) \propto E^{-s} \quad \text{For electrons in radio sources, } s \sim 2-3 \\ \text{cosmic rays: } s \sim 2.6$$

Later will get to how this is inferred.

So usually nonthermal ⇒ synchrotron;

thermal ⇒ brems.

## Radiation Processes

4.

So restrict attention to Brems.  $\rightarrow$  SR, from electrons.

Brems: Almost always concerned with thermal (TB).

R+L

pp 155-165

Ensemble of ionized gas atoms, els, thermal vel  $\Rightarrow$  collisions, radiation.

[Note:  $v^2 \propto T$  so higher  $T \Rightarrow$  more frequent colls  $\Rightarrow$  greater emissivity  $j_v$  in  $\text{erg cm}^{-3} \text{s}^{-1} \text{Hz}^{-1} \text{sr}^{-1}$ , by integrating over colls & over dist. of vel's given by Maxwellian distribution.]

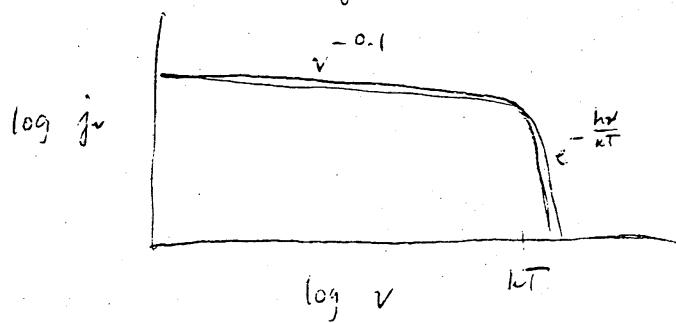
1 collision: High  $v \Rightarrow$  brief  $\Rightarrow$  smaller effect; power per Hz  $\propto \frac{1}{v} \propto \frac{1}{\sqrt{T}}$ .

Many collisions:  $j_v \propto n_e^2$  (binary collisions)

$\propto T^{-\frac{1}{2}}$  as above

$\propto e^{-\frac{hv}{kT}}$  (run out of fast electrons;  
can't have  $E(\text{photon}) = hv \gg kT$ )

So  $j_v \propto n_e^2 T^{-\frac{1}{2}} e^{-\frac{hv}{kT}}$   $g_F \leftarrow$  "Giant factor"



Roughly Constant

So If absorption is small,  
emitted spectrum is

$$L_v = (\text{Vol}) (4\pi j_v) \text{ erg s}^{-1} \text{Hz}^{-1} \text{ which has same } v\text{-dep.}$$

Total power: Int. over  $v \Rightarrow P_{\text{tot}} \propto \frac{n_e^2 T^{\frac{1}{2}}}{(2\pi \sim (hv) j_v)} \text{ erg s}^{-1}$

## Radiation Processes

5.

Absorption?  $\frac{j_\nu}{\kappa_\nu} = B_\nu$  (Lockman notation:  $I_{BB}$ )

so radio:  $B_\nu \approx \frac{2kT}{\lambda^2} \propto T\nu^2$

So since  $j_\nu \propto \nu^{-0.1 - \frac{1}{2}}$ ,  $\kappa_\nu \propto \frac{j_\nu}{B_\nu} \propto \nu^{\frac{-2.1}{2} - 3/2}$

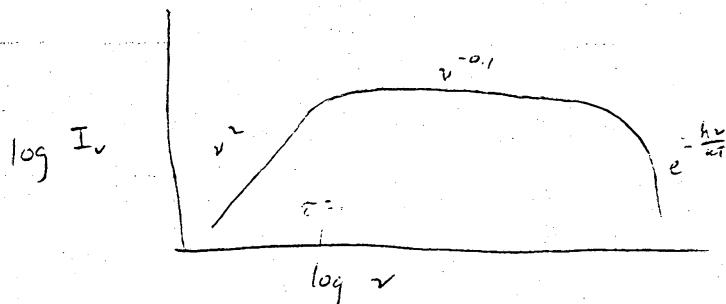
Can calculate  $\tau$  of a cloud  $\propto \nu^{-2.1 - 3/2} \ln n_e^2$

What if  $\tau \gg 1$ ? (When? For given cloud, look to low  $\nu$ .)

$\tau \gg 1$  means photons experience many absorptions/re-emissions  
- thermalized  $\Rightarrow BB$ !

So Low Freqs. ( $\tau \gg 1$ ):  $I_\nu \propto \nu^2$

High Freqs ( $\tau \ll 1$ )  $I_\nu \approx j_\nu l \propto \nu^{-0.1}$



## Radiative Processes

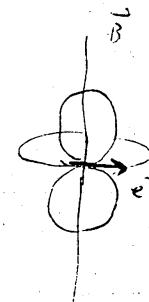
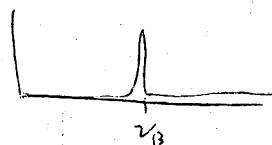
6.

### Synchrotron Radiation.

Cyclotron: Radial at  $\frac{\text{freq.}}{\text{period}}$  of orbit, avg. dist.  $\propto \sin^2\theta$ .

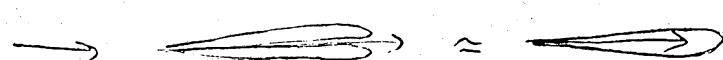
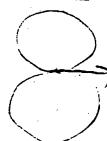
$$\text{Freq. is } \nu_B = \frac{eB}{2\pi m c}$$

So a line.



