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Low Noise Pre-Amplifiers for Radio Astronomy

by

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Summary

A brief theoretical and practical description is given of the principles of design and construction of low noise IF amplifiers, operating on a frequency of 30 Mc/s, suitable for use in radio astronomy.

1. Introduction

A very important branch of radio astronomy requires that good receivers, of high sensitivity, low noise and flat bandwidth be used in the range of frequencies between 1000 Mc/s and 1425 Mc/s. The need for low noise receivers in this range is still met best by using a crystal mixer followed by an intermediate frequency amplifier. The noise figure (referred to as NF) of such a receiver is determined by the noise generated in the crystal mixer, the mixer loss and by the noise figure of the IF amplifier.

This note describes the theory and practice of constructing an IF amplifier of low NF. This is best achieved by making what is called a "pre-amplifier". This is a small IF amplifier which is normally an integral unit with the mixer. The IF signal from the pre-amplifier is sent by RF cable to the receiver proper. The pre-amplifier should have more than enough gain to overcome the cable losses.

The requirements for a good pre-amplifier may be summarized:

(a) It should give, when properly matched to the crystal mixer, the lowest possible NF for the mixer - pre-amplifier combination over as wide a range of frequencies as possible.

(b) The pre-amplifier band-pass characteristic should, in many requirements, be very flat.

(c) The stability of the gain and band-pass of the pre-amplifier should be good.

2. Design considerations for pre-amplifiers

The pre-amplifier, although contributing only a relatively small part to the NF of the mixer - pre-amplifier combination, can and should be designed in the best possible way. This choice of design requires decisions on the following points:

(a) What is the best pre-amplifier circuit design?

(b) What tubes give the best performance?

(c) The various components in the circuit which influence the NF should be known and chosen to minimize the NF.

(d) The way of achieving the best match for low NF between the mixer and pre-amplifier must be understood.

(e) The band-pass of the pre-amplifier may have to be wide and flat and the NF must still be low. This requires special design.

(f) The ways for stabilizing gain and band-pass of the pre-amplifier must be known.

This present note covers the design points (a) (b) (c) and (d). The two points (e) and (f) are somewhat linked with the other four, and these will be considered in a subsequent note. The work described is first a theoretical study of low noise pre-amplifiers, and second a description of the details of design and construction of such a pre-amplifier. The practical part of this work was not designed to develop the lowest possible NF, but rather to determine how the critical components in such a pre-amplifier may be designed, constructed and, in practice, adjusted.

Paragraph 3 outlines all the essential theory and Paragraph 4 gives practical design results and the results of measurements on a pre-amplifier. For convenience, an IF of 30 Mc/s has been considered throughout.

3. The theory of low noise pre-amplifiers.

Most of the following theory is derived from the book "Vacuum Tube Amplifiers" by Valley and Wallman - Volume 18 of the Radiation Laboratory Series. This will be referred to as V&W. The theory is only reproduced here in order to select the most important results for easy reference. Equations in this note will be referenced by their numbers in V&W.

(a) Choice of the best pre-amplifier circuit

This is considered in great detail in V&W (see pages 643-666). The conclusion reached at page 656-7 is that the best circuit is a grounded cathode neutralized triode input stage followed by a grounded grid triode stage. This solution, reached by 1948, still seems to be valid in practice, since all the available low-noise pre-amplifiers (known to the author) still use the same basic circuit.

