A Test of a Polarization Observing Mode on the NRAO Spectral Processor

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April 15, 1994

Abstract

A test of a spectral line polarization observing mode on the NRAO spectral processor was conducted by varying the signal phase in one of its two inputs with a trombone line. The test used an IF frequency of 250 MHz, a bandwidth of 5 MHz, and all 1024 frequency channels of the spectral processor. For the test configuration, the Stokes parameters S_2 and S_3 for each frequency channel, as computed from the detected outputs of an ideal polarimeter, will vary sinusoidally with trombone setting, and the phase computed from S_2 and S_3 will vary linearly with both trombone setting and IF frequency. The spectral processor exhibited this ideal behavior for all frequency channels except for channels 0-31 and 1023, and the experimental results better agreed with predicted results when the first 32 and last 64 channels of the spectrum were eliminated from the data analysis.

1 Introduction

A spectral line polarization observing mode was implemented on the NRAO spectral processor (SP) in early 1993 by J. R. Fisher. This memorandum documents the results of a test of the mode.

The amplitudes and relative phase of two orthogonal components of polarization are needed to completely describe the polarization of an electromagnetic wave. A polarimeter produces four quantities related to the self- and cross-products of the two components. The total intensity of the electromagnetic wave is always the sum of the two self-products, and the relative phase of the two components can be derived from the two cross-products.

Successful SP measurements of total intensity in the past indicate that the self-products are properly generated, so the test was designed to explore how well the SP cross-products responded to a change in signal phase. The test and its underlying theory are discussed in section 2, the results of the test are presented in section 3, and concluding remarks are given in section 4.

2 Theory of the Experiment

To test the polarization observing mode of the SP, the output of a broadband noise source was split between two inputs of the SP. The vector of Stokes parameters which describes the polarization of the signal entering the SP is

$$\mathbf{S} = K \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{1}$$

where K is a constant. The polarization was altered by introducing a variable phase offset into one SP input with a constant-impedance trombone line (General Radio Co. Type 874-LT). The phase offset for a trombone setting of d_{t} is

$$\theta = 2\pi \frac{\nu}{c}(n_{\mathrm{t}} d_{\mathrm{t}} + L) \tag{2}$$

where ν is channel frequency, c is the speed of light, and $n_{\mathrm{t}} = 1$ is the refractive index of the trombone line. The term $L = n_{\mathrm{c}} d_{\mathrm{c}}$ accounts for the difference in lengths, d_{c}, of a coaxial cable with refractive index, n_{c}, between the SP inputs. The introduction of a phase offset modifies the Stokes vector in equation 1 by a transmission matrix (Mott 1992)

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\theta) & \sin(\theta) \\ 0 & 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} \tag{3}$$

such that the polarization of the signal entering the SP is

$$\mathbf{Sm} = \mathbf{M} \cdot \mathbf{S} = K \begin{bmatrix} 1 \\ 0 \\ \cos(\theta) \\ -\sin(\theta) \end{bmatrix} \tag{4}$$

Equations 2 and 4 show that the Stokes parameters Sm_2 and Sm_3, as measured by an ideal polarimeter, will vary sinusoidally with d_t. The phase, as computed with $\arctan(Sm_3/Sm_2)$, will decrease linearly with both increasing frequency and increasing d_t. The Stokes parameters and phase measured by the SP should reflect this ideal behavior.

