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A COMPARISON BETWEEN PRIME FOCUS AND CASSEGRAIN ANTENNAS

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I. INTRODUCTION

This report deals with a comparison between radio telescopes with paraboloidal reflectors and different ways of obtaining the desired aperture illumination. We shall consider mainly the two most used feed systems:

1. The horn feed in the focal point of the paraboloid; we call this the prime focus case.
2. The Cassegrain type where the feed system consists of a horn close to the vertex of the paraboloid and a hyperboloidal second reflector with its focal point in the focus of the paraboloid. The horn is placed in the other focal point of the hyperboloid.

The prime focus paraboloid is the most common antenna for radio astronomy in the frequency range over 200 MHz. Generally the front end of the receiver is mounted close to the feed behind the focal point of the paraboloid. This type of antenna is simple and moreover it is theoretically fairly well understood.

In recent years another type of antenna has gained in interest, viz., the Cassegrain type. Here again the paraboloid is used but a second reflector with hyperboloidal shape is placed near the focus of the paraboloid. It reflects the radiation impinging upon it towards the feed horn, which is now located near the vertex of the paraboloid.

The Cassegrain type has mainly been used for relatively small antennas for very high frequencies (above 10 GHz). Its success there has caused a considerable quantity of publications, whose main conclusion is that the Cassegrain is superior to the prime focus antenna. However, many of these conclusions are based on guesses and a conclusive experimental proof has not yet been given. The main feature of a Cassegrain is said to be the lower spillover and the higher aperture efficiency.

It is the purpose of this report to compare the two types of antennas and to show their specific characteristics with the help of computations and experimental results.

First we give the formulae for the geometry of the Cassegrain antenna and a discussion of the so-called "minimum blocking condition". In the next section on the electrical characteristics we deal with the effects of aperture blocking, F/D ratio and displacement of the feed from the correct focal point on the radiation pattern. Further, the aperture efficiency, spillover radiation, sidelobe level and feed design are investigated. Then typical mechanical features of the Cassegrain are compared with the prime focus antenna. An appendix with numerical results for three different antennas and curves of different important characteristics complete the report.

II. THE GEOMETRY OF THE SYSTEM

A. Basic Properties

The basic property of the paraboloidal reflector follows directly from the definition of a parabola. It transforms spherical waves originated in the focal point into a plane wavefront traveling parallel to the axis of rotation of the paraboloid. Or reasoning the other way around, which is possible by virtue of the reciprocity relation, it focuses a plane wave arriving on the reflector parallel to its axis of rotation into the focal point.

This property has to exist also in the case of a two reflector antenna. The hyperbola is defined as the locus of points from where the difference of the distances to two points, the foci, is constant. With this in mind, it is easy to show that the path length of a ray originating in point F (fig. 1) after reflection on the hyperboloid and the paraboloid to the aperture plane of the paraboloid is constant. One of the foci of the hyperboloid is F_1, which is also the focal point of the paraboloid; point F is the other focus of the hyperboloid, being the focus of the Cassegrain system also. Hence, a combination of a paraboloid with a hyperboloid, whose focal point coincides with that of the paraboloid, focuses a plane wave coming parallel to the paraboloid's axis into the second focus of the hyperboloid.

The system parabola/hyperbola can be replaced by an "equivalent parabola", that is, the surface S in figure 1. S is the equivalent focusing surface for the combination parabola and hyperbola. It is the principal surface of the system and is defined as the points of intersection of rays incoming parallel to the rotation axis of the parabola and the extension of the rays arriving in the focal point F. Thus the surface S will focus a wave arriving from the left parallel to the axis into point F. So we see that S is a paraboloid with diameter D as the paraboloid P and focal length F_e. The ratio of the focal length of the equivalent paraboloid S to the focal length of P is called the magnification of the Cassegrain system, denoted by m.

From simple geometry we can deduct the following formulae connecting the different quantities of the Cassegrain (fig. 1):

main reflector P $$\tan\frac{\psi}{2} = \frac{D}{4F} \qquad (1)$$

equivalent parabola S $$\tan\frac{\varphi}{2} = \frac{D}{4F_e} \qquad (2)$$

hyperbola H $$\cot\varphi + \cot\psi = \frac{2f}{d} \qquad (3)$$

eccentricity of H $$e = \frac{f/2}{f/2 - \ell} = \frac{\sin 1/2\,(\psi + \varphi)}{\sin 1/2\,(\psi - \varphi)} \qquad (4)$$

magnification $$m = \frac{F_e}{F} = \frac{\tan\psi/2}{\tan\varphi/2} = \frac{e + 1}{e - 1} \qquad (5)$$

The prime focus paraboloid has only one free parameter, the ratio F/D. The Cassegrain however has three free parameters. Having chosen the ratio F/D of the primary reflector P we can choose the diameter d of the subreflector H; this choice depends on the maximum blocking area that we want to allow. Now we can still choose the angle φ, or in other words the eccentricity e of the hyperbola. Or we can choose a certain magnification m and find from (5) the eccentricity e and from (4) the angle φ. So the Cassegrain system has three free parameters. From equation (2) we see that the Cassegrain is equivalent to a prime focus paraboloid of focal length F_e, where $F_e = m.F$. This is the reason for calling m the magnification.

It should be noted that the magnification applies only to some characteristics following from the geometry. In optical telescopes the magnification is defined in a different way. There it is the ratio of the focal lengths of the objective and the eyepiece.

B. Aperture Blocking

The subreflector in the Cassegrain will cause a certain aperture blocking, i.e., part of the incoming radiation will be intercepted by the subdish and will not be focused towards the feed in the focal point of the system. Hannan discusses the so-called "minimum blocking condition" [2]. In that case the shadow of the hyperboloid on the main reflector is just as big as the shadow of the aperture of the feed as seen from the focal point of the paraboloid (fig. 2). From figure 2 we see that $d \approx 2\varphi f$. When we denote the feed aperture by d_f and the angle under which it is seen from F_1 as $2\varphi_f$, the feed shadow on the paraboloid is

$$D_{min} \approx 2\varphi_f F \approx d_f \cdot \frac{F}{f}.$$

It follows directly that with the feed in the vertex of the paraboloid the minimum blocking condition is an equally large subdish and feed aperture.

With a 10 dB illumination taper on the edge of the subreflector we can take $2\varphi \approx 1.4 \frac{\lambda}{D}$. Combining the different formulae we arrive at a rather simple approximation for the minimum blocking area D_{min}:

$$D_{min} \approx \sqrt{3\lambda F}. \qquad (6)$$

This formula differs slightly from Hannan [2] since we have taken into account the illumination taper. Another way of writing the result (6) is:

$$\left(\frac{d}{D}\right)^2 \approx 3\lambda \frac{F}{D^2} \approx 2.5\,\Theta_A \frac{F}{D} \qquad (7)$$

where Θ_A is the half-power beam width of the antenna.

These formulae are only approximations, but are useful to obtain a first impression on the blocking. We see from (7) that the minimum blocking area can be small if the beamwidth is small, and further it turns out that a small ratio F/D for the paraboloid is advantageous. However, in most cases the minimum blocking condition is not fulfilled in an actual design because there are some other specifications on the size of the subdish. We will deal with these in the section on the electrical characteristics.

A feed designed for a particular ratio F/D can be used with any value of D as long as the percentage blocking stays constant. The value of e remains also unchanged in this case. If we want to use this feed on a reflector with another ratio F/D, the value of e for the subreflector has to be changed, because ψ changes and φ stays constant. Keeping the same blocking percentage we obtain a different value for f according to equation (3). The position of the feed must be changed; in its new position it will see the subdish again under an angle φ.

With an invariant main reflector and a flatter subdish this dish becomes larger, the feed beamwidth larger and the axial dimension of the antenna shorter. With an invariant feed beamwidth and a subreflector becoming more flat the main reflector becomes flatter and the axial dimension of the antenna increases.

III. ELECTRICAL CHARACTERISTICS

We shall now deal with the electrical properties of the two types of antennas and point out their specific advantages and disadvantages.

A. Aperture Blocking

Let us first consider the influence of aperture blocking on the gain and the sidelobe level of the antenna. The large aperture blocking of a Cassegrain is often considered to be a big disadvantage.