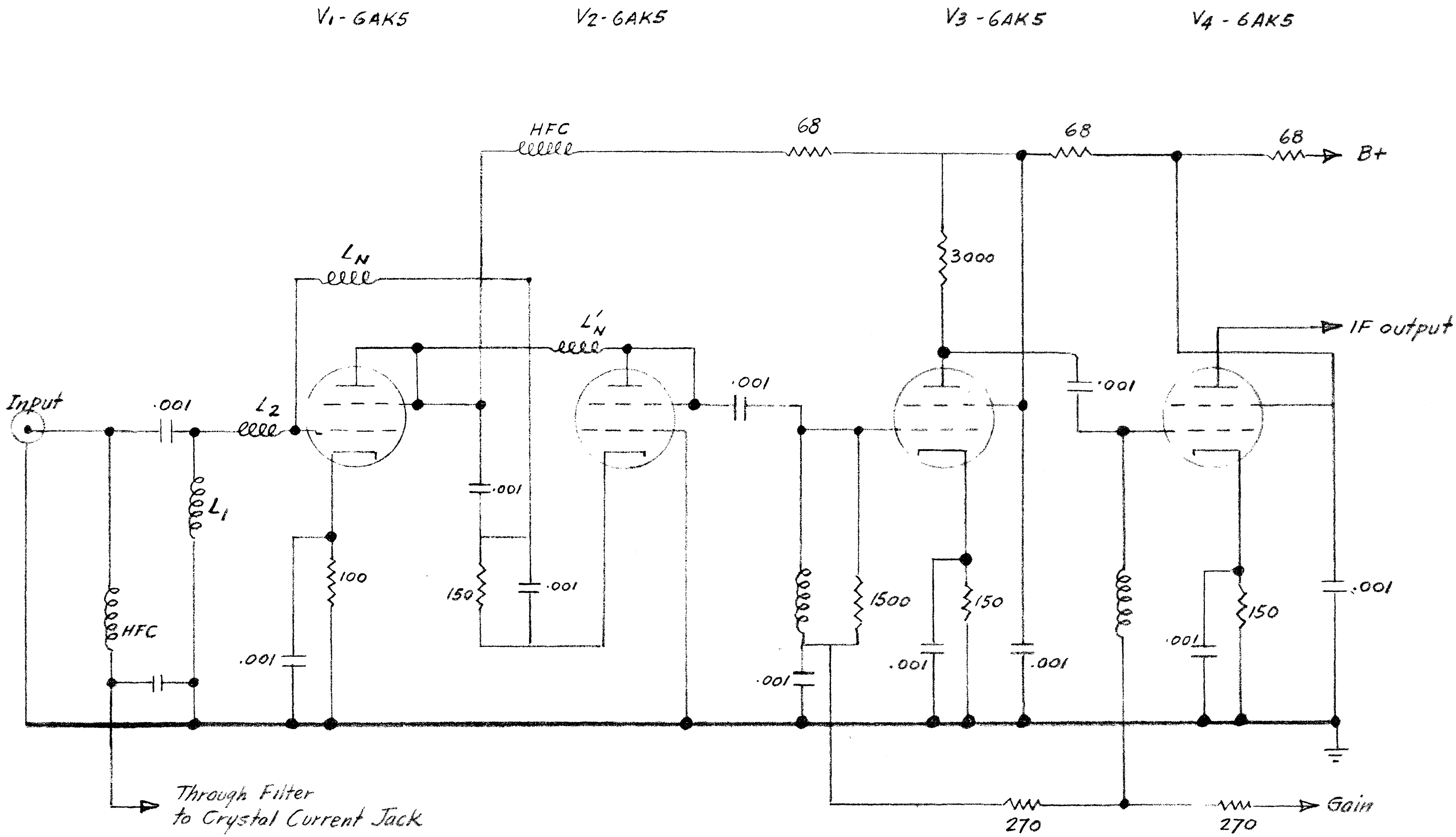

FIGURE I

Circut of a grounded - cathode - triode grounded - grid - triode pre-amplifier -

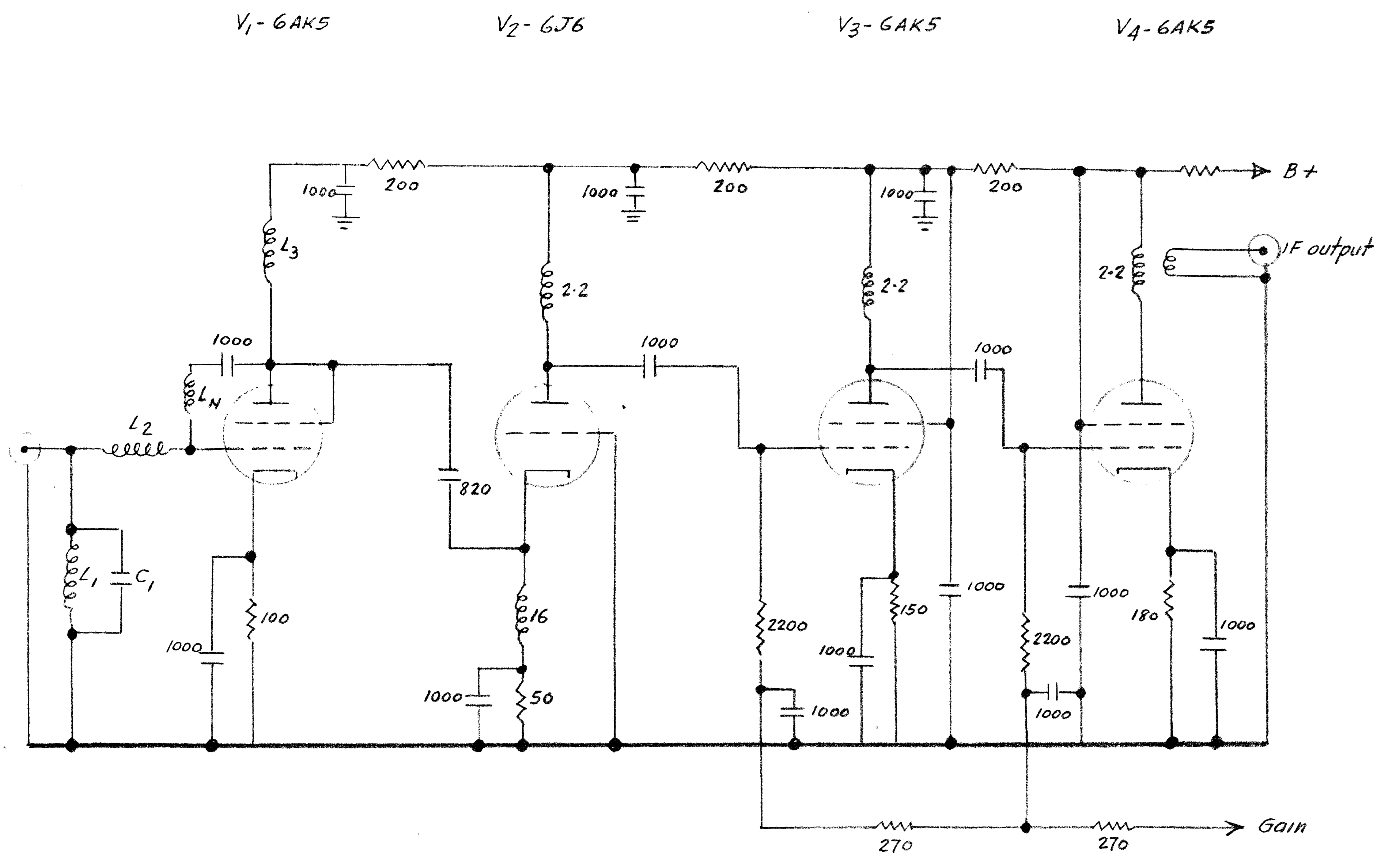

FIGURE 2 Circuit of experimental pre-amplifier used for testing.