3 Test Results

The test of the polarization observing mode used an IF frequency of 250 MHz, a bandwidth of 5 MHz, and all 1024 frequency channels of the SP. Detected outputs from the SP were recorded for 10 different trombone settings, d_t, spaced at roughly 10 cm intervals. For complex inputs A and B, the detected outputs of the SP are AA^*, BB^*, $\mathrm{Re}(AB^*)$, and $\mathrm{Im}(AB^*)$. The Stokes parameters were computed from the detected outputs with

$$Sm_0 = AA^* + BB^* \quad (5)$$
$$Sm_1 = AA^* - BB^* \quad (6)$$
$$Sm_2 = 2\mathrm{Re}(AB^*) \quad (7)$$
$$Sm_3 = 2\mathrm{Im}(AB^*) \quad (8)$$

The Stokes parameters Sm_2 and Sm_3 measured in SP channel 375 at each d_t are shown in Figures 1a and 1b, respectively. The Stokes parameters display the expected sinusoidal dependence upon d_t. The phase computed from Sm_2 and Sm_3 is shown in Figure 1c, and decreases linearly with d_t as predicted. With the exception of SP channels 0-31 and 1023, the Stokes parameters and phases of all channels exhibited the behavior shown in Figure 1. The Stokes parameters and phases in the bad frequency channels remained constant with d_t.

Figure 2 shows the phase computed for frequency channels 32 – 959 at a trombone setting of $d_t = 0$. The phase decreases linearly with frequency as predicted by equations 2 and 4. The measured phases from the first 32 and last 64 channels of the spectrum deviated from the line shown in the figure, and were not used in the analysis. The elimination of these frequency channels changed the center frequency to 249.922 MHz and the bandwidth to 4.531 MHz.

For a quantitative evaluation of SP performance, the test data were analyzed in two different ways. With the first method, straight-line fits were

