NATIONAL RADIO ASTRONOMY OBSERVATORY
c/o KITT PEAK NATIONAL OBSERVATORY
P. O. BOX 4130
TUCSON, ARIZONA 85717
TELEPHONE 602-795-1191

POST OFFICE BOX 2
GREEN BANK, WEST VIRGINIA 24944
TELEPHONE 304-456-2011
TWX 710-938-1530

EDGEMONT ROAD
CHARLOTTESVILLE, VIRGINIA 22901
TELEPHONE 703-296-0211
TWX 510-587-5482

March 20, 1973

MEMORANDUM

From: B.L. Ulich

Subject: Absolute Thermal Calibration of Spectral Line Observations

The thermal calibration signal in a millimeter wavelength spectral line receiver is generally derived from a rotating half-disk covered with absorbent material. When the chopper wheel is rotated, the synchronously detected power is proportional to the difference in the temperature of the absorbent material and the sky brightness temperature. We now derive the absolute temperature change seen by the receiver feed horn when the chopper is rotated, and thus the equivalent antenna temperature of the calibration signal. Since millimeter wavelength spectral line receivers are double sideband, we allow the possibility of different gains and atmospheric optical depths at the signal and image frequencies.

When the absorbent material is in front of the feed horn, the antenna temperature is:

$$T_A = G_S T_{AMB} + G_I T_{AMB} \tag{1}$$

where T_A = Antenna temperature looking at absorbent material (^oK)

T_{AMB} = Ambient temperature (^oK)

G_S = Receiver gain in signal sideband

G_I = Receiver gain in image sideband

Since the receiver is near the ground, we can safely assume that the absorbent material is at ambient temperature. When the chopper wheel is not in front of the feed horn, the antenna temperature is:

$$T_A' = G_S \{\eta_L T_{SKY}(\mathrm{SIGNAL}) + (1 - \eta_L) T_{AMB}\}$$
$$+ G_I \{\eta_L T_{SKY}(\mathrm{IMAGE}) + (1 - \eta_L) T_{AMB}\} \quad (2)$$

where T_A' = Antenna temperature looking at sky (^oK)

η_L = Antenna loss efficiency

T_{SKY} = Sky brightness temperature (^oK)

The antenna loss efficiency η_L includes ohmic loss, blockage by spars and radiometer, and spillover. It is assumed that all absorbed, blocked, or spilled over radiation falls on a blackbody at ambient temperature. The synchronously detected chopper wheel antenna temperature is given by

$$T_{CAL} = T_A - T_A'$$
$$= G_S\eta_L\{ T_{AMB} - T_{SKY}(\mathrm{SIGNAL})\}$$
$$+ G_I\eta_L\{ T_{AMB} - T_{SKY}(\mathrm{IMAGE}) \} \quad (3)$$

where T_{CAL} = Chopper wheel antenna temperature (^oK)

Neglecting the small effect of the 2.7^{o}K cosmic background radiation, the sky brightness temperature is adequately modeled by (Falcone et al., 1971, and Ulich, 1973)

$$T_{SKY} = T_M(1 - e^{-\tau x}) \quad (4)$$

where T_{SKY} = Sky brightness temperature (^{o}K)

T_M = Mean atmospheric temperature (^{o}K)

τ = Zenith optical depth of atmosphere

x = Air mass ($\approx$ secant of zenith angle)

The mean atmospheric temperature T_M depends slightly on the zenith optical depth (Kislyakov, 1966) since for large τ only the atmosphere near the antenna is sampled. In this analysis, however, we assume that T_M is independent of τ . The sky brightness temperatures in the two sidebands are given by

$$T_{SKY}(\mathrm{SIGNAL}) = T_M(1 - e^{-\tau_S x}) \tag{5}$$

and $$T_{SKY}(\mathrm{IMAGE}) = T_M(1 - e^{-\tau_I x}) \tag{6}$$

where T_{SKY}(SIGNAL) = Sky brightness temperature in signal sideband (^{o}K)

