NATIONAL RADIO ASTRONOMY OBSERVATORY

VLA Report No. 1

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Draft

Contents

Appendices

1. Introduction

Higher resolution is, and has always been, a recurrent demand of radio astronomy. In recent years instruments have been developed with pencil beam resolutions of the order of a few minutes of arc, and with interferometric - partial information - resolutions of seconds of arc. The latter type instruments are all extremely slow in their information gathering rates.

The concept of a very powerful, versatile high resolution instrument, a radio counterpart of the 200 inch telescope, and ways of achieving that concept, has been the subject of much discussion in recent years. Achievement of such an instrument was given first priority in the recent report of the Panel on Astronomical Facilities of the National Academy of Sciences (the Whitford Report).

Spurred by the interests of scientists both within and without the NRAO, serious consideration of the problem was begun at the Observatory in the fall of 1963. Development of the variable baseline, tracking interferometer was undertaken partly to gain experience in what appears to be a most promising technique for the desired instrument. The development work has been given additional impetus by the Whitford Report.

The purpose of this report is to present the current status of work on the development of a very large array (VLA), and to solicit comments, criticism, ideas, and assistance for further work. The report is based primarily on studies by the following group: L. C. Chow, B. G. Clark, D. E. Hogg, H. Hvatum, G. W. Swenson (University of Illinois), W. C. Tyler, and C. M. Wade. Their work has been strongly influenced by the report of The Panel on Astronomical Facilities for the National Academy of Sciences, by a meeting of astronomers held at Green Bank in the fall of 1964, and by many discussions with radio astronomers around the world.

The report is being distributed in a somewhat rough-draft form, in the interest of getting it out at this time. We hope it will serve as a starting point for further discussions and design work.

2. Astronomical Need and General Performance Requirements

The National Academy of Sciences' report on Ground-Based Astronomy (1964) comments as follows upon the world-wide situation with regard to radio telescopes:

> ... The major factor that limits the advance of radio astronomy today is not particularly lack of observing time with frontier instruments, as in the case of optical astronomy, but rather the lack of instruments of the proper design to meet problems now recognized.
>
> ... None of the proposed or existing instruments will provide the versatility, the speed, and particularly the resolution demanded for progress with the prime astronomical problems.
>
> ... Contrary to the situation in optical astronomy, radio telescopes have not nearly approached the ultimate limitations on performance produced by inhomogeneities of the earth's atmosphere.
>
> ... Clearly ... no definitive knowledge of the radio sources throughout the universe can be obtained until resolution of the order of seconds of arc is available to radio astronomers.

While the need for much greater resolution is apparent in almost every branch of radio astronomy, from studies of the sun and planets to examination of the most distant galaxies, it is in the latter area that present limitations are most keenly felt. The principal hope for progress in Cosmology appears to rest on the expectation that radio telescopes can examine galaxies at much greater distances than can optical telescopes. Indeed, the galaxies whose spectra indicate that they are the most distant ones ever examined optically have been identified with some of the strongest extragalactic radio sources. Cosmologists need large numbers of data on radio sources at the greatest distances possible, and for many years a major activity of radio astronomers has been the attempt to provide accurate and systematic catalogs, free from selection effects, of radio sources with the smallest possible lower limits on flux densities. The pioneer catalogs of this type were produced at Cambridge, England and Sydney, Australia. These catalogs immediately demonstrated that the sensitivity of the radio telescope was not necessarily the limiting factor governing the number of detectable sources. Confusion, due to the blending of weak sources, is the more common limitation on the number of sources observable. Thus, antennas with smaller beamwidths, as well as greater sensitivities, are needed to study sources at greater distances.

It is not enough to count numbers of distant sources and measure their flux densities. Recent comparisons of optical and radio features, of those radio sources which have been identified, have shown that extragalactic radio sources have a wide variety of characteristics. In order to make meaningful use of the statistics produced by the deep-sky surveys, it is essential that a realistic classification of radio galaxies be developed, including the evolutionary sequence and intrinsic luminosities. The best hope for success in this effort lies in the detailed examination of nearby galaxies. This, in turn, will require the highest possible angular resolution, of the order of a few seconds of arc.