The NF of this pre-amplifier is almost completely determined by the NF of the first stage. This stage is very stable, and the amplifier is comparatively simple to design and adjust.

An example of such a circuit is given in Figure 1, which is a copy of V&W figure 13.13 on page 661.

This circuit is not completely typical of the best practice. V&W admit that V2 would have better been a 6J6. This would avoid the need to neutralize the cathode-plate capacity of V2, which is high in a 6AK5, and so L_N would not be needed. V&W quote an average NF of 1.6 db for the circuit of Figure 1 in production.

In the present work a circuit very smiliar to Figure 1 was used. The final form of this circuit is shown in Figure 2.

(b) Choice of the best tube type

The choice of tubes is quite critical for both V1 and V2, since this pair determines the overall NF of the pre-amplifier. For V_1 a large transconductance (g_m) and a small inherent noise are important. Unfortunately, there seems to be no adequate theory (none is known to the author) for choosing the best tube. As will be shown later, the tube noise is approximately minimized by having a large g_m (see equation (3)) and a small value of the grid cathode conductance due to transit-time effects. (See equation (1)) Equation (1) also shows that a small value of B is desirable. There seems to be no way of determining B except by measurement.

As a guide to current practice, Table I shows the tubes used in a few typical pre-amplifiers.

Table I. Some typical 30 Mc/s pre-amplifiers

Pre-amplifier	V_1	V_2	Remarks
V&W page 661 Figure 1 of this note	6AK5-Triode connected	6AK5-Triode connected	V&W admit a 6J6 would be a better choice for V_2. Mean NF was 1.6db
LEL pre-amplifier	6BC4-Triode connected	6CB6-Triode connected	NF measured is about 1.85 db with a 3 db bandwith of 7 Mc/s
AIL Type 13130 pre-amplifier	404 A-Triode connected	417A	NF about 1.5 db deduced from performance with Empire Devices Mixer at 1400 mc
Experimental pre-amplifier used for present work	6AK5-Triode connected	6J6	NF of 1.8 achieved with a 3 db band-with of 3 Mc/s

(c) The theory of a minimum NF circuit

This theory is dealt with fully in Chapter 13 of V&W. Only the salient points will be collected here.

(i) The equivalent noise representation of the first tube

Figure 3 shows the equivalent noise representation of a triode. Sources of noise represented by generators i_p and i_g exist in the tube due to the fluctuations in the flow of electrons. Expressions for i_p and i_g are given in V&W, equations 13.15 and 13.16:

$$\overline{i_g^{\;2}}\ df = 4\ RBTG_t\ \ df \quad \text{\_\_\_\_\_\_\_\_} \quad (1)\ (V\&W\ 13{:}15)$$

Where G_T is the grid-cathode conductance due to transit time loading, B is a factor of about 5, k is Boltzmann's constant and T is room temperature in degrees K.

I_p is conveniently regarded as being equivalent to the thermal noise in a conductance Req. $G_m{}^2$ in the anode-cathode circuit, or, as is equivalent, to the noise in a resistance R_y in the grid-cathode circuit. R_{eq} is then known as the equivalent noise resistor, and equation (1) may be written:

$$\overline{i_p^{\;2}}\ df = 4\ RT\ \ Req\ \ G_m{}^2\ \ df \quad \text{\_\_\_\_\_\_\_\_} \quad (2)\ (V\&W\ 13{:}16)$$

It is possible to calculate Req for a triode, since approximately

$$Req = \frac{2.5}{G_m} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (3)\ (V\&W\ 13{:}17c)$$

However, agreement between calculated and observed values of Req is not good for most tubes, although the value calculated for a 6AK5 (with a G_m of 6700 micro- mhos), of 380 ohms, is quite a good practical figure.

<u>(ii) Other sources of noise in the circuit of V_1</u>

In (i) above we have written down the noise contributions from the tube itself. In addition noise arises from con-