T_{SKY}(IMAGE) = Sky brightness temperature in image sideband (^{o}K)

τ_S = Zenith optical depth in signal sideband

τ_I = Zenith optical depth in image sideband

Inserting Eq.5 and Eq.6 into Eq.3 we get for the chopper wheel antenna temperature

$$\begin{aligned} T_{CAL} = {} & G_S \eta_L \{T_{AMB} - T_M(1 - e^{-\tau_S x})\} \\ & + G_I \eta_L \{T_{AMB} - T_M(1 - e^{-\tau_I x})\} \end{aligned} \tag{7}$$

The antenna temperature of a spectral line in the signal sideband is given by

$$T_A'' = G_S \eta_B \eta_L T_B e^{-\tau_S x} \tag{8}$$

where T_A'' = Antenna temperature of the spectral line (^oK)

η_B = Beam coupling efficiency

T_B = Brightness temperature of the spectral line (^oK)

The beam coupling efficiency η_B is the normalized convolution of the antenna power pattern and the source brightness distribution. For sources very large compared to the half power beamwidth, such as the sky itself, $\eta_B = 1$. Note that the chopper wheel calibration and the line observation are assumed to be made at the same air mass x.

The directly measured quantity in a spectral line observation is the ratio R of the peak line intensity to the chopper wheel calibration signal

$$R = \frac{T_A''}{T_{CAL}} = \frac{G_S \eta_B \eta_L T_B e^{-\tau_S x}}{T_{CAL}} \tag{9}$$

where R = Ratio of spectral line peak intensity to chopper wheel calibration signal

Solving for T_B we get

$$T_B = \frac{Re^{\tau_S x}}{\eta_B G_S \eta_L} \Big[G_S \eta_L \{T_{AMB} - T_M(1 - e^{-\tau_S x})\} + G_I \eta_L \{T_{AMB} - T_M(1 - e^{-\tau_I x})\} \Big]$$

$$= \frac{R}{\eta_B} \Big[(1 + \frac{G_I}{G_S})(T_{AMB} - T_M) e^{\tau_S x} + T_M + \frac{G_I}{G_S} T_M e^{(\tau_S - \tau_I)x} \Big] \tag{10}$$

We define the absolute calibration temperature C as

$$C = T_M + \frac{G_I}{G_S} T_M e^{(\tau_S - \tau_I)x} + \left(1 + \frac{G_I}{G_S}\right)(T_{AMB} - T_M) e^{\tau_S x} \qquad (11)$$

where C = Absolute calibration temperature (^oK)

Thus $$T_B = \frac{RC}{\eta_B} \qquad (12)$$

The first term of C is the constant term independent of optical depth and air mass. The second term corrects for the different gains and zenith optical depths of the two sidebands. The last term accounts for the fact that the sky is always colder than the chopper wheel and thus the sky fails to emit enough radiation to completely correct for absorption of the spectral line radiation.

The mean atmospheric temperature T_M exhibits a small dependence on the surface ambient temperature. The relationship derived by Altshuler et al. (1968) is

$$T_M = 1.12\ T_{AMB} - 50 \qquad (13)$$

Substituting Eq.13 into Eq.11 we get

$$C = (1.12T_{AMB} - 50) + \frac{G_I}{G_S}(1.12T_{AMB} - 50)e^{(\tau_S - \tau_I)x} + \left(1 + \frac{G_I}{G_S}\right)(50 - 0.12T_{AMB})e^{\tau_S x} \qquad (14)$$

Eq.14 should be used to derive the calibration temperature when T_{AMB}, G_I/G_S, τ_S, τ_I and x are known.