The radio galaxies, and quasars, are of great intrinsic interest in themselves, in addition to their implications for cosmological studies. The observed characteristics of these objects suggest that physical phenomena of fundamental and far reaching importance are involved. Again, high resolution observations, particularly of the radio surface brightness distribution and polarization, are needed to further the studies of these interesting objects.

The "Whitford Report" of the National Academy of Sciences quoted above, calls for 10 seconds of arc resolution at a wavelength of 10 cm, with sufficient sensitivity to detect 25 sources per square degree of sky. There is no reason to think that 10" resolution will satisfy the demand for very long. Observations at Cal Tech, Jodrell Bank, and NRAO have shown that structural features on a scale of the order of a second of arc exist in many sources. These will ultimately have to be studied in detail, if possible. In addition, much smaller structural features have been observed or inferred in four recent, independent lines of research; the Jodrell Bank 600,000 wavelength interferometer observations; studies of scintillations of radio sources; the low frequency cutoff in radio source spectra, interpreted as synchrotron self-absorption; and the rapid time variations in the radio and optical luminosity of some quasi-stellar sources. Clearly the quest for higher resolution needs to be pushed until some fundamental limit is reached. In the optical case the limit is determined by atmospheric inhomogeneities, and has already been reached. In the radio case, the limit is probably also set ultimately by atmospheric inhomogeneities, but these inhomogeneities apparently do not produce appreciable degradation of performance in any antennas constructed to date. Experiments at 11 cm. wavelength with a long-baseline interferometer at Green Bank, West Virginia, during 1964-65 show no discernable atmospheric effects with baselines as long as 25,000 wavelengths (8" resolution). Experiments by the University of Manchester radio astronomers, using very long baselines at longer wavelengths, indicate that resolution of one second of arc is attainable.

If it can be done within reasonable budgetary limits, it seems highly desirable to produce an instrument capable of the greatest resolution permitted by atmospheric limitations. Although these limitations are not well understood, it appears to be possible to achieve one second of arc, at least, which would be comparable with the capabilities of ground-based optical telescopes.

Quite clearly, if resolving power of the order of seconds of arc is to be achieved, a drastic change in the conventional observational procedure is called for. An instrument with a single pencil-beam 10" in diameter capable of tracking a source for six hours, would require a minimum observing time of 3600 hours, over a period of 600 days, to map five square minutes of arc, an area of sky roughly equal to that of the field of view of the 200 inch telescope. This assumes an integration time of one hour, and instantaneous re-positioning of the beam after each observation. It is thus obvious that a faster technique of obtaining information must be found.

A pencil beam instrument accepts information from only one direction at a time, rejecting all information impinging upon it from other directions. In this sense it is highly inefficient. On the other hand, an antenna comprising many parts, from each of which the data are available for separate processing, can produce as many separate pencil beams as there are separate parts. The ultimate development of this latter technique is the so-called "correlation array," which will be discussed in detail later in this report.

In examining a cosmic radio source, it is obviously necessary, insofar as practical techniques permit, to measure all the physical properties that can be determined at radio wavelengths; namely, spatial distribution of brightness, polarization, spectrum, and temporal variation. Temporal variations of galactic and extragalactic sources appear to be sufficiently rare, and at slow enough rates when they actually do occur, to permit this parameter to be ignored in the design of the instrument. Spatial distribution will be discussed in great detail later in this report, as it is the source property most critical to the choice of design philosophy. Polarization is an important clue to the physical process producing the radiation; the instrument should be designed with high priority for this requirement. The spectrum is likewise of great importance to an understanding of the physical nature of a source. The variation of spectral flux density with frequency can be classified into two parts; line radiation, arising from atomic or molecular processes; and continuum radiation, arising from macroscopic processes. The latter type of variation can be determined by measurements at well-spaced frequencies, say at octave intervals. Spectral line observations, however, involve continuous scanning of relatively narrow frequency ranges and require intricate and elaborate equipment. Provisions for continuum spectrum measurements, in the form of receiving equipment for two or more discrete wavelengths, can and should be made. The addition of line spectrometer equipment to the already formidable array of data processing apparatus required for seconds-of-arc angular resolution appears to increase the complexity of the whole system to a point not consistent with the present state of the electronic art. Possibly providions can be made for a very limited spectroscopic capability to permit exploratory observations.