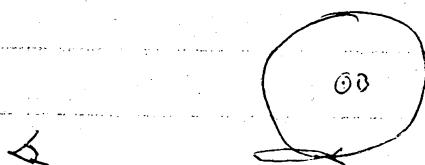
But if el. is relativistic,  $E \gg m_ec^2$ , 2 differences:

1) Beaming:



(~~appreciable~~ at NR where ~~ever~~ <sup>never</sup> radiation along  $\vec{a}$ )  
unlike

So see electron only for brief part of orbit.



2)  $v$  is almost  $c$  so pulse is shorter than it would be if static.

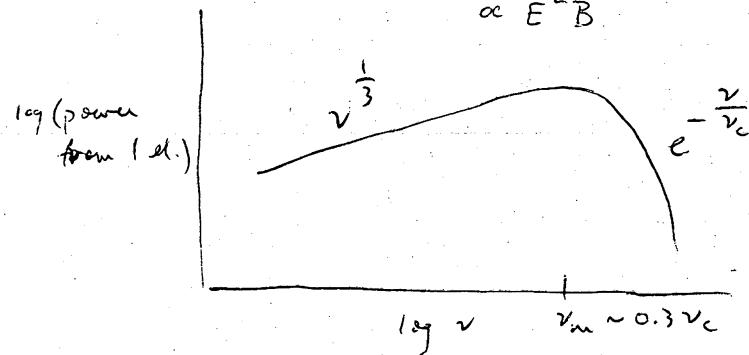
Both effects  $\Rightarrow$  see electron for small fraction of orbital period,  $\Delta t$ .

Fourier theory: Change in  $\Delta t \Rightarrow$  radiation up to  $\nu \sim \frac{1}{\Delta t}$

So get much higher frequencies. Actually, discrete harmonics of orbital freq, but so close that essentially a continuum is produced.

## Radiative Processes

Result:  $\nu_m \approx \frac{\gamma^2 \nu_B}{\alpha E^2 B}$ .  $\gamma = \frac{E}{m_e c^2} \gg 1$  for this to work at all



Numbers: Interstellar field

$$B \sim 3 \mu G \Rightarrow \nu_B = \underline{8.4 \text{ Hz}}$$

But  $\gamma = 10000 \Rightarrow \underline{840 \text{ MHz}}$

Galactic SR background.

More extreme: Pulsar light cylinder:  $B \sim 10^6 G \Rightarrow \nu_B \sim \underline{3 \times 10^{12}}$ .  
 $\gamma \sim 1000 \Rightarrow \nu_m \sim 3 \times 10^{18} \text{ X-rays!}$

~~Electron~~ This radiation is polarized  $\perp$  to projection of  $\vec{B}$ .

Ensemble of electrons:

When we look at SR sources (how do we know:  $T_B \gg$  reason pol, not a thermal spectrum), see power-laws  
 $S_\nu \propto \nu^{-\alpha}$ ,  $\alpha \sim 0.3 - 1.5$  or so.

Inter power-law spectrum of electrons,

$$N(E) = \underline{E^{-s}} \text{ els cm}^{-3} \text{ erg}^{-1}$$

SR theory  $\Rightarrow s = \underline{2\alpha + 1}$  so Tycho's SNR has  $\alpha = 0.6 \Rightarrow s = 2.2$

Very different from Maxwellian, all els have mean  $kT$ . If  $s = 2$ , total energy is spread equally over all electrons.

$$j_\nu \propto \underline{K B^{\frac{s+1}{2}} \nu^{-\alpha}}$$

## Radiative Processes

8.

### Synchrotron Self-Absorption.

Can we say  $\kappa_v = \frac{j_v}{B_v}$ ? No - particles do not have thermal distr.! But certainly  $j_v \leq B_v ("T")$

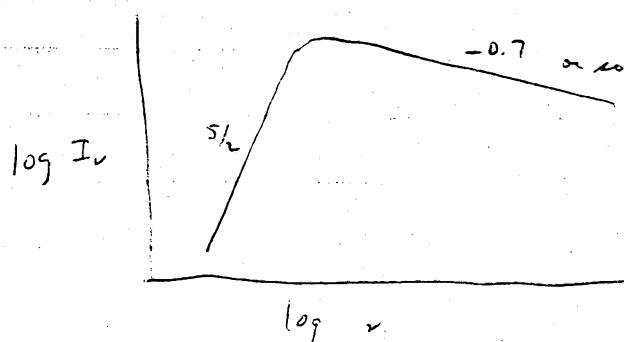
At each  $v$ , eff. "kT" is energy of electrons producing radiation at that  $v$ .

Since  $\gamma_m \propto E^2 B$ ,  $E "T" (v) = E(v) \propto v^{+ \frac{1}{2}}$

$$\text{So } K_v \propto j_v \frac{1}{B(T)} \propto v^{-\alpha} [ "T" v^2 ]^{\alpha} \underline{v^{-\alpha + 5/2}} \\ (K_v \propto \underline{B^{\alpha + 3/2} v^{-\alpha - 5/2}})$$

Note: If  $v$  low enough,  $\alpha = \int K_v dv > 1 \rightarrow$

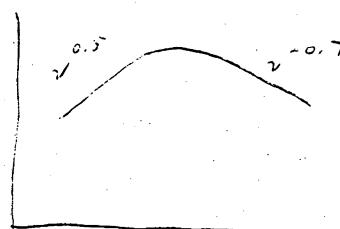
$$I_v \propto = \frac{j_v}{n_v} \propto v^{-\alpha + \alpha + 5/2} = v^{\frac{5}{2}} \text{ independent of } \alpha \\ (\text{why not } \propto v^2?)$$



Never see this.

Why?

- a) Inhomogeneous sources
- b) Superpos of different sources
- c) Other effects as  $v \rightarrow \infty$  decrease



more typical.

Can relate  $v_B$ ,  $S_v$  (then),  $\alpha$  to infer B. (but dangerous!)