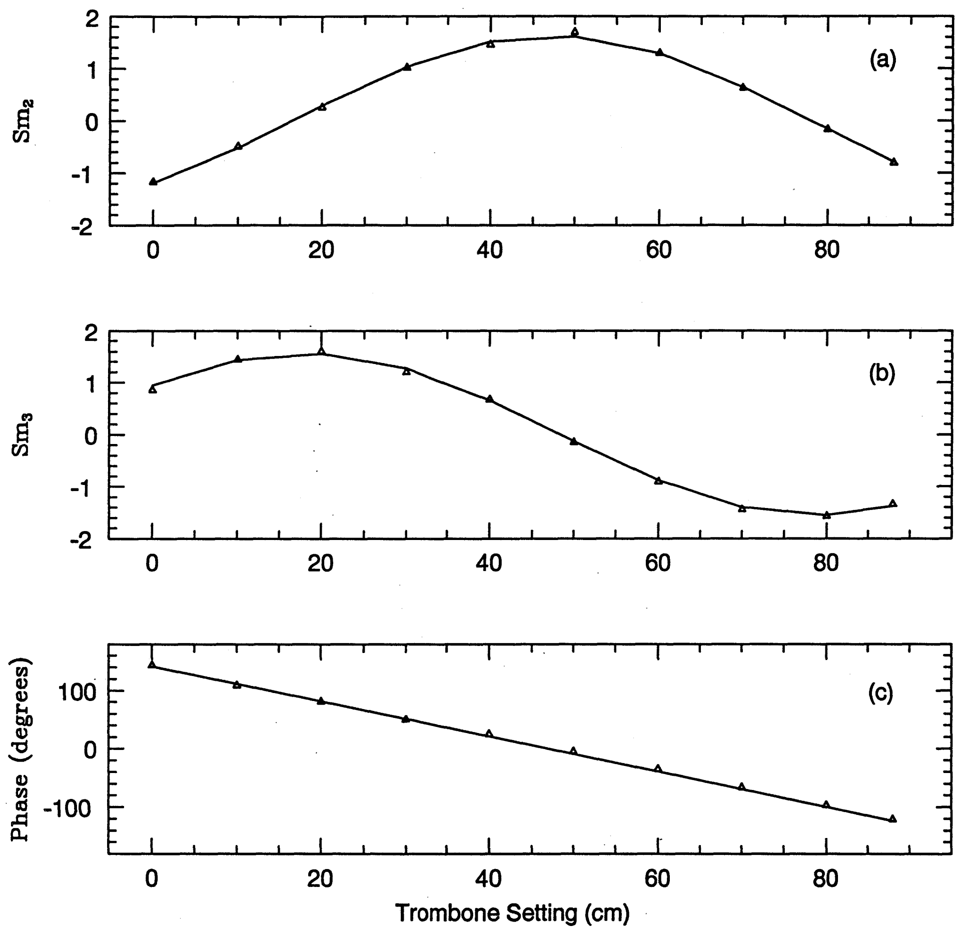

Figure 1: The change in the signal polarization of SP channel 375 as a function of trombone setting, d_t. The Stokes parameters Sm_2 (a) and Sm_3 (b) vary sinusoidally with d_t, and the signal phase (c) decreases linearly with d_t. Points are measured data, and solid lines are model fits.

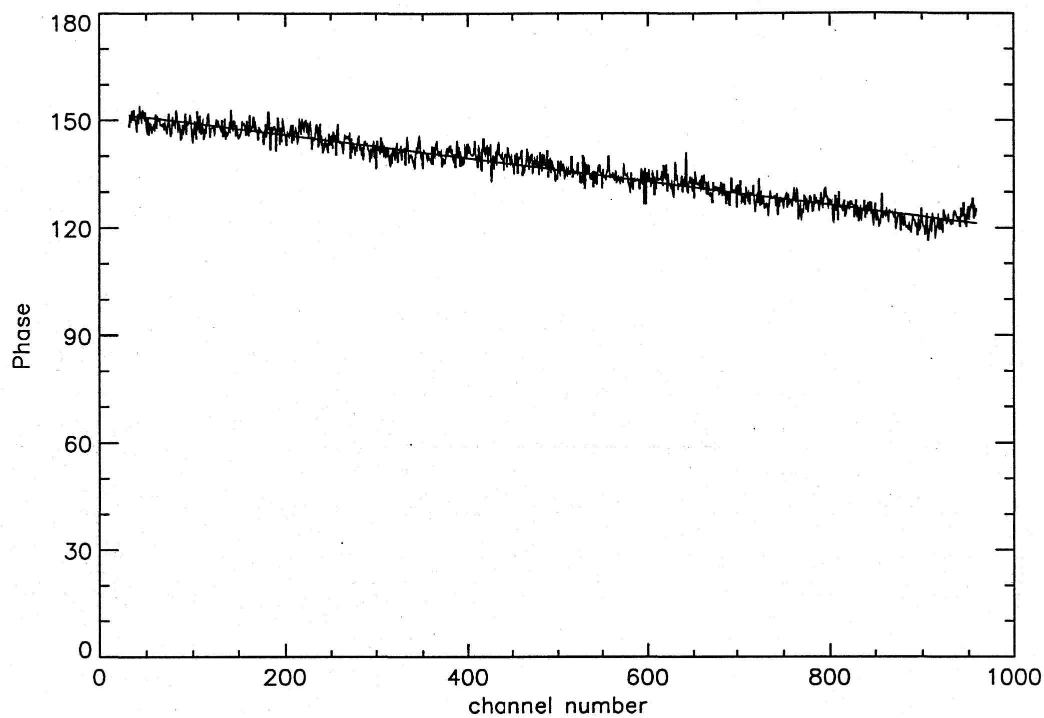

Figure 2: The measured phase across the passband at a trombone setting of $d_t = 0$ cm. The phase slope is due to an additional 374 cm of cable in one input of the SP.