Another parameter that must be defined is the "field of view." By this is meant the area of sky, for a given observation or set of observations, that can be mapped without danger of confusion from "grating lobes." Grating lobes are an inevitable concomitant of a highly-thinned array antenna; no nonuniform spacing or other technique produces any significant improvement in this situation. It is important to have no grating lobes within the area of sky occupied by a source under observation; thus, the design of the instrument depends strongly upon the angular size of source to be investigated. The smaller the source, the more economical the array, and the faster the source can be mapped to a given resolution. It is believed that the vast majority of extragalactic sources are less than 5 minutes of arc in diameter, and this has been chosen as the field of view for the telescope. This is not to say that larger sources cannot be mapped; such observations will merely take longer and will involve moving some of the array elements to different positions, in the manner to be described hereinafter.

In summary then, the following can be said to be desirable characteristics for the instrument under discussion:

(1) Wavelength: 11 centimeters. State-of-the-art techniques exist for the necessary receivers, antennas, and other electronic appurtenances. The atmosphere is known to be sufficiently well-behaved at this wavelength. This wavelength represents a good compromise between the space required for a high-resolution array and the well-known tendency for source flux densities to decrease with decreasing wavelength. The system design should be flexible enough to accomodate a limited number of changes of wavelength when sufficiently urgent demand arises.

(2) Resolution: 10 seconds of arc (Rayleigh's criterion)

(3) Field of view: 5 minutes of arc This is the size of source that can be mapped with the full array, to 10 seconds resolution, without moving or re-observing. Again, the system should be flexible enough to allow larger fields of view to be obtained when necessary, at the expense of additional observing time.

(4) Sensitivity: A point source whose flux density is 2×10^{-28} Joule/m^2 should yield a signal-to-noise ratio of 5 For extended sources, a difference in flux density between adjacent picture elements of the source of 2×10^{-28} should be detectable with a signal-to-noise ratio of 5. Assuming a static, Euclidean universe, and extrapolating from existing, low-sensitivity source surveys, there should be about 25 sources per square degree with this flux density.

(5) Versatility: While the parameters defined above have been adopted

to fit the principal mission of the telescope, namely, the mapping of extragalactic sources, it is essential that it also be capable of application to other problems, such as the mapping of low-brightness extended sources at lower resolution, the mapping of very small sources with a resolution of 1 second, and the measurement of angular diameters of large number of sources down to diameters of 1 second.

(6) Expandability: The telescope should be capable of expansion, to increase its resolution, sensitivity or flexibility as future developments in radio astronomy dictate. Radio astronomy is a rapidly changing field. The instrumental needs of 10 or 15 years hence cannot be reliably predicted, but the instrument of 5 or 10 years hence should be capable of adapting to those needs. This becomes particularly important for instruments that are costly and require long times to develop.

3. Possible Systems

The performance specifications enumerated in the previous Chapter clearly demand that an array of antennas be used, rather than a single reflector. The primary specifications can be satisfied by an array of linear dimensions of about 2.5 kilometers. To extend the resolution to 1 second of arc requires about 25 kilometers. The specifications can be met in principle at least, by any of the following types of array:

1) Continuous, completely filled, phased array.

   Examples: The MIT Lincoln Laboratory solar radar antenna at El Campo, Texas, and the incoherent ionospheric scatter radar antenna at Jicamarca, Peru. Both of these enormous antennas have been eminently successful; however, this system utilizing a continuous distribution of dipoles appears to be impossibly impractical for the high resolutions envisaged here. To achieve 10" resolution would require approximately 10^9 dipoles, and to achieve 1" resolution would require 10^{11} dipoles, even at the maximum feasible spacing.

   This array, and the Mills-Cross type discussed below, have the disadvantage that polarization observations are, at best, extremely difficult to make.

2) Continuous cross-type arrays.

   Examples: The Mills Crosses at Fleurs and Molonglo, Australia; the Mills Cross formerly at the Carnegie Institution River Road Station in Maryland; the Serpukhov Cross in the USSR; the Italian Cross at Bologna.

   This highly successful type of antenna can probably be expanded to achieve the extreme resolution desired. Whether it can do so economically remains to be demonstrated. For a symmetrical cross of the "classical"type, approximately 70,000 dipoles would be needed to achieve 10" resolution, together with a very large number of preamplifiers and an intricate network of phase shifters. For 1" resolution, 700,000 dipoles would be needed.