Two arguments set a lower limit on the dimension of the subreflector. First there is the fact that the subdish must be at least 10 wavelengths wide in order to prevent too much spillover due to diffraction effects on the edge of the subdish. In other words, the subreflector must be sufficiently large so that we can use geometrical optics in considering its effect. Secondly a very small subreflector would require a very narrow primary pattern. This means a very long feed horn, which is undesirable from the standpoint of manufacturing and sometimes also of available space. As a consequence of these requirements the minimum blocking condition is in general not met; the subreflector is larger and the feed smaller.

We can calculate the influence of blocking on the gain and the sidelobe level of an antenna as follows. The blocking causes essentially a gap in the aperture illumination of the paraboloid. Analytically we can allow for aperture blocking by subtracting the negative voltage pattern of the blocking area from the voltage illumination pattern of the undisturbed aperture (fig. 3). We can take the illumination over the relatively small blocking area uniform and so the voltage pattern of this region is:

$$g_b = -\pi \frac{d^2}{4} \frac{2J_1(u_b)}{u_b} = -\pi \frac{d^2}{4} \Lambda_1 (u_b) \approx -\pi \frac{d^2}{4} \tag{8}$$

with d the diameter of the subdish, $u_b = \frac{\pi d}{\lambda} \sin \Theta$. Because $\Lambda_1(u_b)$ hardly changes over the region of interest, we take the voltage pattern of the blocking area as a constant negative voltage.

The radiation pattern of the undisturbed aperture depends on the illumination. For an illumination of the form $(1 - r^2)^p$ the voltage radiation pattern has the form

$$g_a = \pi \frac{D^2}{4} \frac{\Lambda_{p+1}(u_a)}{p+1} \tag{9}$$

where $u_a = \frac{\pi D}{\lambda} \sin \Theta$.

Bearing in mind that $\Lambda_n(0) = 1$ for any n the ratio of the peak voltages is

$$\frac{g_b(0)}{g_a(0)} = \frac{\pi \frac{d^2}{4}}{\pi \frac{D^2}{4}} (p+1) = (p+1) \frac{d^2}{D^2} \tag{10}$$

The peak voltage of the resultant pattern g(0) is, when we normalize it to the peak of the undisturbed pattern g_a

$$g(0) = 1 - (p + 1)\frac{d^2}{D^2} \tag{11}$$

Let the peak voltage of the first sidelobe, normalized to the mainlobe, in the undisturbed aperture be g_s. This is a negative voltage (fig. 3). Taking the blocking into account, this level changes to $(g_s + g_b)$. The sidelobe level for the undisturbed case in dB is

$$SLL(0) = 20 \log g_s \tag{12}$$

This changes in the case with blocking into

$$SLL(B) = 20 \log \frac{g_s + g_b}{g(0)} = 20 \log \left[\frac{g_s + (p + 1)\left(\frac{d}{D}\right)^2}{1 - (p + 1)\left(\frac{d}{D}\right)^2} \right] \tag{13}$$

N. B. Silver and his book [12] calculates, along the same line of reasoning, the blocking of a line source. However, his final result (eq. 6.67, p. 191) for the sidelobe intensity is in error. In fact, p' should be replaced by 1/p'.

Examples. Uniform illumination of paraboloid, hence $g_a = \pi \frac{D^2}{4} \Lambda_1 (u_a)$; theoretical sidelobe level is -17.6 dB. With 1 percent blocking (d/D = 0.1) we find the gain $g(0) = 1 - d^2/D^2 = 0.99$, or a decrease in the gain of 0.1 dB. The sidelobe level is

$$SLL = 20 \log \left[\frac{0.132 + 0.01}{0.99} \right] = 20 \log [0.1435] = -16.85 \text{ dB.}$$

So the sidelobe rises about 0.75 dB.

The same results for an illumination of the form $(1 - r^2)$ have been drawn in figure 4. In this figure also the sidelobe level for a 10 dB edge taper has been drawn. This curve is calculated from radiation patterns given by Nihen and Kay [7] obtained by machine computations. From the curves in figure 4 it is seen that a subreflector diamter d = 0.1 D is a practical choice. The decrease in gain is small and the increase in sidelobe level is a few dB. The theoretical sidelobe level stays under 20 dB, which is generally accepted to be satisfactory in radio astronomy. Moreover the diameter of the subdish will be more than 10 wavelengths in this case and the angle φ will be of the order of 10°. The construction of the feed horn is not too difficult and the length of the feed stays in a convenient range.

Let us compare the subdish blocking of one percent with the blocking of a front end box in a prime focus instrument. The pillbox of the NRAO 85-foot telescopes gives a blocking of about 0.4 percent. Together with the feed support legs the blocking is 2 percent. As the physical size of the front end box hardly changes with frequency we see that especially at relatively small telescopes (10-40 feet) the blocking of the subdish is comparable with that of the pillbox. As the subdish can be light the support can be less heavy than in a prime focus antenna, which diminishes the blocking area. The support must be strong enough to meet the stability requirements. As we shall see these are about as stringent as for the feed in a prime focus antenna. On the other hand, for large telescopes, as the 300-foot, the subdish will be heavier than the pillbox and the supports must be even stronger. In that case the blocking of the Cassegrain will be more serious than that of the primary focus paraboloid.

B. Spillover and Efficiency

An important consideration in the design of a radio telescope is the spillover radiation, that is, the radiation which enters directly into the feed horn due to the fact that the illumination pattern is not zero at the edge of the reflector.

In the case of a prime focus antenna, this radiation originates at the earth's surface; it is the thermal radiation of the earth and the radiation temperature is about 300 °K. In the Cassegrain, however, the feed looks into the sky and the radiation temperature of the sky is very much lower than that of the earth. So the spillover radiation is much lower. Actually there is some radiation of the earth entering the feed via reflection at the subreflector, but the contribution is very small. In any case where we are concerned with low noise, as in measurements of the background radiation of the galaxy, the Cassegrain type is superior to the conventional paraboloid alone. The spillover depends on the F/D ratio of the paraboloid; it increases from about 4 percent at F/D = 0.3 to 15 percent for a value F/D = 1.

It has often been stated that the obtainable aperture efficiency of a Cassegrain is higher than in the case of a primary focus paraboloid. There is, however, no experimental clue as to the validity of this statement.

As the effect of spillover is less serious in the Cassegrain, it would be possible to use a less heavy illumination taper on the paraboloid. This will increase the aperture efficiency slightly. In fact this seems to be the case in some actual Cassegrains. But the experimental proof of an increase in aperture efficiency has not been given in the literature.

As an example, we mention here the efforts of the Jet Propulsion Laboratory to optimize an 85-foot Cassegrain for aperture efficiency and low noise [8]. Taking as the maximum theoretical aperture efficiency 0.60, the efficiency of the prime focus instrument derived from the published random errors [4] in the parabolic surface at the used frequency of 960 MHz is about 0.54. Using a special shaped subreflector in order to optimize the aperture efficiency gave a calculated efficiency for the Cassegrain of 0.59. The measured value was 0.50 ± 0.08. It is clear that there is no big improvement. On the other hand the zenith antenna temperature with the special shaped subreflector was less than 10 °K, which is a real improvement compared to a typical value of 30 °K as measured at NRAO for a prime focus telescope.

The influence of the random errors in the surfaces of the antenna is far more important than the illumination. The aperture efficiency of a paraboloidal antenna with random deviations of the best fitting paraboloid of the rms value d is given as

$$\eta_A(\lambda) = \eta_{A_0} \exp\left[\frac{-16\,\pi^2\,d^2}{\lambda^2}\right] \tag{14}$$

where η_{A_0} is the maximum theoretical efficiency of the undisturbed aperture and λ the wavelength [6] [9]. In this respect the Cassegrain is even more sensitive than the prime focus antenna because now we have two surfaces each with its own random error. As these errors are independent their contribution to the phase error adds. If we allow the subreflector to contribute 10 percent of the decrease in η_A from the maximum value η_{A_0} as a result of random errors, it follows that the surface accuracy of the subreflector has to be 3 times better than that of the paraboloid. For telescopes at very high frequencies it may be necessary to use glass mirrors for the secondary reflector.