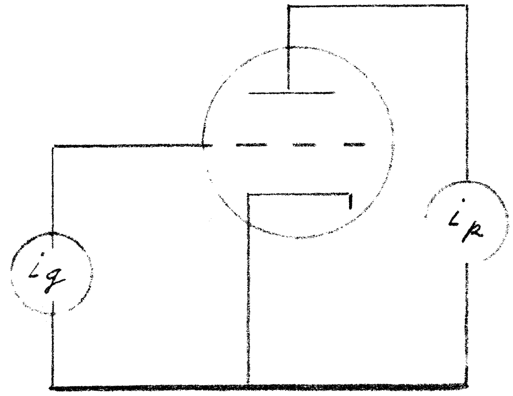

FIGURE 3 — The equivalent noise representation of a triode

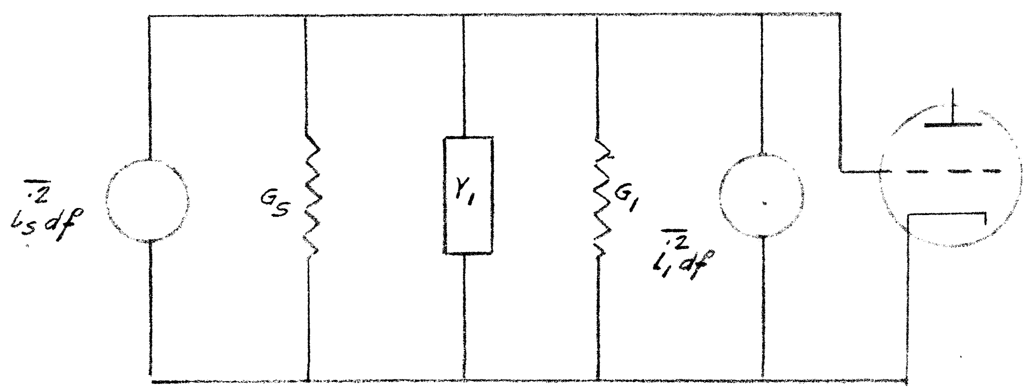

FIGURE 4 – Equivalent circuit for input network

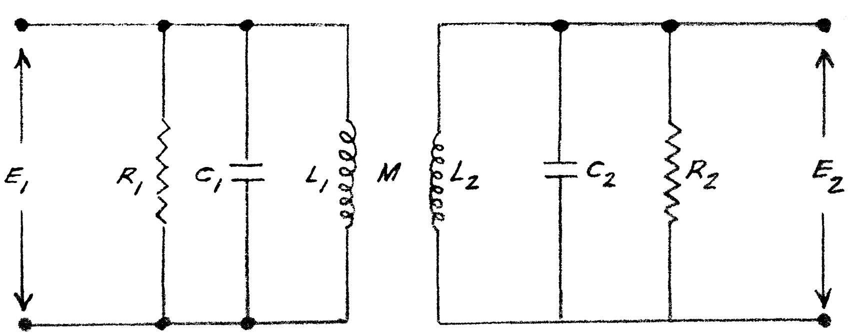

FIGURE 5 Inductance - coupled - double-tuned circuit.

ductances connected to the input of V_1. Figure 4, which is V&W Figure 13.2 , shows what these noise sources are. In Figure 4

G_s is the transformed value of the source conductance which appears across the input of V_1. G_1 represents the network ohmic losses across the input of V_1. Y_1 is the total susceptance across the input of V_1. Thus the total admittance presented to the input of V_1 is

$$Y_s = G_s + G_1 + J\,Y_1 \qquad (4) \quad (\text{V\&W } 13\ \text{X}9)$$

(iii) Minimizing the NF

The first step in the theory of minimizing the NF is to obtain an expression for the NF itself. The NF for the first stage is found by dividing the total noise power at the tube output by the noise power arising from a thermal noise generator of conductance equal to the transformed source conductance G_s. Thus, V&W add the two thermal noise generators which exist at the input of V_1:

$$\left.\begin{aligned} \overline{i_s^2}\, df &= 4kTG_s\, df \quad \ldots \\ \overline{i_s^2}\, df &= 4kTG_1\, df \quad \ldots \end{aligned}\right\} \quad (5) \qquad \text{V\&W } 13.10$$

(Note - V&W include a factor α in the second of these expressions. With the input losses due to pure resistance at room temperature $\alpha = 1$).

Using equations (1), (2) and (5) V&W derive an important expression for the Noise Factor of the first stage (F_1).

$$F_1 = 1 + \frac{G_1}{G_s} + \frac{BG_t}{G_s} + \frac{R_{eq} \; | \, Y_s + G_t \, |^2}{G_s} \quad ..(6)$$
V&W 13.52

This equation is useful, for from it we can derive several important rules for getting a low NF and also we can find the best value of G_s to use for a minimum NF. To find the best value of G_s, the transformed source admittance, the expression for F_1 is differentiated and $\frac{AF_1}{AG_s}$ set = 0. Then, after some approximations, two simple results emerge:

$$G_s \text{ opt} = \sqrt{\frac{cG_{B1}}{Roq}} \quad \_\_\_\_\_ (7) \quad (V\&W\ 13.58)$$

$$F_1 \text{ opt} = 1 + 2 \sqrt{cG_{B1} \, Req} \quad \_\_\_\_\_ (8) \quad (V\&W\ 13.59)$$

where $\quad G_{B1} = G_1 + G_T \quad 1 \quad c = \frac{G_1 + BG_T}{G_{B1}}$ _____ (9) (V&W 13.56)

Equation (7) is very useful since it defines the best admittance that the source should present to the input of V_1 for the lowest noise figure. F_1 opt is the lowest possible noise figure achievable.

This can all be made easier by an example.

<u>Example 1</u>

Take the case of a 6AK5 used as a triode for V_1. The source is to be a crystal mixer, of 300 ohms resistance, and with a shunt capacity of 40 µuf.

First, determine G_1 and G_T for the 6AK5. G_1 is the conductance of the total network losses at the input of V_1. Suppose the input circuit has a Q of 150 and a capacity C_s of 10 µuf, then G_1 is given by

$$G_1 = \frac{2 \pi f C_s}{Q} \quad \_\_\_\_\_ \quad (10) \quad (V\&W\ 13.47)$$

and so in our example $G_1 = 12$ μ mhos at 30 Mc/s.

G_t is the conductance of the tube input due to transit time damping. It is not well determined for any tube. For a 6AK5 at 30 Mc/s V&W (page 638) say it is about 12 u mhos and that B is about 5.

Hence, from the equation (9), $G_{B1} = 24$ μ mhos and, taking B = 5, c is found to be 3.

Now in equation (7) use the value of Req from (3) and get G_s opt = 435 μ mhos.

That is, the optimum transformed resistance that the source of 300 ohms should present to the input of V_1 is $\frac{10^6}{435} = 2300$ ohms.

The same values put into equation (8) show that the best value of noise figure for the first stage only will be

$$F_1 \text{ opt } = 1.24 \text{ db}$$

To sum up this theory of minimum noise figures, let us look again at equation (6), (7) and (8).

