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TO: Scientific Staff

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SUBJECT: FLUX ESTIMATE AND OPTIMUM INTEGRATION TIME

I. INTRODUCTION

The flux of a radio source can be measured as the result of the integration of the area under the curve resulting from the transit of the source across the antenna beam.

Actually, two factors limit the use of this technique. First, the time of useful observation is limited by such factors as the sky background or the receiver drift and fluctuations.

Secondly, the signal-to-noise ratio does decrease beyond a certain time of observation, as will be shown below. This last point is independent of the fact that the longer the observation time, the better is defined the curve. This simply means that, as far as signal to noise ratio is concerned, integration of the curve is not the best way to use the available information.

Actually, should neither the sky background nor the receiver drift be significant, a better technique would be first to smooth the record (i.e. to filter out the noise) over the whole observation time which must be as long as possible, and then to integrate the area under the best fit curve obtained in this way.

When either sky background or receiver instabilities do affect the quality of the record, no optimum technique of reduction can be defined, except by a statistical comparison of results corresponding to different techniques.

In that manner, I. Pauliny-Toth and C. M. Wade have shown, by means of histograms, that results appear more consistent when only the maximum amplitude of the transit curve is used than when integration is performed over the main lobe (provided that the source apparent diameter is much smaller than the antenna beam).

II. Signal-to-Noise Ratio as a Function of the Integration Time.

In the following we assume that the main limiting factor in the precision of the result of an integration is the receiver noise. The antenna beam is approximated by a gaussian curve, the apparent diameter of the source being assumed much smaller than the antenna beam (the method could be extended to other particular cases). As we are interested in determining the flux of the source by integration of the transit curve, we will calculate its signal-to-noise ratio as a function of the time of integration.

Let

$$s(t) = a\ e^{-t^2/2\tau^2} \tag{1}$$

be the output signal as a function of the time, under the preceding assumptions. The result of the integration of this signal from $t = -T$ to $t = +T$ is:

$$S(T) = 2a \int_0^T e^{-t^2/2\tau^2}\ dt$$

For the noise, its rms increases as the square root of the integration time:

$$\sigma\ (T) = \sqrt{2}\ \sigma_0 \sqrt{T} \tag{2}$$

where σ_0 is the rms noise over 1 second integration time. The optimum integration time is therefore defined as the value T_0 of T for which:

$$\frac{S(T)}{\sigma(T)} = \frac{a\sqrt{2}}{\sigma_0} \frac{\int_0^T e^{-t^2/2\tau^2}}{\sqrt{T}} \tag{3}$$

is maximum.

Let: $x = \frac{t}{\tau}$

$X = \frac{T}{\tau}$

(1) becomes, after nominator and denominator have been multiplied by $\frac{1}{\sqrt{2\pi}}$:

$$\frac{S(X)}{\sigma(X)} = \frac{a\sqrt{4\pi}}{\sigma_o}\sqrt{\tau}\;\frac{\frac{1}{\sqrt{2\pi}}\int_o^X e^{-x^2/2}\,dx}{\sqrt{X}} \qquad (4)$$

The function $\frac{1}{\sqrt{2\pi}}\int_o^X e^{-x^2/2}dx$, which is directly related to the "error function" is tabulated (see for instance "CRC Standard Mathematical Tables" pp. 247-249).

In Fig. 1, $\frac{S(X)}{\sigma(X)}$ is plotted as a function of X, in normalized units.

After a sharp raise, this function reaches its maximum for:

$$X = \sqrt{2} \rightarrow T_o = \sqrt{2}\,\tau$$

then decreases as $\frac{1}{\sqrt{X}}$ for $X > 3$.

III. Optimum Integration Time and Signal-to-Noise Ratio

The half-width at half intensity points of the gaussian curve is:

$$\theta = 1.18\,\tau$$

Therefore the optimum integration time is:

$$T_o = \sqrt{2}\,\tau = 1.2\,\theta \,. \qquad \text{(fig. 2)}$$

For a uniformly illuminated dish, the half-width at half-intensity points is:

$$\theta' = [13750 \cos\delta\,]\,[0.45\,\frac{\lambda}{d}\,]$$

where δ is the declination of the source.

For both the 85-ft. and the 300-ft. dishes, the half-width at half-intensity points is 1.4 times larger, due to the tapering of the illumination:

$$\theta'' = [13750 \cos\delta\,]\,[0.63\,\frac{\lambda}{d}\,].$$

Approximating the antenna beams to a gaussian curve having same width at half-intensity points:

$$\theta'' = \theta \rightarrow \tau = [13750 \cos \delta] [0.53 \frac{\lambda}{d}].$$

and:

$$T_o = \sqrt{2}\, \tau = [13.750 \cos \delta] [0.75 \frac{\lambda}{d}] \qquad (5)$$

<u>Example</u>:

For: $d = 300$ ft.