C. Feed Design and Sidelobe Level

In designing a feed which will give a certain illumination taper the concept of the equivalent paraboloid is very useful. One designs the feed for this paraboloid and it will give the right illumination of the Cassegrain. The dimensions of the feed are about m times as big as in the prime focus antenna. This can lead to rather clumsy feed dimensions, with also a larger attenuation along the long horn. For example, a study made by TRG for a multifrequency Cassegrain feed system on the 300-foot telescope yields a minimum feed length of 52 feet (16 m) and an aperture width of 12 feet (3.6 m) when used with a 20-foot diameter subdish. At the highest frequency of 1400 MHz the phase error over the feed aperture is even half a wavelength. The use of a hornlens, a short horn with a lens in the horn aperture to correct the phase front of the outcoming waves, can be of advantage in some cases. The feed for our 3 mm radiometer is a hornlens only 5 cm long with a total loss of 0.35 dB. According to the manufacturer this loss is less than that of a horn without a lens, which would have to be about 80 cm long in order to obtain the same illumination.

Calculations have shown (especially [7] gives much information) that the sidelobe level does not depend on the ratio F/D of the paraboloid nor on the choice of the distance f, which determines the position of the feed. As to this last point, we have to mention here that f has to be large enough in order that the subreflector is in the far field of the feed. This means that $f > \frac{2D^2}{\lambda}$, where D is the aperture dimension of the feed horn. The sidelobe level does depend on the illumination taper in the same way as the paraboloid with the feed in its focus. For a ratio d/D = 0.1 the sidelobe level for a 20 dB taper on the edge is 2 dB lower than for a 10 dB taper.

D. F/D Ratio

In this section we deal with some properties of paraboloidal reflectors which are a function of the F/D ratio. The cross polarization radiation which can harm especially the sidelobe level, becomes less with increasing F/D ratio. This has already been noticed by Silver [12] and also by Kay. Because the paraboloid has a curved surface, the polarization of the field of the feed (primary pattern) will be affected by the reflection on the surface and the aperture field will have a different polarization in different points. Analytically it is expressed by the plane wave boundary condition on the surface

$$\underline{n} \times (\underline{e}_0 + \underline{e}_1) = 0 \qquad (15)$$

where $\underline{n}$, $\underline{e}_0$ and $\underline{e}_1$ are unit vectors defining the normal on the surface, the polarization of the primary and secondary pattern, respectively. In words it means that the tangential electrical field must be zero at the surface. We can resolve the electrical aperture field into the principal polarization (parallel to E-plane) and cross polarization (perpendicular to E-plane) components. The aperture distribution is indicated in figure 5a. It is noted that by symmetry the cross polarization vanishes in the principal planes. The cross polarization pattern has its maximum in the planes under 45° with the principal plane (fig. 5b). It turns out that the maximum of the cross polarization lobe has the same position as the first minimum of the main beam. So one sees that the main beam can be deteriorated by the cross polarization radiation, especially in the directions outside the principal planes. Calculations and measurements [3] indicate that the level of the cross polarization lobe goes from -16 dB for F/D = 0.25 to -28 dB if F/D = 0.60. It is clear that a long focal length is of great advantage here. This is caused, of course, by the smaller curvature of a paraboloid with longer focal length.

For the case where the feed is displaced in the radial direction the cross polarization is more serious because the symmetry in the illumination disappears.

A long focal length is also desirable if an off-axis feed has to be used. The sidelobe level, especially the coma lobe, rises much slower for a long F/D ratio. We shall deal with the off-axis characteristics in more detail later.

On the other side, however, we found already that the spillover radiation of a long focal length antenna is larger than that of one with short focal length. In fact there is an optimal F/D ratio as far as aperture efficiency is concerned which depends on the illumination taper and lies between 0.35 and 0.55 for a prime focus antenna. All NRAO telescopes have an aperture angle of 60°, i.e., a F/D ratio of 0.43. The optimum taper for this ratio gives sidelobes below -20 dB and the cross polarization is about -23 dB below the maximum level of the main beam, which gives virtually no trouble. It has to be noted that the spillover depends mainly on the F/D ratio of the paraboloid and hardly on the magnification of the Cassegrain system.

We now turn to an investigation of the system characteristics in the case that the feed is displaced from the correct focal point. We distinguish between axial and radial defocusing. It is clear that any defocusing can be resolved in a radial and axial component. It is not known analytically, however, whether the effects of the two components can simply be added to obtain the influence of an arbitrary feed displacement. The integrals needed for the computation are of the same type as those for the computation of the Fresnel field of a radially displaced feed and are being studied by the author at the present time.

In the Cassegrain antenna we have to deal with a displacement of the feed and of the subreflector.

E. Axial Defocusing

The effect of an axial displacement of the feed on the radiation pattern is approximately that of a quadratic phase error over the aperture. There will be a decrease in the gain G, and hence in the effective aperture A, of the antenna and a broadening of the main beam. The gain of the antenna is connected to the maximum phase error over the aperture β by the formula:

$$\frac{G}{G_0} = \left[\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\right]^2 \qquad (16)$$

where G_0 is the gain without phase error and β is the maximum phase error over the aperture (between vertex and edge). This formula is valid for uniform illumination. The more difficult derivation of the formulae for the tapered illumination and the beam broadening is dealt with in another report [1].

It is clear that β is proportional to the maximum change in path length difference δ, taken from the phase center of the feed to the aperture plane of the antenna, between the edge ray and the vertex ray due to the axial defocusing. Let us calculate the values of δ for the different cases. When we move the feed of a prime focus antenna from the focal point over a distance ϵ, the vertex ray becomes ϵ longer and the edge ray $\epsilon \cos \psi$. By the definition of δ we find

$$\delta = -\epsilon\,(1 - \cos \psi), \text{ with } \psi \text{ the aperture angle.} \tag{17}$$

Thus given a maximum tolerable δ we find the maximum tolerable ϵ from (17).

In the Cassegrain we have two possibilities, communicated to us by Ruze:

1. Displacement of the feed. Bearing in mind that the length of the rays from subreflector to paraboloid stays constant we easily find

$$\delta = -\epsilon\,(1 - \cos \varphi), \text{ with } \varphi \text{ the aperture of the subdish at the feed.} \tag{18}$$

2. Displacement of the subreflector. A more tedious but otherwise straightforward calculation yields

$$\varphi = -\epsilon\,[(1 - \cos \psi) + (1 - \cos \varphi)]. \tag{19}$$

The corresponding phase difference β follows from $\beta = (2\pi/\lambda)\,\delta$, and the change in gain can be found from (16).

We see that we would have found (18) by using the concept of the equivalent paraboloid. In figure 6 the curves from (16) are drawn using equations (17) - (19) for a telescope with $F/D = 0.35$, $m = 11.5$, the feed in the vertex ($F = f$) and a wavelength of 9 mm.

The following conclusions can be made:

1. The position of the feed is not very critical in the Cassegrain. In fact it is about m times less critical than in the prime focus case.
2. The position of the subdish, however, is very critical, even more than the position of the feed in the primary focus of the paraboloid. In general φ is small (about 10°) and so $(1 - \cos \varphi)$ will be nearly zero, that is, the situation where the position of the feed is not critical at all and the position of the subdish is about as critical as the feed position in the prime focus. The subdish position is more critical for a higher eccentricity of the hyperboloid.

Taking as an example a Cassegrain with F/D = 0.35, so $\psi = 71°$ and $\varphi = 7°$, and allowing a value for $\delta = \lambda/16$, which means $G/G_0 = 0.99$, we find for the tolerable defocusing:

paraboloid (eq. 17) $\epsilon = 0.095\ \lambda$.

Cassegrain

feed displacement (eq. 18) $\epsilon = 8.33\ \lambda$

subdish do. (eq. 19) $\epsilon = 0.091\ \lambda$

F. Radial Defocusing

The effect of radial displacement of the feed has been studied by several authors. Ruze [10] has investigated the defocusing of the prime focus case. A report written by Nihen and Kay [7] deals with the Cassegrain and the Schwartzschild antenna, as does the work of White and de Size [13].