The following rules emerge for minimum noise figure:

I. Make $Y_1 = 0$. That is, always tune out any reactance at the input to the first stage of the pre-amplifier.

II. Keep G_1, the losses in the input circuit small. G_1 contributes directly to F_1, but also indirectly by increasing Y_s.

III. Present an admittance to the input of V_1 as given by equation (7).

Many reasons may prevent the achievement of a 30 Mc/s NF as low as 1.24 db. Nevertheless, V&W (page 645) report a NF as low as 1.1 db and a median value of 1.35 for a very carefully built

30 Mc/s amplifier using a 6AK5 input, followed by a 6J4.

(d). The input network

(i) The theory of the network

It is necessary now to describe the theory of the design of an input circuit which will transform the source conductance to the optimum value at the tube input. For the example chosen in (c) above, this implied transforming the crystal of 300 ohms resistance to the 2300 ohms the 6AK5 would like to see in order to give a minimum NF.

This subject is treated in V&W Chapter 5. In that chapter, the properties of the double tuned interstage coupling are considered in detail. The part of this work which we need is that which deals with the circuit of Figure 5 (V&W Figure 5. 1a). This circuit is, of course, not the only one which might be used, but it, or one of its derived equivalents, proves the most useful in practice.

The only results which we need are that:

(a) It is assumed that L_1 and C_1 and L_2 and C_2 are resonant at the mid-band frequency.

(b) The driving point admittance Y_{11}, which is the ratio of $\frac{\text{Input current}}{\text{Input voltage}}$ is given by

$$Y_{11} = \frac{(1 + S^2 - u^2) + ju\sqrt{b + 2}}{R_1 \left(1 + j\sqrt{Q_2/Q_1}\, u\right)} \qquad (11) \text{ (V\&W 5.23)}$$

This driving point admittance is the quantity we need, since it tells us the input admittance to the network for any termination.

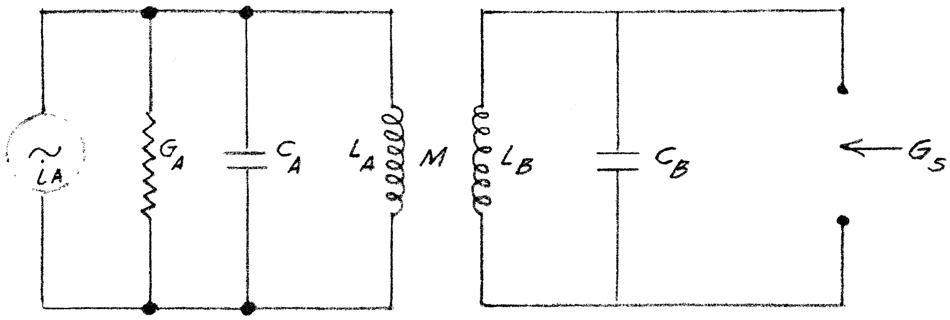

FIGURE 6 — Practical case of matching circuit

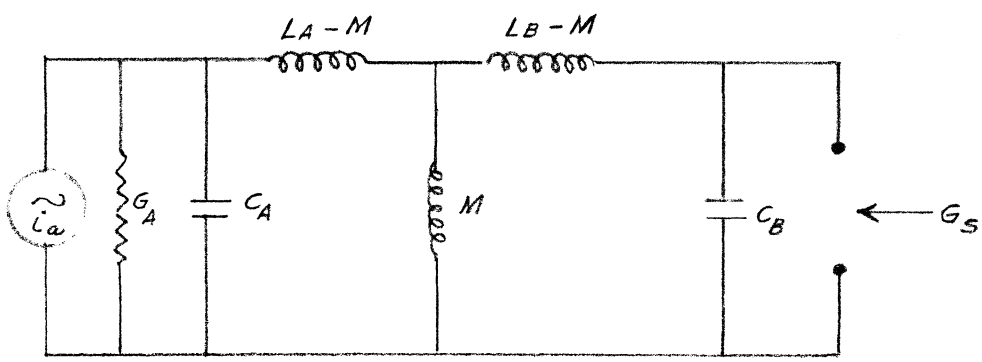

FIGURE 7 — An equivalent network to Figure 6

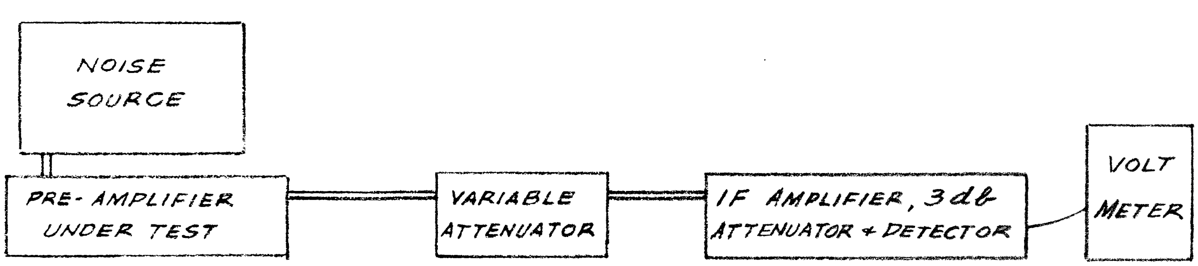

FIGURE 8 — The system used for measuring noise figure

Figure 6 shows the practical circuit we need to analyze. An expression is needed for G_s, the conductance presented to the tube, in terms of G_a. Equation 11 is simplified when we are concerned only with the mid-band frequency f_o and the circuit of Figure 6. V&W derive the simple form for G_s as

$$G_s = \frac{K^2 Q_A^2 \; (C_B/C_A)}{1 + Q_A^2 \alpha^2} \; G_A \quad \text{\_\_\_\_\_\_\_\_\_\_} (12) \quad (\text{V\&W } 13.117)$$

where $\alpha = (f/fo - fo/f)$ so that the mid-band case becomes

$$G_s = K^2 Q_A^2 \; \left(\frac{C_B}{C_A}\right) \; G_A \quad \text{\_\_\_\_\_\_\_\_\_\_} (13)$$

This equation is all we need to get the source impedance correctly transformed. An example will show the method.