made to the phase-d_t data for each channel. Figure 1c shows the fit to the data in channel 375. The slopes of the fits to SP channels 32 − 959 are shown in Figure 3, and the intercepts are shown in Figure 4. As can be seen from the figures, both quantities vary linearly with channel number. Straight-line fits were then made to these data. The solutions are annotated in each figure. This data analysis method effectively eliminates the d_t-dependence of the phase, and isolates the frequency dependence. The second data analysis method explored how the phase, when averaged over the passband, varied with d_t. Assuming the phase is weighted uniformly with frequency, the phase averaged over the passband is

$$\phi = -2\pi \frac{\nu_c}{c}(n_t d_t + L) \tag{9}$$

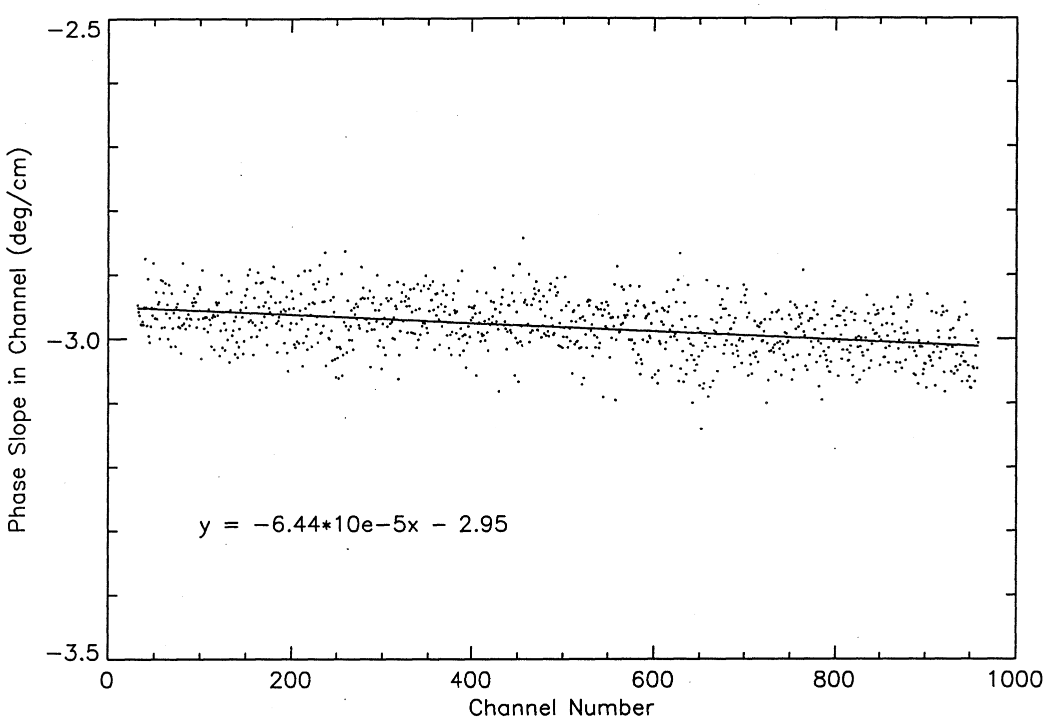

Figure 3: The slope determined from a straight-line fit to the phase-d_t data for each SP frequency channel.

where ν_c is the frequency at the center of the band. The phase slope across the band introduces an uncertainty, σ_ϕ, in the calculated phase given by

$$\sigma_\phi = \pi \frac{\Delta\nu_c}{c\sqrt{3}}(n_t d_t + L) \tag{10}$$

Figures 5a and 5b show ϕ and σ_ϕ computed at different values of d_t, and, as predicted by equations 9 and 10, the quantities vary linearly with d_t. Again, each figure is annotated with the equation of the best-fit straight-line to the data.

The fit parameters in Figures 3- 5 can be used to evaluate SP performance by comparing their measured values with those predicted by theory. The comparison is made in Table 1. The table lists the theoretical expression for the slope, m, or intercept, b, of the line in the specified figure. Consult