The radial displacement of the feed causes a tilt of the beam, a decrease of the gain, an increase in the half-power beam width and an enhancement of the side-lobe level at the side of the axis of rotation of the system, that is, the so-called

Comalobe. These effects can be described by a linear and a cubic phase error over the aperture. However, computations in this case are difficult and need in most cases computer help. We shall only give a compilation of results as found in different reports together with some curves.

The most striking characteristic of the Cassegrain system is the very low coma aberration. This is due to the long effective focal length. We can define the scanning range of an antenna as the number of half-power beam widths (HPBW) scanned off-axis until a certain decrease in gain (e.g., 1 dB) or rise in sidelobe level has been reached. Generally (in the prime focus case) the scanning range varies about as the square of the ratio (F/D). However, in the Cassegrain the scanning range varies only little with the F/D ratio; in fact, for $F/D > 0.5$ it decreases slowly due to the increasing spillover. The large effective focal length makes the scanning range nevertheless much larger than in prime focus antennas.

Figure 7 gives a comparison for the relation of gain and sidelobe level, respectively, as a function of scanning angle for the Cassegrain and prime focus antenna with equal F/D ratios of the paraboloid. For example, scanning 4 HPBW causes a drop in the gain of 1.5 dB for the prime focus antenna and only 0.3 dB for the Cassegrain. The comalobe of the prime focus is -10 dB which is an intolerable high value. The Cassegrain gives -20 dB where 1 percent blocking is included.

The increase in the HPBW goes slowly to a scan of about 4 beam widths; if the scan is larger it increases faster. The scanning range has a maximum as a function of the diameter of the subdish at the value $d/D = 0.4$. But this is not of practical importance, as in that case the aperture blocking is too high.

It is well known that the tilt of the beam in a radially defocused paraboloidal antenna is smaller than the angular displacement of the feed. This is due to the fact that the reflector surface is not flat and hence the reflection according to Snell's law is modified. The Beam Deviation Factor (BDF) connects the two quantities. Various authors [5], [11], [12] have published calculations and curves of the BDF (fig. 8).

For F/D ratios larger than one the BDF approaches the value 1, i.e., the reflection is essentially that on a plane mirror. In a Cassegrain antenna the effective F/D ratio is generally much larger than one and so we can take BDF = 1. The radial feed displacement Δf_{rad} of the prime focus antenna gives, with a focal length F and BDF = 1, a beam tilt of about $\Delta f_{rad}/F$. In a Cassegrain with the same F/D ratio for the paraboloid and a magnification m the necessary feed displacement in order to obtain the same beam tilt is $m\,\Delta f_{rad}$. In other words (fig. 1): a displacement Δ of the feed in the Cassegrain is equivalent to a displacement Δ of the feed of the equivalent paraboloid (with focal length $F_e = m.F$); it is also equivalent to a displacement Δ/m of the virtual feed in the focal point F_1 of the paraboloid P.

We found already that the physical dimensions of the feed in the Cassegrain are m times bigger than in the primary focus antenna. Two feeds located just beside each other give the same angular distance between the beams in both cases.

The beamwidth to half-power for an illumination with about 18 dB taper (NRAO feeds) is approximately $\Theta_A = 1.2\ \lambda/D$. Taking the BDF = 1 we can find the radial defocusing per beam deviation of one HPBW from

$$\Delta f_{rad}\,(1\ \text{HPBW}) = 1.2\ m.\lambda.\frac{F}{D}\ . \qquad (20)$$

For a prime focus paraboloid m = 1. Figure 9 gives the beam tilt as a function of the radial displacement for a 0.35 and 0.43 F/D ratio with m = 11.5 and 8.0, respectively, and wavelengths 3.5 and 9 mm for the first and 2 and 6 cm for the second case.

Calculations have shown that the axial position of the feed in an off-axis position is not critical. This is in accordance with the result on the influence of the axial defocusing of the feed. The axis of the feed has to be directed towards the vertex of the subreflector. We can define a depth of focus as the distance over which the feed can be moved in axial direction to obtain a decrease in the gain of 0.5 dB. The depth of focus is found to be inversely proportional to $\operatorname{tg}\varphi$. In the region $\varphi = 10°$ the depth of focus is about 0.1 D.

Another possibility to achieve a beam tilt is to rotate the subreflector. The angle of tilt is about half the desired beam deviation. This will give the same effect as feed displacement. We have to be careful that the axial position of the subreflector stays focused because that is a very critical point as we saw in the last section. Rotation around the point of the subdish on the mechanical axis of rotation of the antenna will be necessary. Although no calculations have been made, it is estimated that for beam tilts of the order of a few degrees the rotation of the subreflector around its vertex without readjustment of the feed would give a beam deflection without deteriorating the beam shape. It would be interesting to check this statement in an experiment. Also, we have plans to perform calculations on this problem.

Subreflector tilt has been suggested by Christiansen as a simple means of obtaining scanning possibilities in a transit radio telescope. According to him this would be far easier than moving the relatively large feed with its connections to the radiometer. On the other hand, however, one has the problem of rotating the subreflector while accurately controlling the axial position of its vertex. Especially at very large telescopes, and these are most likely to be transit instruments, this can give severe mechanical problems. One considers, for example, a 10 m subdish in the 300-foot antenna.

It has to be said finally that the radial displacement of the feed causes some astigmatism in the plane perpendicular to the direction of scan. As a consequence the beam will be broader in that plane. Calculations and experimental results have not been found in the literature.

IV. MECHANICAL CHARACTERISTICS

As the preceding sections have shown, there are some advantages in the Cassegrain type of radio telescope as far as the electrical characteristics are concerned. We shall now point out some features in the field of mechanical construction which will show that the Cassegrain will be of advantage in many cases over the primary focus antenna. First of all there is the fact that the focal length of the paraboloid can be short, while still the desired long effective focal length is achieved. It is clear that it is advantageous as a short construction can be made more rigid.

In a relatively small antenna the support legs for the subreflector can be more light as long as stability is preserved due to the smaller weight of the subdish as compared to the sometimes quite heavy pillbox with front end of the radiometer. Making the legs thinner diminishes the blocking and the scattering on the supports. We have to mention here that the alignment of the system is harder in the case of the Cassegrain, because both the feed and the subdish have to be put in the correct position with respect to the axis of rotation of the system.

The feed in the Cassegrain is conveniently located close to the vertex of the paraboloid. This means that it is possible to mount the front end of the radiometer behind the reflector and make a very short connection to the feed horn. This is especially advantageous for bulky front ends as masers with the cooling equipment or, to take an example outside radio astronomy, in the case of a radar transmitter with monopulse feeds. As a disadvantage, on the mechanical side is the need for two reflecting surfaces with their tolerance requirements. Also the feed has to be much larger, which will in some cases give more trouble to meet the specifications.

If theoretical and experimental results show the feasibility of tilting the subreflector in order to obtain scanning possibilities with a transit telescope, this method is more likely to be used than moving the feed. Although the mechanical requirements imposed on the stability and accuracy of the tilted subreflector are quite severe, it seems that they are easier to meet than the problems of moving the large and heavy feed together with the attached equipment.

V. SUMMARY AND CONCLUSIONS

A comparison of the most important characteristics of the prime focus and the Cassegrain antenna is made in Table 1. The table speaks for itself. The choice of three free parameters makes it easier to combine electrical and mechanical specifications. The aperture blocking for relatively small antennas (< 50 feet diameter) is not worse than that of the pillbox in a prime focus telescope. The choice of 1 percent ($d = 0.1 D$) is good as the theoretical sidelobe level stays under -20 dB. In practice, however, one finds sometimes a small shoulder on the main beam of about -17 dB. One can keep the F/D ratio short to obtain rigidity and low spillover. Due to the magnification the effective focal length is long and hence the coma lobe is weak and the cross polarization unimportant. The possibility of achieving a long effective focal length with a mechanically short antenna is one of the most interesting characteristics of the Cassegrain.

The aperture efficiency increase of a Cassegrain is only marginal. The illumination can be taken a little less tapered, but the random errors of the reflecting surfaces are still the main cause of the final aperture efficiency. The surface accuracy of the subreflector must be very high, about three times better than that of the main reflector.