To achieve adequate speed of operation, many "beams" would necessarily be produced simultaneously, leading to great electronic complexity. This type of system, together with the "grating cross" discussed below, will be analyzed in greater detail in a separate report, and a proposed system set forth.

3) Grating cross arrays.

Examples: The Christiansen Cross at Fleurs, Australia; the Bracewell Cross at Stanford University; the Cross at Nancay, France. Several related instruments have been constructed, some having features of both continuous arrays and interferometers, notably the arrays constructed by Covington at Ottawa and by Erickson at Clark Lake, California.

This type of array shares with the Mills Cross the ability to produce very narrow beams with great economy of materials and equipment. The conventional grating cross employs two sparsely-filled arms correlated with each other. Because of the wide spacing a number of strong secondary lobes appear in the antenna pattern, the so-called "grating lobes." Provided one has some assurance that none of the grating lobes falls upon another source of radiation, the grating cross can be used very effectively to map the brightness distribution of an extended source. It is also required, of course, that the antennas comprising the arms of the cross be spaced closely enough so that the angular separation between grating lobes is greater than the angular extent of the source to be examined.

The grating cross principle can probably be extended to resolutions of 10" or 1". However, as such instruments to date have commonly operated in the transit mode, it will be necessary either to add phase tracking or a multiplicity of separately instrumented beams in both the hour-angle and declination coordinates in order to achieve adequate integration time and speed of observation. Either of these alternatives inevitably lead to great complexity in the electronics system. This system, as applied to very-high-resolution observations, will be analyzed in greater detail in another report.

4) Aperture synthesis antennas.

This type of radio astronomy instrumentation had its origin in early interferometer experiments (A. G. Little and R. Payne-Scott, Aust. J. Sci. Res., A4, 489, 1951) and its most sophisticated development in recent experiments by the Cambridge radio astronomers led by Ryle (M. Ryle, B. Elsmore, Ann C. Neville, Nature, 207, 1024, 1965).

The basic principle of aperture synthesis involves the Fourier analysis of the brightness of a finite area of the sky, and the recording of individual Fourier components or of partial sums of components for later summation in a computing machine. It is easily shown (Bracewell, IRE Trans., AP-9, 59, 1961) that an array of interferometers, each measuring a single Fourier component of a brightness distribution, can be used to simulate a very large antenna. As each interferometer determines and records one Fourier component, independently of all other interferometers in the array, it is unnecessary for all measurements to be made simultaneously. In fact, it is possible to synthesize a large antenna with only two small ones, by moving one of them about for successive observations in such a way that all necessary Fourier components of the brightness distribution are determined. Provided the area being mapped is of finite extent, its brightness distribution can be represented by a Fourier series, and the minimum baseline length and the interval between successive baseline lengths is determined by the finite angular dimension of the source. The maximum baseline length, of course, is determined by the angular resolution sought. Thus, the number of baselines needed is finite, even in a two-dimensional situation.

The aperture synthesis principle has many ramifications. It has been successfully applied by several groups of researchers, including those at Cambridge, Cal Tech, NRAO, and Stanford. A particularly reliable and precise form of instrumentation, suitable to extremely long baselines and highly intricate arrays of antennas, was developed at Cal Tech (R. B. Read, Ap. J. 138, 1, 1963) and further perfected at NRAO (Wade, Clark, and Hogg, Ap. J. 142, 406, 1965).

A large array of antennas, connected into interferometer pairs by correlators, will be called a "correlator array." If N antennas are present, there can be as many as N(N-1)/2 interferometers existing simultaneously. Such an array can be built using a number of steerable reflector antennas which can track a source for a considerable length of time, thus permitting relatively long integration times. As will be discussed later, this hour-angle tracking can be utilized in the "super-synthesis" scheme to reduce the number of antennas required to synthesize a given equivalent antenna aperture. From the standpoints of speed, electronic simplicity, sensitivity, ease of calibration, flexibility and versatility, and overall cost economy, the correlation array appears to be superior to all the other possible schemes for realizing seconds-of-arc resolution. Thus, the balance of this report will be devoted mainly to an investigation of a correlation array to achieve the parameters listed in Chapter 2.