Example 2

In Example 1 we showed that, for minimum NF a source resistance of 300 ohms should be transformed to 2300 ohms, or a conductance of 435 μ mhos. Thus G_s = 435 μ mhos and $G_a = \frac{10^6}{300} = 3333$ μmhos

Use (13) and get

$$\frac{G_s}{G_a} = K^2 Q_A^2 \frac{C_B}{C_A} = \frac{435}{3333} = 0.130$$

If we follow Example 1, we see that C_A and C_B are both fixed, since we chose the mixer capacity (C_A) to be 40 $\mu\mu$f and the tube input capacity, with strays, to be 10 $\mu\mu$f. $K^2 Q_A^2$ can be chosen, but usually the desirable choice is that the coupling should be transitional, and for this $K^2 Q_A^2 = 1/2$. __________ (14) (V&W 13.120)

Therefore, in the transitional coupling case Equation 13 becomes:

$$G_s = \frac{C_B}{2C_A} \; G_A \quad \text{\_\_\_\_\_\_\_\_\_\_} (15) \quad (\text{V\&W } 13.122)$$

So, for our example, $\frac{G_s}{G_A} = \frac{C_B}{2C_A} = \frac{10}{80} = 0.125$

and we desire to have $G_s/G_A = 0.130$

So, chiefly because we chose our mixer capacity so high, we arrive fortuitously at almost the correct value for G_s.

The rest of the design is now easy. L_A must resonate with C_A at 30 Mc/s and must therefore be <u>0.71 μh</u>, and L_B must resonate with C_B and thus in <u>2.84 μh</u>. M is got from the fact that K, the coefficient of coupling is:

$$K^2 = \frac{M^2}{L_a L_B} \qquad \text{——————} \qquad (16)$$

and by using Equation (14).

Using (14) first we see that $K = \frac{.707}{Q_A}$ and $Q_A = \frac{2\pi f_0 C_A}{G_A} = 2.26$ and hence K = 0.313. Hence M = <u>0.45 μh</u>.

<u>(ii) Practical networks</u>

The network of Figure 6 is not as easy to adjust as a T or Π network. Figure 7 shows the T network which is equivalent to Figure 6. The use of such a network may be illustrated by continuing our example further. We found values for L_A, L_B and M. Thus the three inductances in Figure 7 must be:

$L_A - M =$ <u>0.26 μh</u>

$M =$ <u>0.45 μh</u>

$L_B - M =$ <u>2.39 μh</u>

A still further simplification may in some cases be possible in practice; to omit L_A-M, since it is nearly zero. We should have to increase M to <u>0.71 μh</u>, so that L_A-M is zero and L_A still resonates with C_A. We have also to alter L_B-M to <u>2.13 μh</u> so that

L_B still resonates with C_B. The only changes that result are as follows:

From equation (16) K is now 0.50 and so by (13):-

$$\frac{G_s}{G_A} = \underline{0.32}$$

This is perhaps too far removed from the desired value of 0.125 to represent a practical circuit for the example chosen.

4. The design and construction of a 30 Mc/s pre-amplifier

The theory that has been outlined has been tested by work on a 30 Mc/s pre-amplifier. The circuit used in this pre-amplifier varied, but essentially it was that of Figure 2. The mechanical and electrical layout and construction of the pre-amplifier was not perfect, but since the experiments were intended to show how, in practice, the various elements of the circuit should be designed and adjusted, and not to produce the lowest possible NF, these short-comings of the pre-amplifier were overlooked.

(a) Noise figure measurements

The system for measuring NF is shown in Figure 8. The post-amplifier used had a built-in 3db attenuator, and a pass-band about 10 Mc/s wide at the 3 db points. The noise generator was a noise diode with a 300 ohm load. The impedance of this source and its connector was measured at 30 Mc/s to be a resistance of 300 ohms with 12 μμf in parallel. Since the pre-amplifier might eventually be used with a mixer of 40 μμf capacity, a further 25 μμf was added across L_1 (C_1 in figure 2) so that the input circuit would be correctly designed for such a mixer.

The absolute values for NF measured are probably good to 0.2 db, but the relative values are, of course, much more accurate.

Measurements were made by adjusting the variable attenuator until the noise output was an easily read value. The extra 3 db of attenuation was switched in and the noise source current increased from zero until the original noise output was again achieved. If i milliampers is the diode current when this occurs, the NF is given by:

$$\mathrm{NF} = 10 \log_{10} i \qquad \text{———} \quad (17)$$

(b) <u>First stage gain</u>

A simple qualitative example was noted of the importance of high gain in the first stage. As originally built the preamplifier had a rather low gain, due to improper tube bias. The measured NF was 5.0 db. Correction of the bias and coupling condensers (which were rather small) improved this figure to 3 db.

(c) <u>Input coupling network</u>

A coupling network calculated very much along the lines of Example 1 was first designed. As will be described under paragraph on "neutralizing", there is considerable difficulty in adjusting such a network together with L_N and L_3 (all references are to Figure 2). Eventually, the input network used was that sketched in Figure 2. L_1 and L_2 had the following values

$$L_1 = 0.68 \ \mu h$$

$$L_2 = 4.5 \ \mu h$$

The calculated ratio of G_s/G_A for these values is 0.089, which is about 30% smaller than is desirable. Nevertheless, the effect on

NF should not be great, and in fact, using V&W Figure 13.7 suggests it will be only about 0.02 db.