For the feed design one can use the equivalent paraboloid; the feed has to be designed to illuminate the equivalent paraboloid in the desired manner. The actual illumination of the main reflector will be correct in that case. The feed dimension is m times bigger than in the prime focus antenna.

The final conclusion of this investigation is that for normal applications in radio astronomy the Cassegrain antenna is not definitely superior to the prime focus telescope. For big telescopes (> 100 feet in diameter) it is inadvisable to use the Cassegrain; for diameters less than 50 feet the Cassegrain can give a slightly better performance. Only in special circumstances, as the need for very low noise and off-axis feeds, the Cassegrain is in a favorable position.

Many of the characteristics of the Cassegrain antenna are only a result of theoretical investigation. There is a definite need for experimental verification of many points, as aperture efficiency, spillover radiation and off-axis characteristics. Here the idea of rotating the subdish deserves a careful investigation.

The properties of prime focus paraboloids are at the moment fairly well understood and we have gathered a considerable amount of experimental data. It would be very interesting if the next 85-foot antenna at NRAO were a Cassegrain so we could make experiments on the antenna, accurate comparison material for which is available.

In the appendix we have collected numerical material on three Cassegrain antennas in order to illustrate their properties. The antennas are a 30-foot antenna designed and built as a Cassegrain for mm-wave observations, a 85-foot antenna identical to the NRAO telescopes but changed into a Cassegrain and to be used at wavelengths larger than 2 cm, and the 300-foot modified in a Cassegrain.

Acknowledgment -- It is a pleasure to thank Drs. Mezger and Findlay for interesting discussions.

TABLE 1

COMPARISON BETWEEN PRIME FOCUS AND CASSEGRAIN ANTENNA

	Prime focus	Cassegrain
Focal length maximum gain	F/D = 0.35-0.55	F/D = 0.25-0.40
Number of free parameters	F/D	F/D; d; m
Accessibility of equipment	Difficult	Easy
Feed diameters		m times of P. F.
Aperture blocking	85-foot typical 2%	Typical subdish 1% -- supports ≈ 1%
Aperture efficiency with 85-ft.	NRAO 58 ± 5%	JPL 50 ± 8%
Typical zenith T_A	30 °K	10 °K
First sidelobe level (confusion)	-25 dB	-20 dB theoretical -17 dB (shoulder) (experimental)
Gain variation due to axial defocusing		Feed -- m times less than P. F. Subdish -- worse than feed in P. F.
Coma effect due to radial defocusing		For small angle scanning about m times less than P. F.
Radial feed displacement for 1 HPBW tilt		m times larger than P. F.

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APPENDIX

NUMERICAL RESULTS FOR THREE CASSEGRAIN TELESCOPES

1. 30-Foot North American Aviation Cassegrain Antenna

The F/D ratio is 0.35, the focal point lies in the vertex of the paraboloid (F =f), d = 27' = 68.5 cm, and the eccentricity of the hyperboloid e = 1.19. The surface deviations are for the paraboloid 0.1 mm r.m.s. and about 0.02 mm r.m.s. for the subreflector.

We caluclate F = 3.20 m and ψ = 71°, φ = 6.35°, so the feed sees the subreflector under an angle of 12.7°. The magnification is m = 11.5 and hence the effective focal length is 36.8 m. The distance from the phase center of the feed to the vertex of the subreflector is found to be (d/2) cot φ = 3.08 m. Further is d/D = 0.075 and the blocking is 0.563 percent.

From figure 4 we see that this introduces a gain decrease of about 0.2 dB and the sidelobe level is approximately -22 dB. A beam tilt of 4 HPBW decreases the gain 0.3 dB (fig. 7) and the coma lobe is -19.5 dB. The feed displacement is 7 and 17 cm for 3.5 and 9 mm wavelength, respectively (fig. 9).

From equation 14 we find G/G_0 = 0.97 at λ = 9 mm and 0.83 at λ = 3.5 mm.

2. 85-Foot NRAO, Modified to Cassegrain

Let us choose d = 0.1 D = 2.6 m. We also want the focal point in the vertex of the paraboloid. Given is F/D = 0.43 and so ψ = 60°.

From (3) follows φ = 7.1° and hence m = 9.3 and e = 1.24. The distance focal point F to vertex of hyperboloid is (d/2) cot φ = 10.4 m. The effective ratio F/D = 4.0. The blocking (≈ 2%) causes a gain drop of about 0.5 dB and a sidelobe level of -20 dB. Coma lobe with 3 HPBW tilt is -20 dB; the radial feed displacement is then 25 cm for a wavelength of 2 cm.

APPENDIX (CONTINUED)

3. 300-Foot NRAO, Modified After TRG Proposal

The idea was to obtain a multifrequency feed in the vertex of the paraboloid. Chosen was a subdish diameter of 20 feet. The feed dimensions in this case turn out to be 52 feet long and an aperture diameter of 12 feet. Even with this very large feed the phase error at 1400 MHz over the feed aperture is one-half of a wavelength. The magnification of the Cassegrain is 4, so the effective F/D ratio is 1.7.

It is clear that the blocking of a 20-foot subreflector is much more than that of the pillbox. Still the blocking is less than 1 percent with a 20-foot subdish. The weight of the subdish is estimated at 2,000 pounds. As a consequence the support legs have to be made stronger. Moreover the stability of the subreflector and its position with changing elevation of the antenna seems to be insufficient to obtain a stable focused position. So the whole subreflector should be remotely movable in order to put it in the focal point. Considering these problems it cannot be suggested to try the change on our 300-foot telescope.