$\lambda = 21$ cm.

$\delta = 0$

one has:

$$2T_o = 48 \text{ sec.}$$

Using fig. 1, one can compare for example the signal-to-noise ratio corresponding to one gate at the maximum of the transit, to the maximum signal to noise ratio. Let ϵ be the half-width of the gate: $\frac{S_\epsilon}{\sigma_\epsilon}$ corresponds to the abscissa: $X = \frac{\epsilon}{\tau}$.

For: $2\epsilon = 10$ sec, and with:

$d = 300$ ft.

$\lambda = 21$ cm.

$\delta = 0$

one has:

$$\frac{S_\epsilon}{\sigma_\epsilon} = 0.58 \frac{S}{\sigma} \text{ max.}$$

One would get the same signal-to-noise ratio for:

$$2T = 3 \text{ mn } 15 \text{ sec.}$$

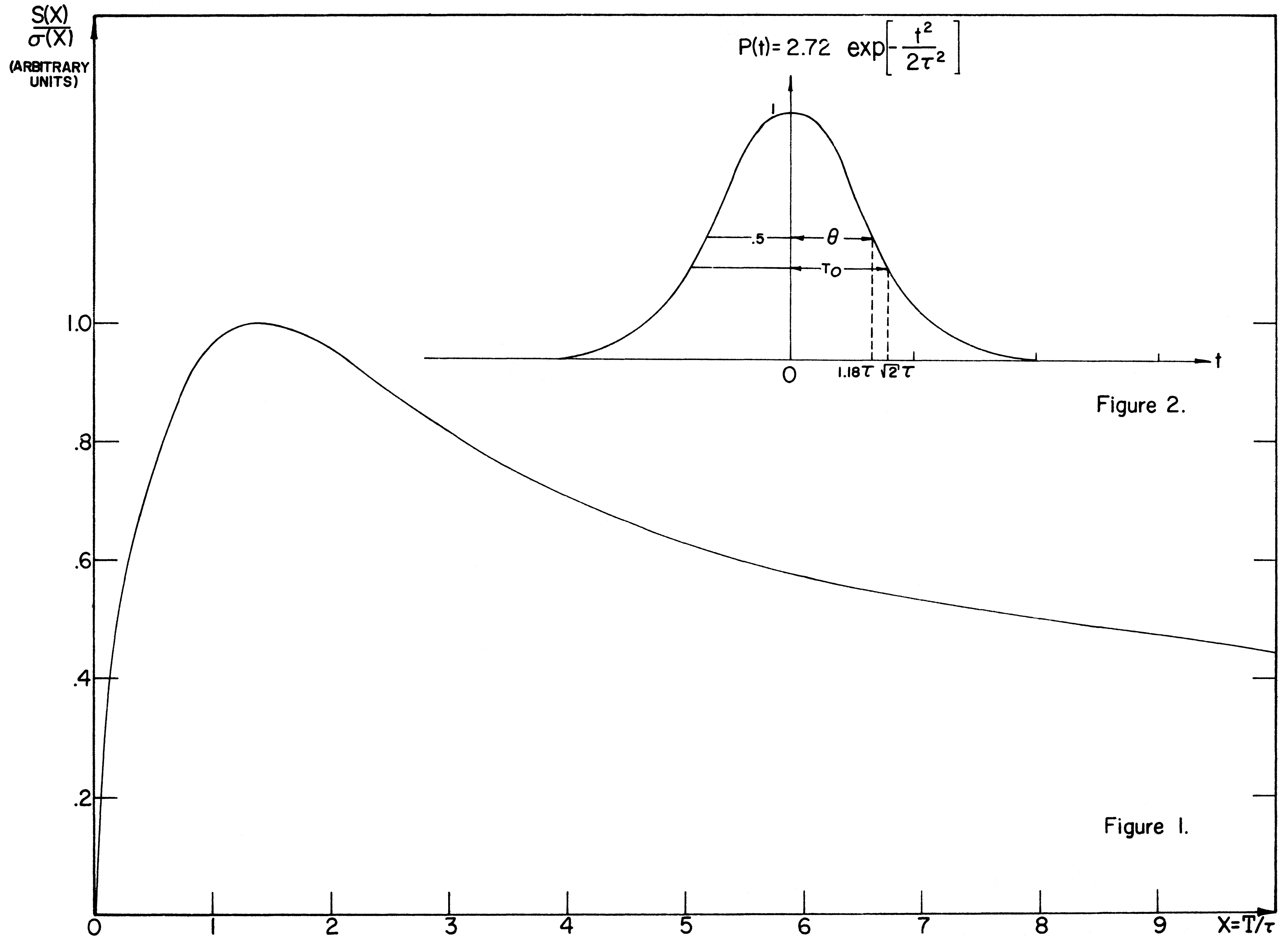

Figure 2.

Figure 1.