The Q of the coils used for L_1 and L_2 was not very high. For ease of experiment, slug-tuned inductors were used. The measured Q's were about 90. An unsuccessful attempt was made to detect the change in NF consequent on using coils of Q about 150, and in fact a consideration of Equation 6 suggests that such an effect would not be found until the NF was considerably less than the 1.8 db achieved.

(d) The neutralizing coil

The purpose of L_N is to neutralize the grid-anode capacity of V_1. This capacity is about 1.2 μμf, so that L_N should be about 24 μh. However, stray capacities can easily alter this value, and so a way of adjusting L_N has to be found. One way is to disconnect one of the heater connections from V_1, thus leaving that tube cold. A signal fed to the input of the pre-amplifier can only reach the output via the grid-anode capacity of V_1. Thus, L_N may be adjusted by connecting a 30 Mc/s signal source to the amplifier input and tuning L_N for minimum output.

This system was adopted, and proved to work. Nevertheless, such an adjustment of L_N leads to several difficulties. Both L_3 and L_2 should be adjusted for resonance. Changes in L_N are reflected, by Miller effect, into the grid-cathode circuit to change the reactance across L_2. Changes in L_3 also effect the values of these reactances. Also, it is difficult to use a slug-tuned adjustable L_N, since stray capacities may affect the value of L_N at

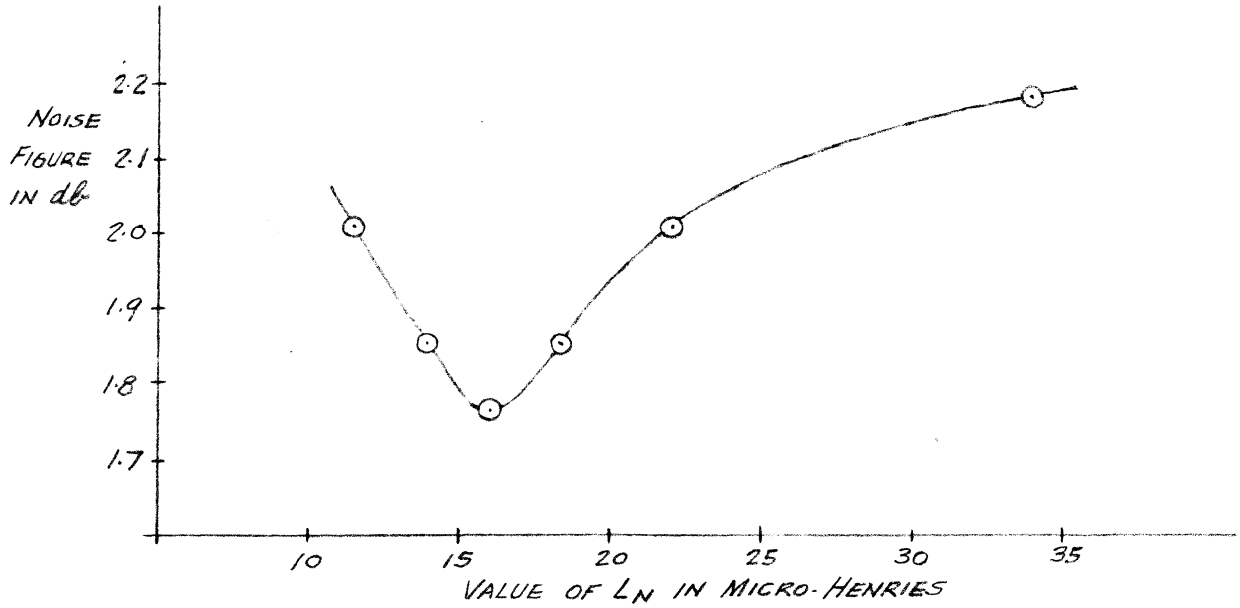

FIGURE 9 — The effect on noise figure of the size of the neutralizing coil.