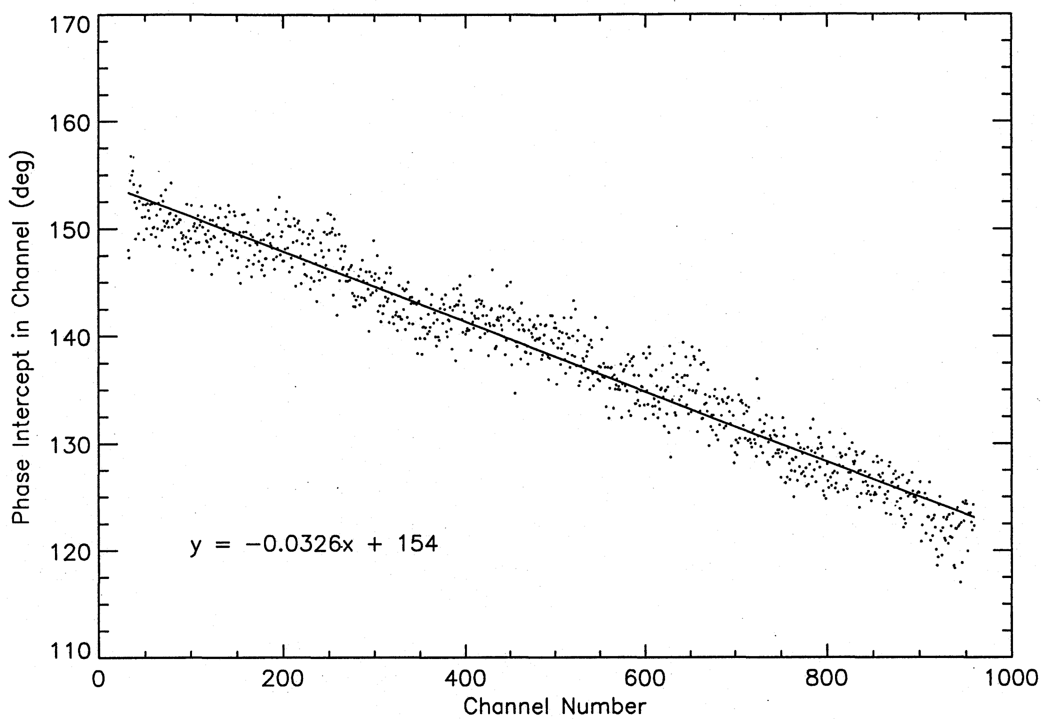

Figure 4: The intercept determined from a straight-line fit to the phase-d_t data for each SP frequency channel.

the appropriate figure for the units of m and b. The frequencies and bandwidths used to compute the predicted values of m and b were $\nu_o = 247.5$ MHz, $\Delta\nu = 5$ MHz (over $N_{ch} = 1024$ frequency channels), $\nu_c = 249.922$ MHz, and $\Delta\nu_c = 4.531$ MHz. The measured and predicted parameters listed in the table agree very well, indicating that the polarization observing mode of the SP worked well. In a similar way, the fit parameters can be used to estimate the difference in path lengths, L, between the SP inputs. Four estimates of L are made in Table 2. The measured values of b in the second and third rows of Table 2 are computed modulo 2π, and the values of L from the other two entries in the table were used to constrain the integer number of 2π wraps in phase through the excess cable. The number of wraps was found to be 4. For a dielectric cable with $n_c = 1.5$, the average value of $L = 561 \pm 8$ cm from Table 2 gives a difference in cable length of

$d_c = 374 \pm 5$ cm.

4 Conclusions

The phase measurements made by the SP are completely consistent with the phase offset introduced by a combination of trombine line setting and 374 cm of dielectric cable, indicating that the SP polarization observing mode worked very well. Deviations from ideal behavior at both ends of the spectrum are most likely due to band edge effects, although problems intrinsic to the SP cannot be ruled out for channels 0-31 and 1023.

I thank J. R. Fisher for assistance with the observations.