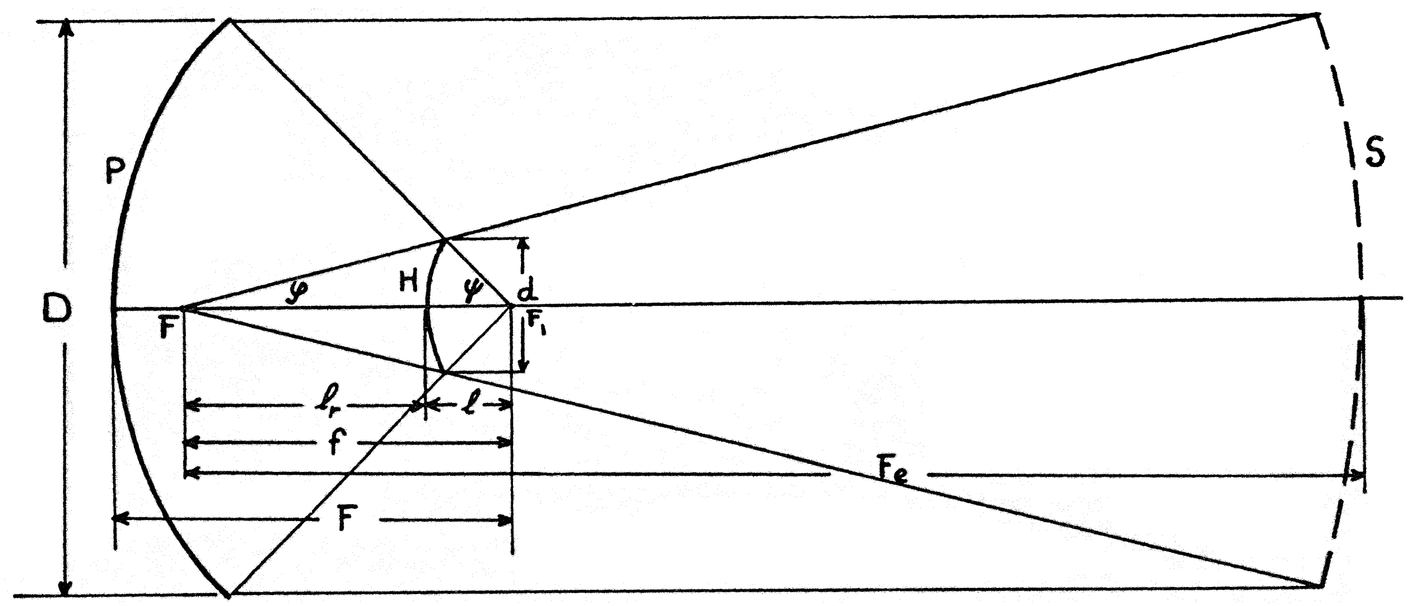

Fig 1. Geometry of Cassegrain Antenna

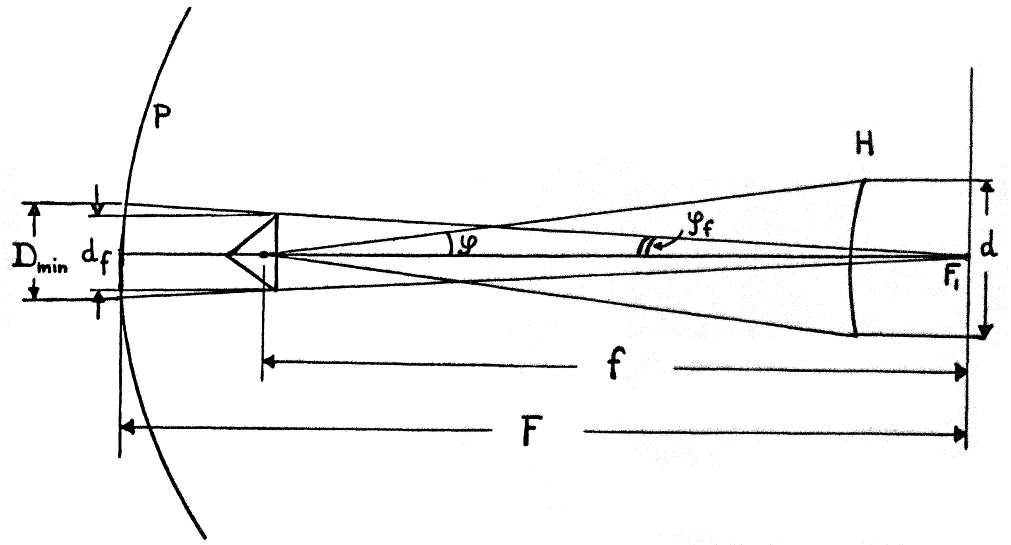

FIG.2. MINIMUM BLOCKING CONDITION

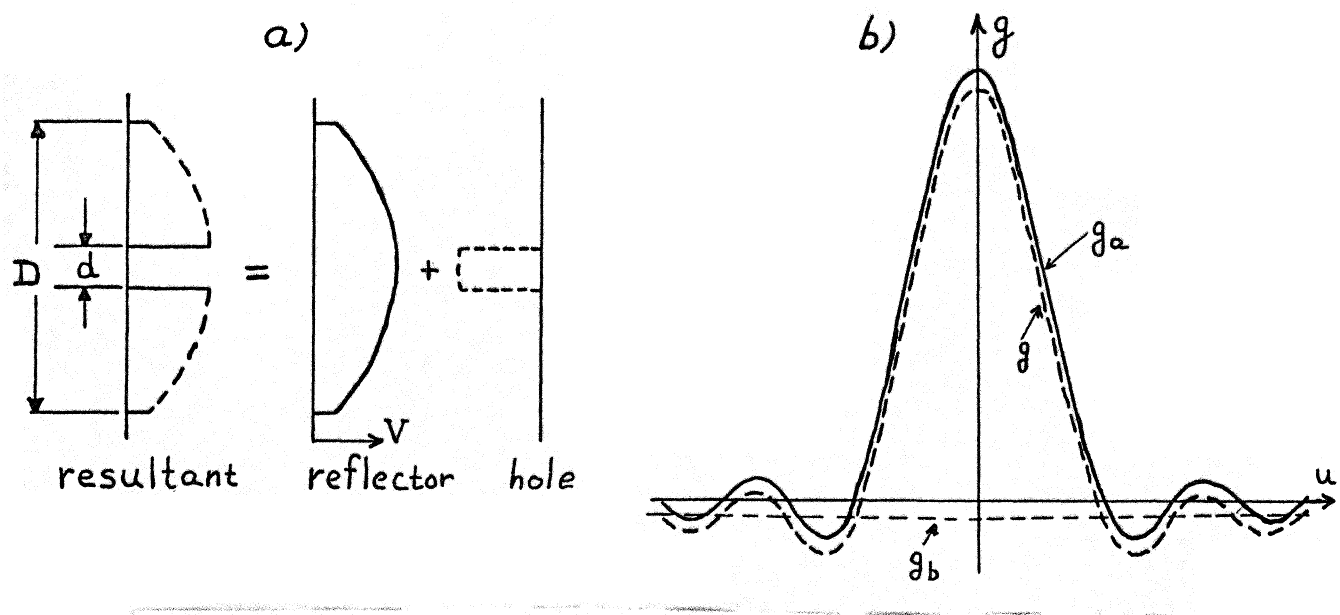

FIG.3 EFFECT OF APERTURE BLOCKING [2]

