A MEASURE ON COMPLIMENTARY ARRAY PAIRS

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Many u-v plane coverages of an array have concentration of holes. It is therefore necessary to derive a complimentary array such that the u-v plane coverages of this latter array cover the holes of the former. When this is done successfully, there should be few holes left and these two arrays form a complimentary pair.

Let us assume that we have an array of p fraction of holes within a given u-v area and at a certain declination and tracking. Then we have another array of q fraction of holes under the same area and conditions. If the two arrays are "truly" complimentary, i.e., no part of one array coverage overlaps that of the other and causes redundancy, then the total fraction of filled cells is

$$(1-p) + (1-q) \quad . \tag{1}$$

The total fraction of holes left is therefore

$$H = 1-((1-p) + (1-q)) = p+q-1 \quad , \tag{2}$$

with the understanding that if H is zero there are no holes.

On the other hand, if the two arrays are "truly" uncomplimentary, they will overlap each other in all areas. The total fraction of holes will be either

$$H = p$$

or (3)

$$H = q$$

whichever is smaller.

Both of these are idealized cases; the practical design cases should lie somewhere in between. A third case can now be singled out. If the two arrays are completely uncorrelated in design, through statistical reasoning, the total fraction of holes is

$$H = pq \tag{4}$$

This may be called the random case and can be used as a measure of success in designing a complimentary array pair. If the fraction of holes of the paired arrays is smaller than (4) and is approaching (2), we can say the array pair is highly complimentary. If the reversed situation exists and is approaching (3), we can say that the array pair is not complimentary. Improvement should be possible to make it more complimentary.

To end, we may say that a more quantitative measure should be derived. Conway has suggested to define a figure of regression ρ, such that

$$\rho = \frac{H-pq}{D} \tag{5}$$

where D is the no normalizing factor. It should be desirable to have $\rho = -1$ for truly complimentary case, $\rho = 0$ for random case and $\rho = 1$ for truly uncomplimentary case. However, for the above conditions, D is hard to define.

To the actual problem of getting minimum holes within a given u-v area, it is obvious that the measure of complimentary is only a partial answer to how much improvement can be possible. The baseline lengths and their orientations have to be taken into account. Take the examples of array pairs studied by Hogg and Chow. While those of Hogg's design are about the measure of the random case and those of Chow's are better than the random case, the actual numbers of holes in the examples of both are about the same.

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