4. The Correlator Array

The number of antennas needed to yield complete brightness distribution information about a source can be determined by transforming from the aperture plane into the fringe visibility plane. The interrelationships among the various quantities is given in Fig. 4-1. The fringe visibility plane (u-v plane) representation, or "transfer function" of an array is the autocorrelation function of the aperture plane illumination. The "brightness spectrum" or complex coherence function of the brightness distribution is the Fourier transform of the brightness distribution. Only those components of the complex visibility of the source can be measured whose spatial frequencies correspond with spatial frequencies (vector spacings) represented in the u-v plane representation (transfer function) of the aperture illumination. The Fourier transform of the observed brightness distribution, as observed at the output of the array, is the product of the transfer function and the F.T. of the actual brightness distribution. It is necessary, therefore, to study the transfer function of any proposed correlator array in order to know how effectively it gathers the information needed to synthesize a portion of the sky.

The number of antennas needed to obtain all the necessary information in a finite area of the sky is determined by the angular extent of the area and by the angular resolution to be achieved. As the area of interest is finite in extent, it can be represented by a Fourier series whose fundamental term has its frequency determined by the angular width a of the area. It is, in fact, $(a)^{-1}$ cycles per radian. In order to accommodate any arbitrary distribution of brightness within the area it is necessary that all harmonics be present; thus, spacings of antennas in the array must be at intervals of $(a)^{-1}$ wavelengths. In other words, every antenna spacing