a) APERTURE ILLUMINATION b) VOLTAGE RADIATION PATTERN

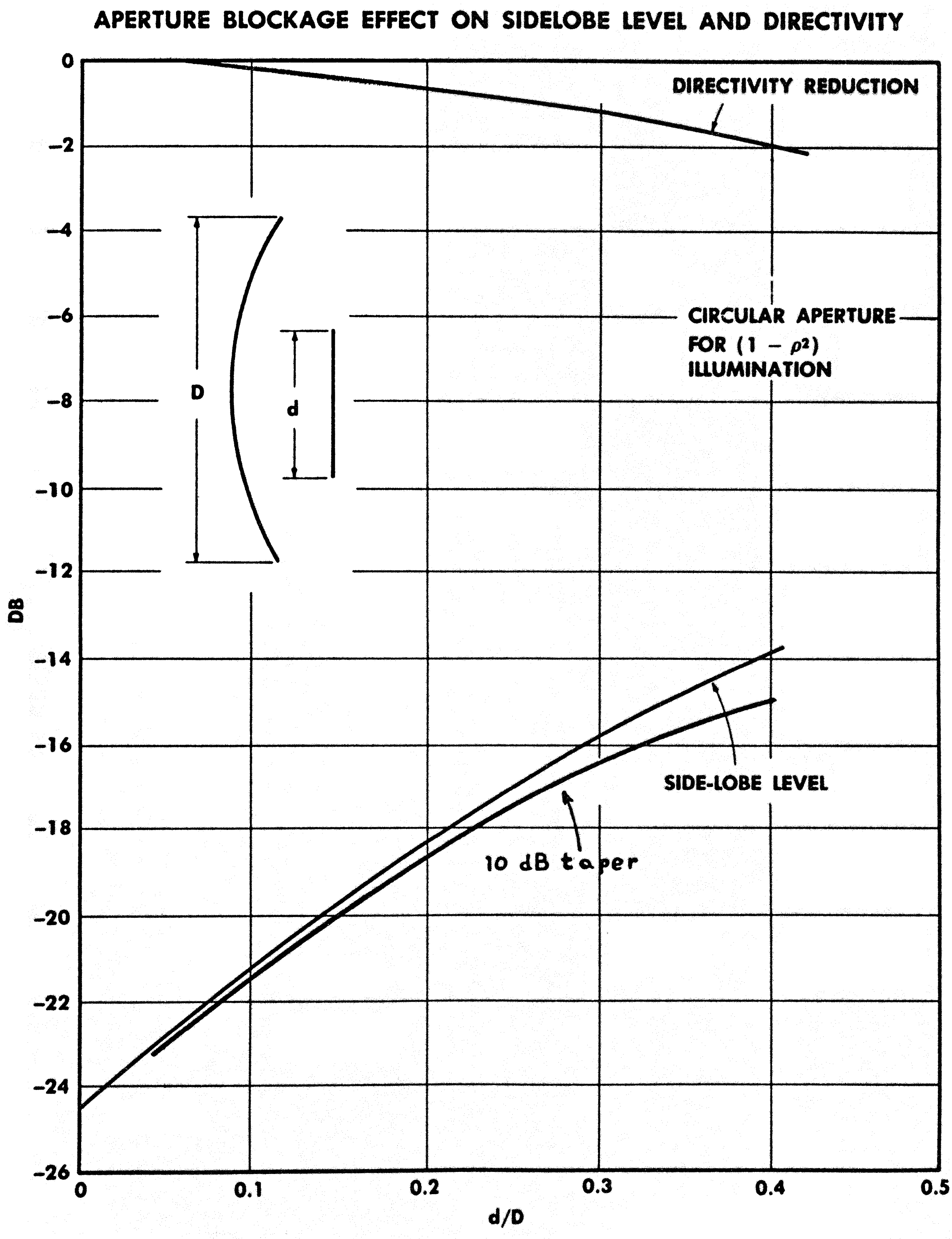

FIG.4 GAIN AND SIDE LOBE LEVEL AS A FUNCTION OF APERTURE BLOCKING

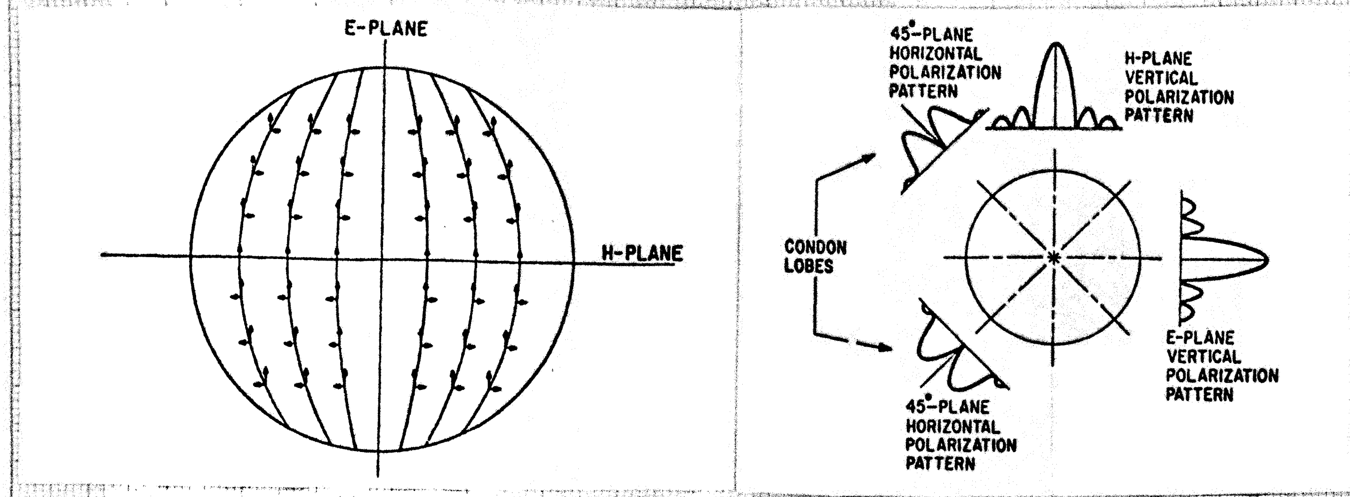

FIG.5 CROSS POLARIZATION a) APERTURE FIELD b) RADIATION PATTERNS [3]

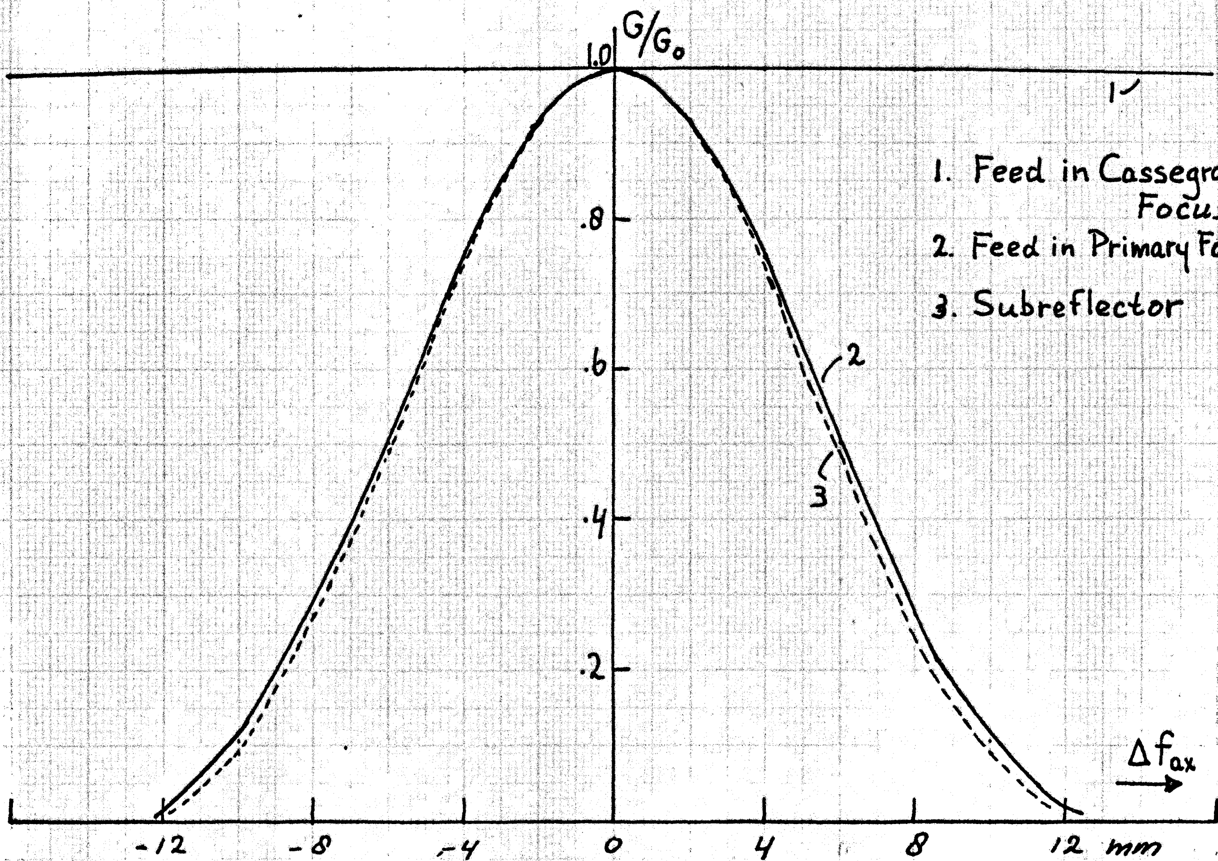

FIG.6 GAIN AS FUNCTION OF AXIAL DEFOCUSING. 1) FEED IN CASSEGRAIN FOCUS 2) FEED IN PRIMARY FOCUS. 3) CASSEGRAIN SUBREFLECTOR

