HOLE DENSITY VERSUS ARRAY PHASE ERRORS

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Studies on complementary array have pushed hole density below the 5% mark. Spurious sidelobes due to these holes will therefore be low. In comparison, spurious sidelobes due to array phase errors may become important. To see whether further decrease of hole density is warranted, this report tries to study and compare these spurious sidelobes.

Sidelobes due to Holes

Assuming that we have H holes in a total of T cells, we should have a hole density of H/T. Clark points out that if most of these holes are concentrated on low spatial frequencies, they tend to add in phase near the array main beam. Hence we would expect an upper bound of spurious sidelobes of

$$s_u = \frac{H}{T} \qquad (1)$$

Lo[†] points out that if the holes are scattered randomly throughout the u-v plane area, they would add randomly except at the array main beam. In this case, we should expect a lower bound of sidelobe level to equal to its rms value, i.e.

$$s_\ell = s_u \frac{1}{\sqrt{H}} = \frac{\sqrt{H}}{T} \qquad (2)$$

Now, if we assume that we have 4% holes, i.e. 1,000 holes in a 24,000 cells u-v plane, we would expect an upper bound of

$$s_{udB} = 10 \log (1000/24000) = -14 \text{ dB} \qquad (3)$$

† Y. T. Lo, A Mathematical Theory of Antenna Arrays with Randomly Spaced Elements, IEEE Trans. on Antenna and Propagation, vol.AP-12, pp.257-268.

and a lower bound of

$$s_{\ell dB} = -14 \text{ dB} - 5 \log H = -14 - 15 \text{ dB} = -29 \text{ dB} \qquad (3a)$$

It is noted that the smaller the number of holes, the closer are the upper and lower bounds.

Sidelobes due to Array Phase Errors

We shall assume that we have a uniformly weighted u-v plane. The radiation pattern is

$$P(x,y) = (1/T) \sum_n \sum_m^T \cos(mx+ny+\phi_{mn}+\Delta\phi_{mn}) \qquad (4)$$

where T is the total number of cells. If the phase errors $\Delta\phi_{mn}$ are small, we can expand the above equation in a first order approximation; namely,

$$P(x,y) \doteq P_o(x,y) + \sigma(x,y) \qquad (5)$$

where the pattern without error is given by

$$P_o(x,y) = (1/T) \sum_n \sum_m^T \cos(mx+ny+\phi_{mn}) \qquad (6)$$

and the first order spurious sidelobe pattern is given by

$$\sigma(x,y) = (1/T) \sum_n \sum_m^T [\sin(mx+ny+\phi_{mn})]\Delta\phi_{mn} \qquad (7)$$

Let us assume that the rms value of $\Delta\phi_{mn}$ is given by σ_{mn}. The mean square level of the spurious sidelobes is given by

$$\overline{\sigma}^2 = (1/4\pi^2) \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \sigma^2 (x,y)\ dx\ dy \qquad (8)$$

$$= (1/2T^2) \sum_{n} \sum_{m}^{T} \sigma^2_{mn}$$

If the rms values of the phase errors are equal for all cells, i.e. let $\sigma_{mn} = \sigma_o$, the rms level of the spurious sidelobes is then given by

$$\sigma_\ell = \sqrt{\overline{\sigma}^2} = (1/\sqrt{2}) \quad (\sigma_o/\sqrt{T}) = 0.7\ (\sigma_o/\sqrt{T}) \qquad (9)$$

Here, σ_ℓ indicates the above is a lower bound of the spurious sidelobes. Sidelobe levels will follow the above equation closely, if we can assume that $\Delta\phi_{mn}$ between cells are not correlated. In other words, we can assume that we have T independent phase errors.

In many cases, this is not necessarily true. We have L elements in the array, and therefore we have only L independent phase errors at any one time. We may assume that each phase error has a correlation time of τ. Within an observation period of P, we shall have a total of

$$L \cdot (P/\tau)$$

independent phase errors. Therefore, we have to modify (9) by

$$\sigma = \sigma_\ell \sqrt{\frac{T}{L\ (\frac{P}{\tau})}} = 0.7\ \frac{L_o}{\sqrt{L\ (\frac{P}{\tau})}} \qquad (10)$$

If the phase error is due to misalignment of the L.O. to each element, the correlation time is equal to the period of observation. Thus we have an upper bound of the spurious sidelobe level due to phase error:

$$\sigma_u = 0.7 \; (\sigma_o/\sqrt{L}\;) \tag{11}$$

Let us assume that we have 36 elements in the array and have 24,000 cells u-v plane coverage. Let us also assume an rms phase error of 3° for each element or each cell. The lower bound of the spurious sidelobe is

$$\begin{aligned} \sigma_{\ell dB} &= 10 \log \; (0.70) \; (3x0.01745)/\sqrt{24000} \\ &= 10 \log 2.4 \times 10^{-4} = - 36 \text{ dB} \end{aligned} \tag{12}$$

The upper bound of the sidelobe is:

$$\sigma_{udB} = 10 \log \; (0.7) \; (3x0.01745)/\sqrt{36} \tag{12a}$$

$$= - 12 \text{ dB}$$

If we compare, upper bound to upper bound and lower bound to lower bound, the spurious sidelobes due to hole density and phase errors, we find that the levels are comparable. Hence, we may say that 4% holes are compatible with phase errors of 3° in the array. Any further decrease of holes in the u-v plane may not be warranted if the phase errors of the array are not changed.