If $G_I = G_S$:

$$C = (1.12T_{AMB} - 50)(1 + e^{(\tau_S - \tau_I)x}) + (100 - 0.24T_{AMB})e^{\tau_S x} \tag{15}$$

If $G_I = G_S$ and $\tau_S = \tau_I = \tau$:

$$C = (2.24T_{AMB} - 100) + (100 - 0.24T_{AMB})e^{\tau x} \tag{16}$$

Eq.15 is valid when the signal and image gains are equal. Eq.16 holds for equal gains and equal optical depths in both sidebands. Note that in the center of a broad atmospheric window τ_S and τ_I are very nearly identical, but close to an atmospheric absorption line they can be quite different. In this case, the second term in Eq.14 may become appreciable, and values of C larger than $2T_{AMB}$ are possible.

Davis (1973) has conducted two experiments at the Millimeter Wave Observatory in an attempt to absolutely calibrate the $^{12}C\ ^{16}O$ $J = 1 \longrightarrow 0$ line at 2.6 mm in the galactic center source Sagittarius A. The following is a summary of his results. The first experiment involved simply measuring the ratio of the peak line intensity to the chopper wheel signal at the same elevation angle. He found the ratio R = 0.0288 at 2.3 air masses. In this case, the klystron was set below the $^{12}C^{16}O$ line so the signal came in the upper sideband. The line strength was measured at several elevation angles and the zenith extinction was found to be 2.3 db. From the shape of the theoretical attenuation curves in Thompson and Haroules (1968), he derived a zenith extinction in the lower sideband of 0.5 db by comparison with the measured upper sideband value. Eq.15 yields a value for $C = 1089^\circ K$ at an ambient temperature of $290^\circ K$. Since Sagittarius A is greatly extended compared to the antenna beamwidth (2.2'), $\eta_B \approx 1$. Eq.12 then yields $T_B = 31.2^\circ K$.

The second experiment involved using an IF amplifier whose center frequency was equal to one-half the difference between the $^{12}C^{16}O$ line at 115.3 Ghz and the atmospheric oxygen absorption line at 118.8 Ghz. The klystron was set between the two so that the $^{12}C^{16}O$ line came in the lower sideband and the upper sideband was centered on the opaque oxygen line. Then the $^{12}C^{16}O$ line in Sagittarius A was measured relative to the Moon at the same elevation angle. Now both the line and the Moon's thermal radiation are received in only the lower sideband and a direct comparison is possible. The brightness temperature of $212^\circ K$ for the Moon, at 300° phase angle, was obtained from the lunation curve of Ulich (1973) at 97 GHz. Since the Moon was observed when the brightness temperature was close to its mean value, the brightness temperature is almost independent of wave-length, and the 97 GHz value is valid at 115 GHz. The measured ratio of 0.145 yields a line brightness temperature of $30.7^\circ K$, in excellent agreement with the previous experiment. Both the Moon and Sagittarius A are large enough in angular extent so $\eta_B \approx 1$. Note that if the "traditional" value of $400^\circ K$ is used for the chopper wheel calibration, the calculated brightness temperature is only $11.5^\circ K$, which is too low by a factor of 2.7. For the Orion nebula the factor is about 2.1.

Thus it appears that the published antenna (or brightness) temperatures for $^{12}C^{16}O$ are considerably in error due to incorrect analysis of the chopper wheel calibration.

REFERENCES

ALTSHULER, E.E., FALCONE, V.J., AND WULFSBERG, K.N. (1968). Atmospheric effects on propagation at millimeter wavelengths. IEEE Spectrum 5, 83-90.

DAVIS, J.H. (1973). Private communication.

FALCONE, V.J., WULFSBERG, K.N., AND GITELSON, S. (1971). Atmospheric emission and absorption at millimeter wavelengths. Radio Science 6, 347-355.

KISLYAKOV, A.G. (1966). Effective path length and mean temperature of the atmosphere. Radiofizika 9, 451-461.

THOMPSON, W.I., AND HAROULES, G.G. (1968). A review of radiometric measurements of atmospheric attenuation at wavelengths from 75 centimeters to 2 millimeters. NASA TN D-5087.

ULICH, B.L. (1973). Absolute brightness temperature measurements at millimeter wavelengths. The University of Texas at Austin, Electrical Engineering Research Laboratory Technical Report No. NGL-006-73-1.