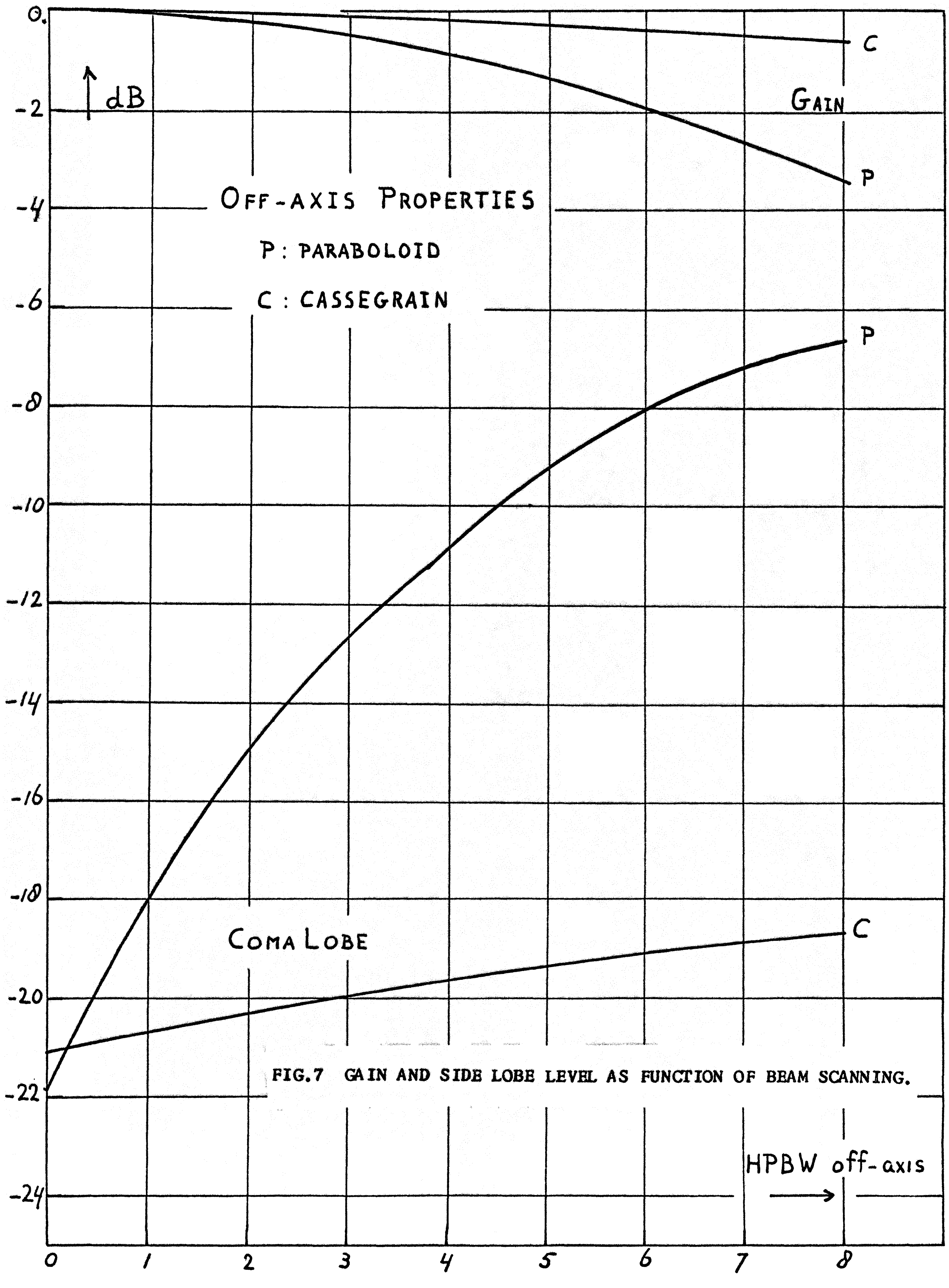

FIG.7 GAIN AND SIDE LOBE LEVEL AS FUNCTION OF BEAM SCANNING.

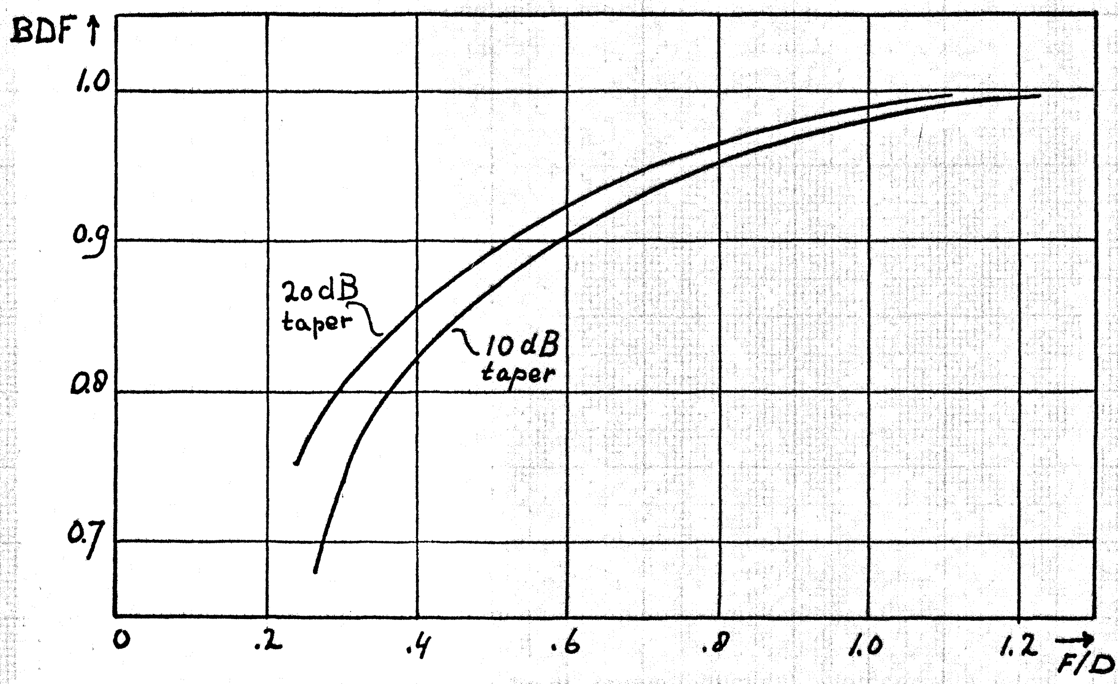

FIG.8 BEAM DEVIATION FACTOR AS FUNCTION OF F/D RATIO

FOR TWO ILLUMINATION TAPERS

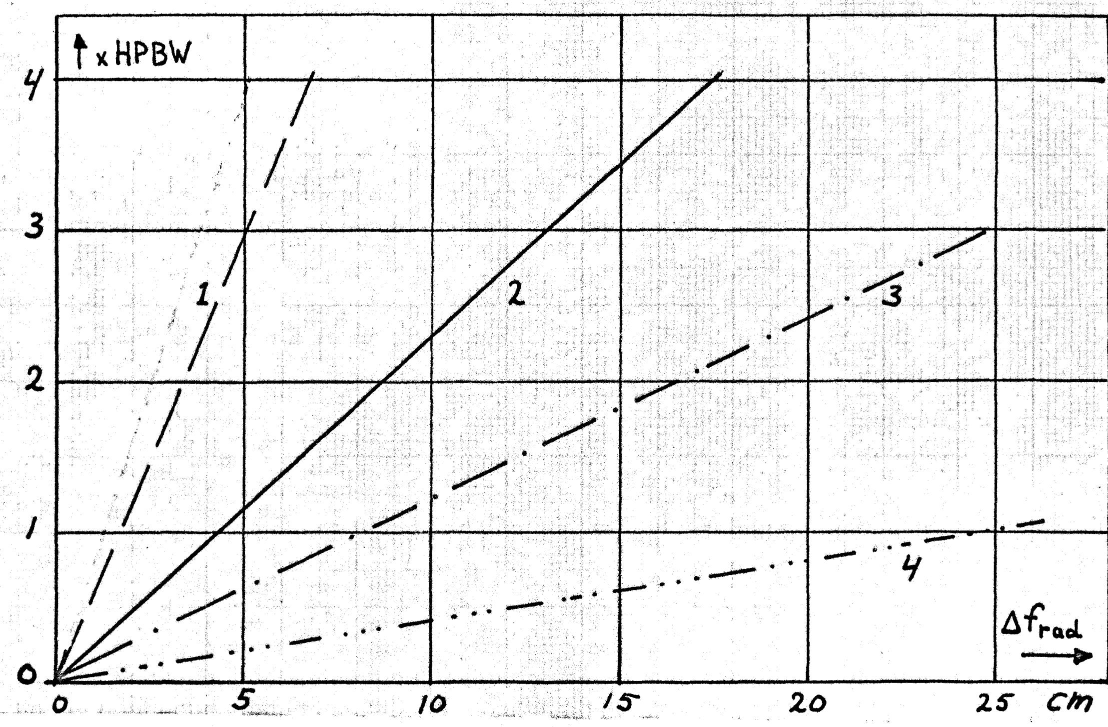

FIG.9 BEAM TILT IN HPBW AS FUNCTION OF FEED DISPLACEMENT IN CASSEGRAIN ANTENNA

1 AND 2: F/D = 0.35; m = 11.5; = 3.5 AND 9 MM RESPECTIVELY

3 AND 4: F/D = 0.43; m = 8.0; = 2 AND 6 CM RESPECTIVELY