References

[1] Mott, H., 1992, Antennas for Radar and Communications: A Polarimetric Approach, New York: Wiley

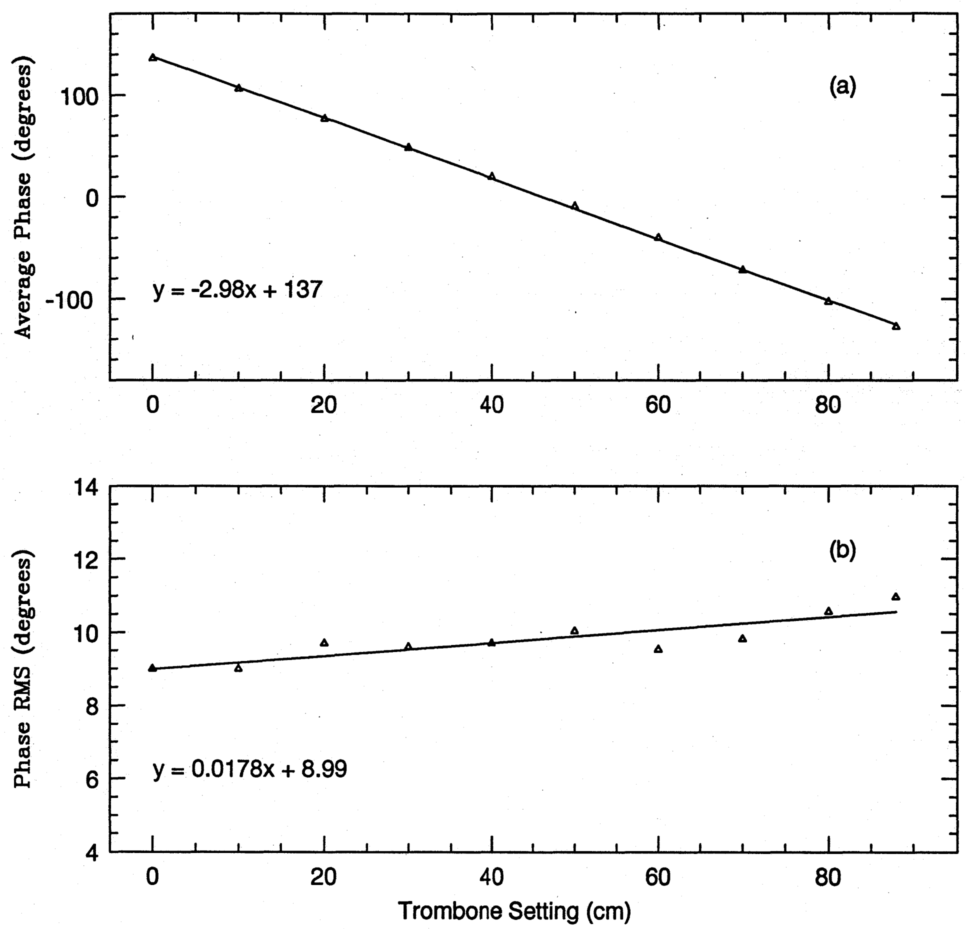

Figure 5: (a) The phase averaged over the passband at each trombone setting. (b) The error caused by the phase slope across the band when averaging in frequency.

Figure	Quantity	Expression	Predicted	Measured
3	m	$-2\pi\Delta\nu/cN_{\text{ch}}$	-5.86×10^{-5}	-6.44×10^{-5}
3	b	$-2\pi\nu_o/c$	-2.97	-2.95
5a	m	$-2\pi\nu_c/c$	-3.00	-2.98
5b	m	$\pi\Delta\nu_c/c\sqrt{3}$	1.57×10^{-2}	1.78×10^{-2}

Table 1: A comparison of the predicted and measured fit parameters to the experimental data.

Figure	Quantity	Expression	Measured	L (cm)
4	m	$-2\pi\Delta\nu L/cN_{\text{ch}}$	-3.26×10^{-2}	556
4	b	$-2\pi\nu_o L/c$	154	558
5a	b	$-2\pi\nu_c L/c$	137	557
5b	b	$\pi\Delta\nu_c L/c\sqrt{3}$	8.99	573

Table 2: Estimates of the excess path length from the fit parameters to the experimental data.