SYSTEM LINEAR IN INTENSITY

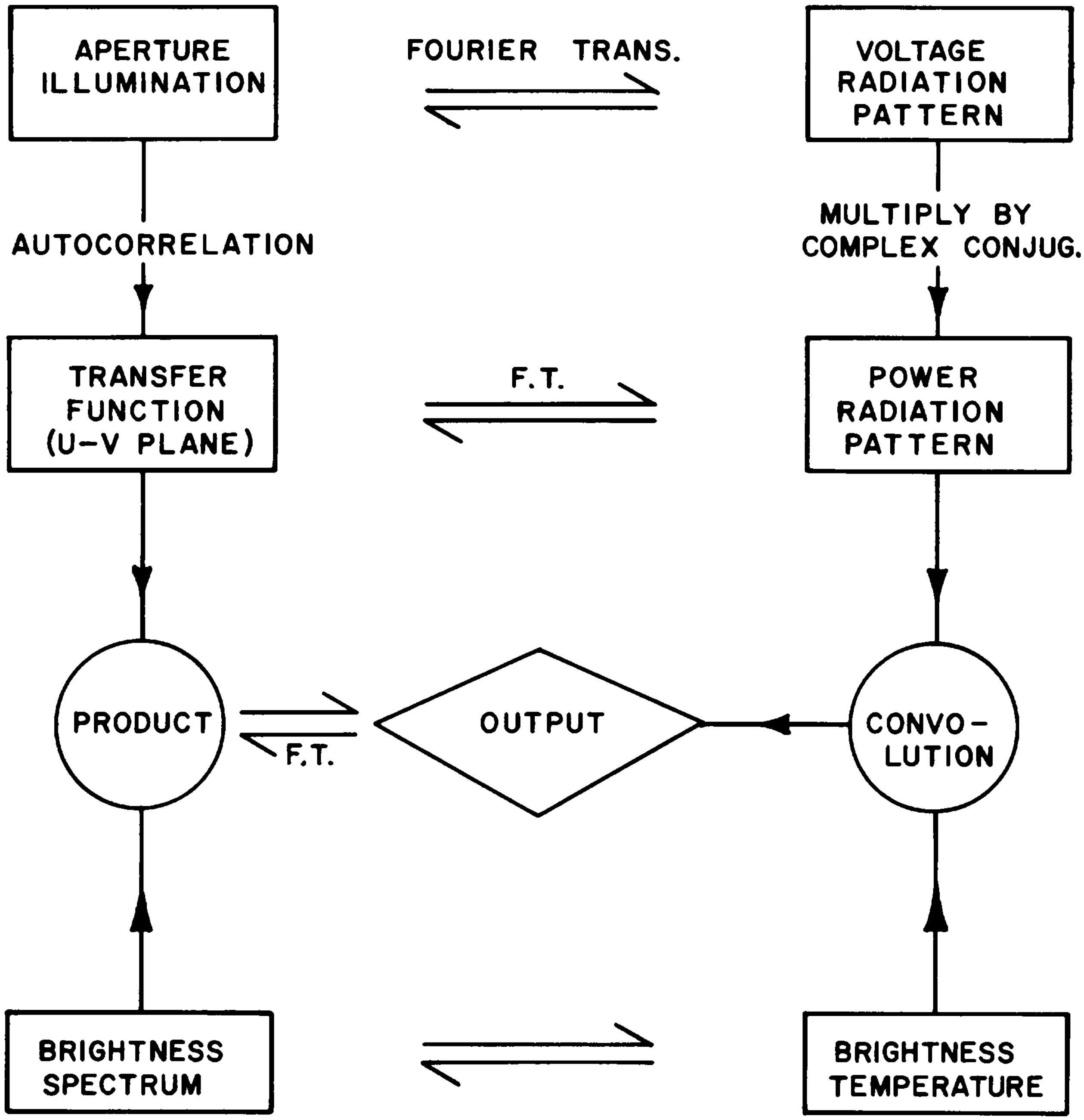


FIG. 4-1

corresponding with a term of the two dimensional Fourier series must be present. The finest detail that can be represented in the scanned area is determined by the highest-frequency term in the series, and this, in turn, is determined by the longest spacing in the array.

It is assumed that "resolution" is defined in the Rayleigh sense; that is, that two point sources can just be resolved if they are separated by half the distance between the first-order nulls in the antenna's diffraction pattern. The separation of the first-order nulls (the "beamwidth between nulls") is determined not only by the width of the aperture but also by the shape of the illumination function of the aperture. The aperture illumination function also controls the sidelobe level (as distinguished from the grating lobes) and it is customary for the illumination (or excitation, or weighting) of the aperture to be "tapered" from the center to the edges in order to achieve low sidelobes. This tapering increases the beamwidth over that obtainable with, say, uniform illumination. The relationship between beamwidth (between nulls) and ratio of the first-order sidelobes to the main beam, for well-chosen illumination functions, is given by the experimental relationship

$$(Bw) = [0.12\ (SL) + 0.12]\ \lambda/a$$

in which (B_w) is the beamwidth between nulls; (SL) is the ratio between the main beam response and the first sidelobe response, in dB; λ is the wavelength; and <u>a</u> is the aperture width. This relationship is for rectangular apertures, and is illustrated in Fig. 4-2.

These facts permit calculation of the general features of an array, to meet the specifications listed in Chapter 2. Assume initially that it is desired to synthesize in one instant an equivalent antenna capable of