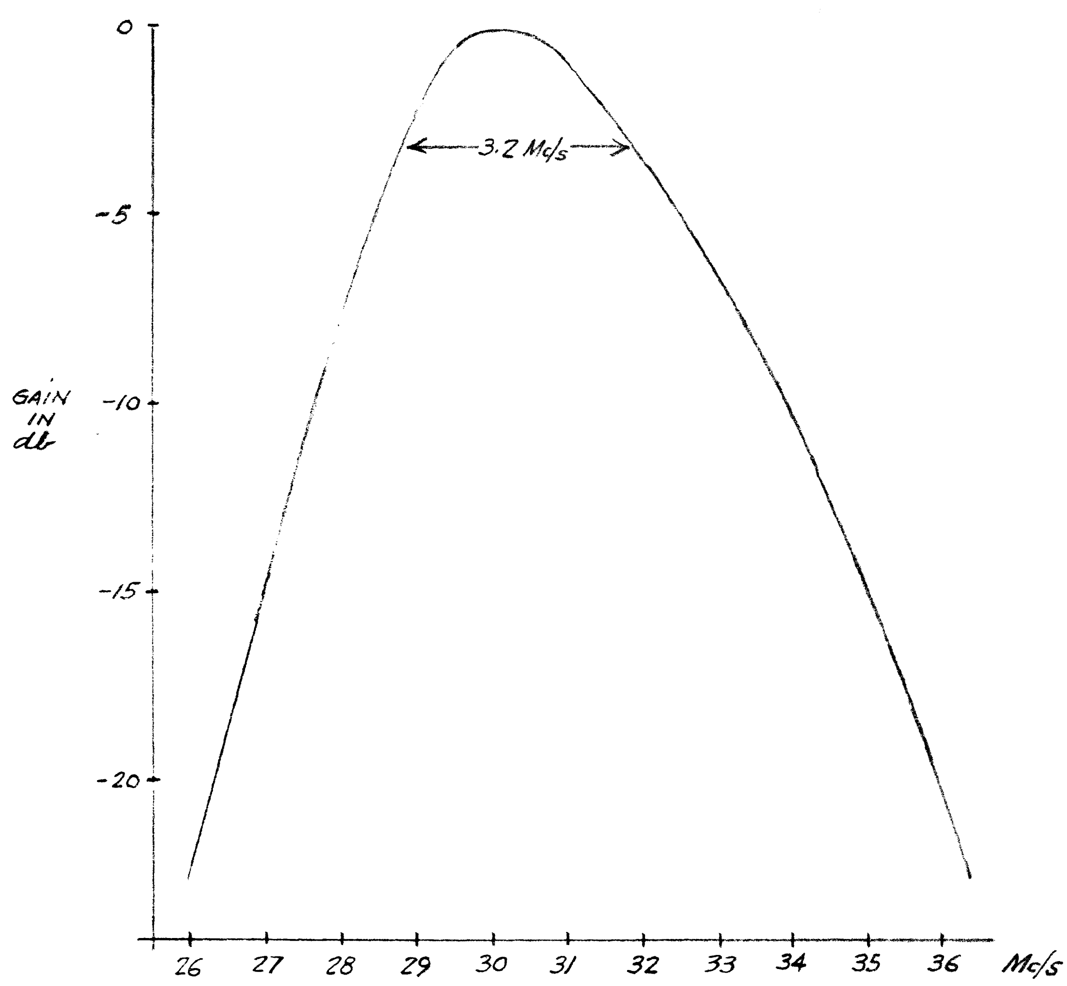

FIGURE 10 — Band-pass characteristic of the experimental pre-amplifier

resonance. For example - using a slug-tuned L_N suspended in the wiring led to a value of L_N at resonance of about 10.5 µH. The value found for an L_N of a simple coil type was 16 µH. The difference arose from the stray capacities due to the slug-tuning.

After some experimenting, a good way for adjusting L_N was found. First, an adjustment of L_3 was needed. This could be made by removing V_2 from its socket. With V_2 in place, L_3 is highly damped and resonance is hard to detect. L_3 can be adjusted accurately enough with V_2 absent. Alternatively, L_3 may be replaced without serious loss of NF by a radio-frequency choke.

Then L_1 is adjusted to be resonant with the input capacity to 30 Mc/s (L_2 is of course disconnected from L_1 for the check). L_2 is reconnected and L_N is then altered over a range of values and the NF measured for each value. L_2 is adjusted to resonance for each value of L_N. This is essential, since for best NF L_2 must be resonant and changes in L_N are reflected across L_2 by Miller effect and thus alter the tuning of L_2.

Such a test was made, and the results are shown in Figure 9. Various coils were made for L_N, varying from 11.5 µH to 34 µH. These coils all had Q's of between 50 and 80. They were used in turn, in the circuit of Figure 2. L_3 was a 16 µH choke and L_2 was adjusted each time for resonance.

Figure 9 shows a marked dip in NF at L_N = 16 µH, a result which confirms both the resonant value of L_N and also the improvement in NF consequent on good neutralizing.

Lastly, a note on the Q of L_N should be made. L_N can contribute, by its losses, to the NF, and so its Q should be reasonably high. V&W on page 649 state that the contribution of the conductance G_N of L_N to the NF is given by $^{G_N}/_{G_S}$. In our present case $G_S = 435$ μmhos. G_N may be related to the Q_N of L_N by

$$G_N = \frac{1}{2 \Pi fo\, Q_N\, L_N}$$

and so $G_N = 3.3$ μ mhos for $Q_N = 100$. Hence there is going to be a very small contribution to the NF, of the order of less than 1% in power ratio, due to an L_N which has a Q of about 100.

(e) <u>The band-pass characteristics</u>

No attempt was made to broaden or flatten the pass-band of the pre-amplifier. The pass-band used is shown in Figure 10.

(f) <u>Varying tube characteristics</u>

The following Table II shows the effects of using a number of different tubes in the pre-amplifier. The 6AK5 (V_1) was altered first. The results for changing 6J6's were taken with the best 6AK5 (marked *) as V_1. The results show, as would be expected, that the tube used as V_1 is of greatest importance. They also show that the contribution from V_2 to the NF is very small, since the NF hardly varies with changing tubes in V_2.

Table II. The effect of changing tubes on NF

Tube type	Position	Number	Manufacturer	Noise Figure db
6AK5	V_1	1	RCA	2.10
		2	"	2.25
		3	"	2.33
		4	"	2.68
		5*	WE	1.85
		6	"	2.01
		7	"	1.85
		8	"	2.62
		9	TS	2.33
		10	"	2.41
		11	"	2.10
		12	"	2.18
6J6	V_2	1	SO	1.85
		2	"	1.76
		3	"	1.76
		4	"	1.85
		5	"	1.85
		6	"	1.85

5. Summary

The theory and experiment reported in this note show how the first steps may be made in good pre-amplifier design. The next stage will be:

(a) The achievement of wider bandwiths

(b) The improvement of NF still further by careful attention to the details of design and layout and by using better tubes.