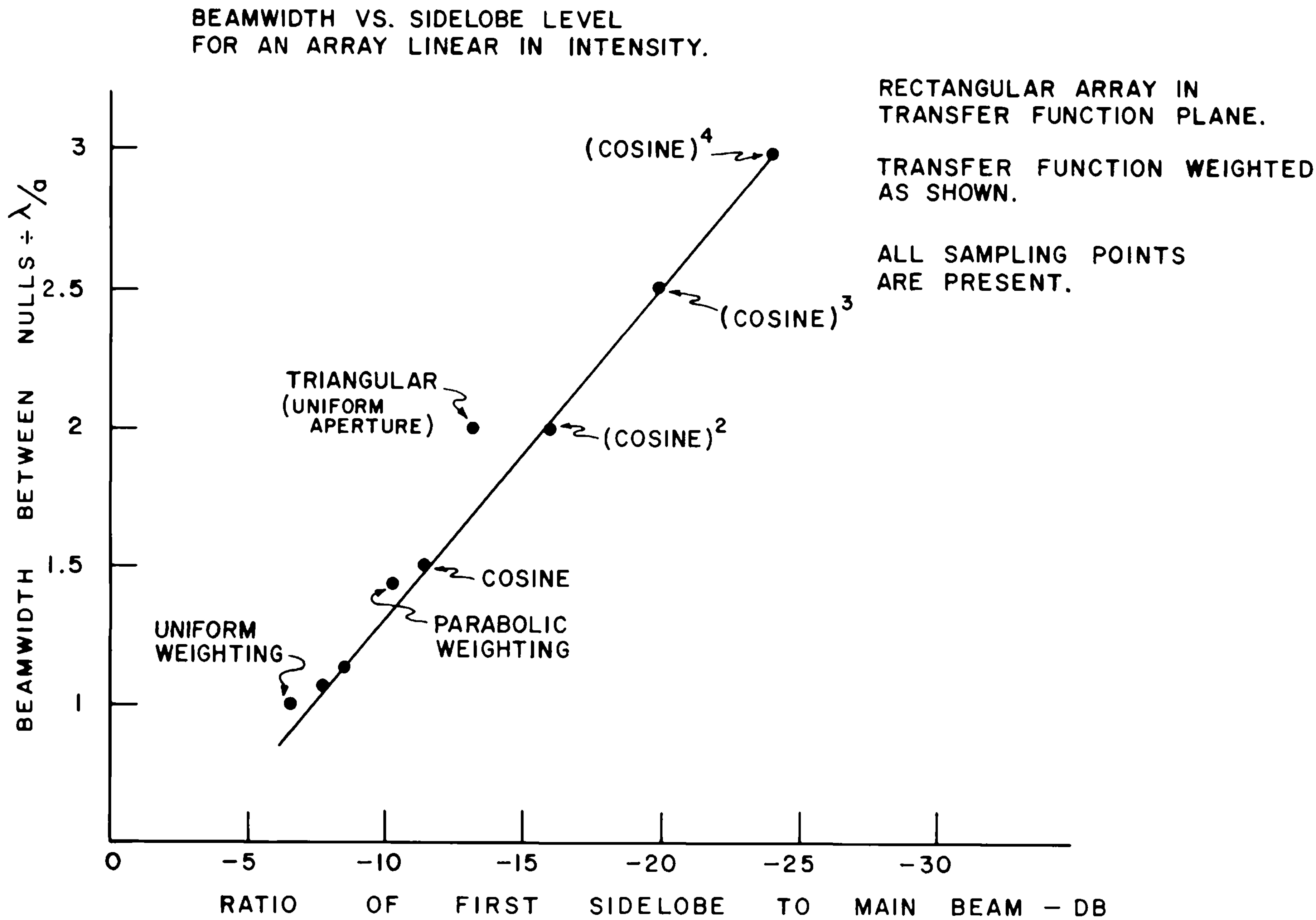

BEAMWIDTH VS. SIDELOBE LEVEL
FOR AN ARRAY LINEAR IN INTENSITY.
RECTANGULAR ARRAY IN
TRANSFER FUNCTION PLANE.
TRANSFER FUNCTION WEIGHTED
AS SHOWN.
ALL SAMPLING POINTS
ARE PRESENT.
(COSINE)^4
(COSINE)^3
(COSINE)^2
TRIANGULAR
(UNIFORM
APERTURE)
COSINE
PARABOLIC
WEIGHTING
UNIFORM
WEIGHTING
BEAMWIDTH BETWEEN NULLS ÷ λ/a
3
2.5
2
1.5
1
0
−5
−10
−15
−20
−25
−30
RATIO OF FIRST SIDELOBE TO MAIN BEAM − DB

FIG. 4−2

meeting the criteria of Chapter 2. The antenna spacing corresponding to 5' (.00146 radian) is 688 wavelengths. The points in the transfer function plane (fringe visibility plane) should therefore be arranged in a square grid with 688 wavelength intervals. Assuming that -13.6 dB sidelobes are acceptable, the aperture width is determined from the fact that twice the Rayleigh resolution angle equals 1.7 times the aperture width in wavelengths. The transfer function is thus a rectangular area 17,600 x 35,200 wavelengths. (Note that only one half of the transfer function plane need be considered because of the symmetry relation $W(u,v) = W^*(u,-v)$). Therefore, the total number of sample points in the transfer function is 1310. If two points in the transfer function fall closer together than $\frac{\lambda}{a}$, the points are said to be redundant. B. G. Clark (Appendix I) has shown that the minimum, theoretically-attainable, redundancy in an array is approximately 20%. However, investigation of practical situations suggest that the typical redundancy is of the order of 100%, and that it will be difficult to reduce it much below this value. Thus, the minimum number of sample points is 2620, including two-fold redundancy. The number of points in the transfer function is the number of combinations of antenna stations in the aperture function, taken two at a time. Thus, for the present case $N(N-1)/2 = 2620$, so that $N = 73$ antennas. This is a lower limit to the required number; however, it appears to be possible to realize this value, approximately, with a T-shaped array with 25 antennas on each of the three arms of the T, and with several other arrays with similar numbers of antennas, such as circular, Y-shaped, and "random" arrays.

Examples of the transfer functions corresponding with Tee and Wye-shaped arrays for 10" resolution are shown in Figures 4-3 and 4-4, respectively. Each array has 73 elements, existing simultaneously. In the

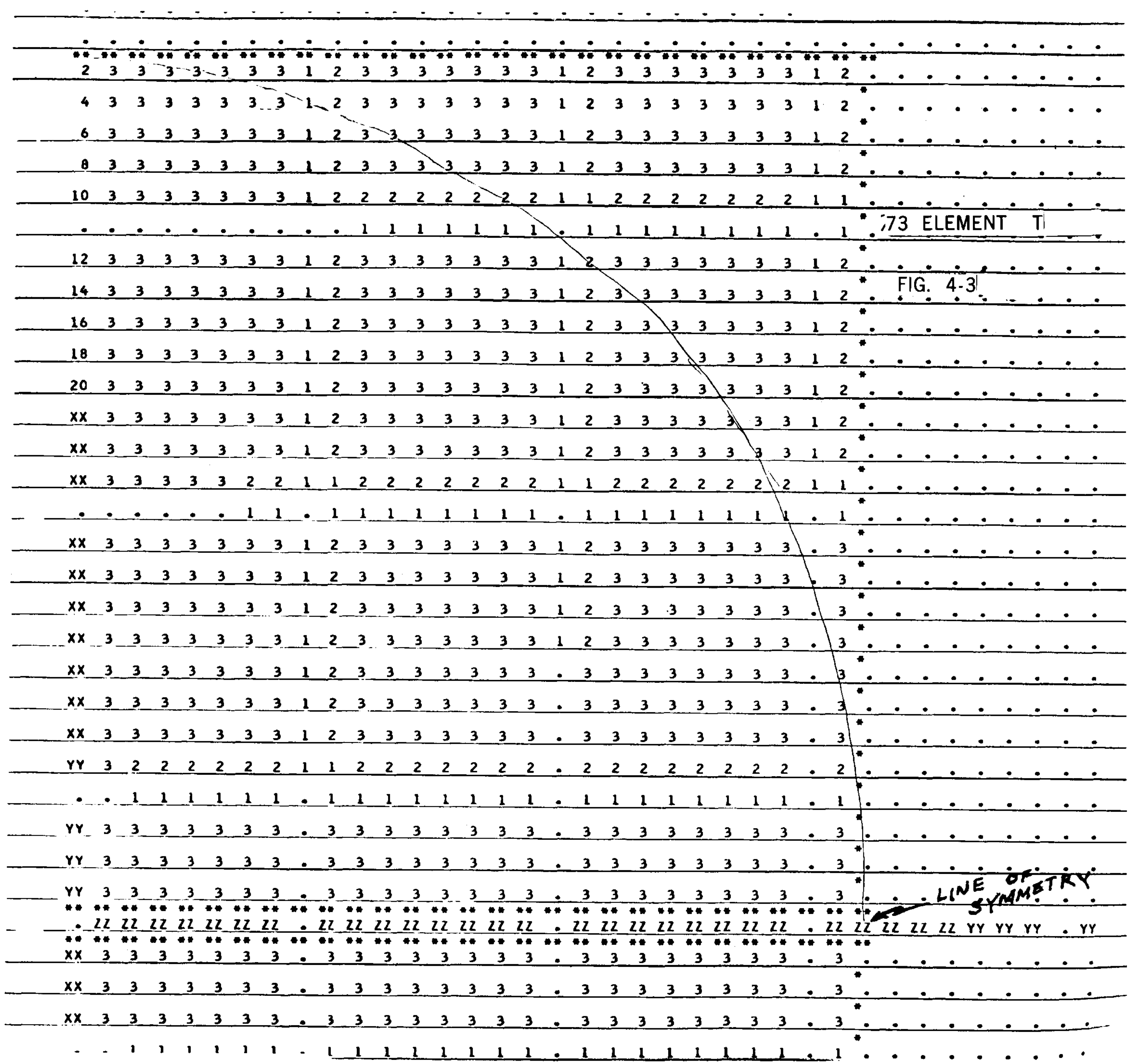


FIG. 4-3

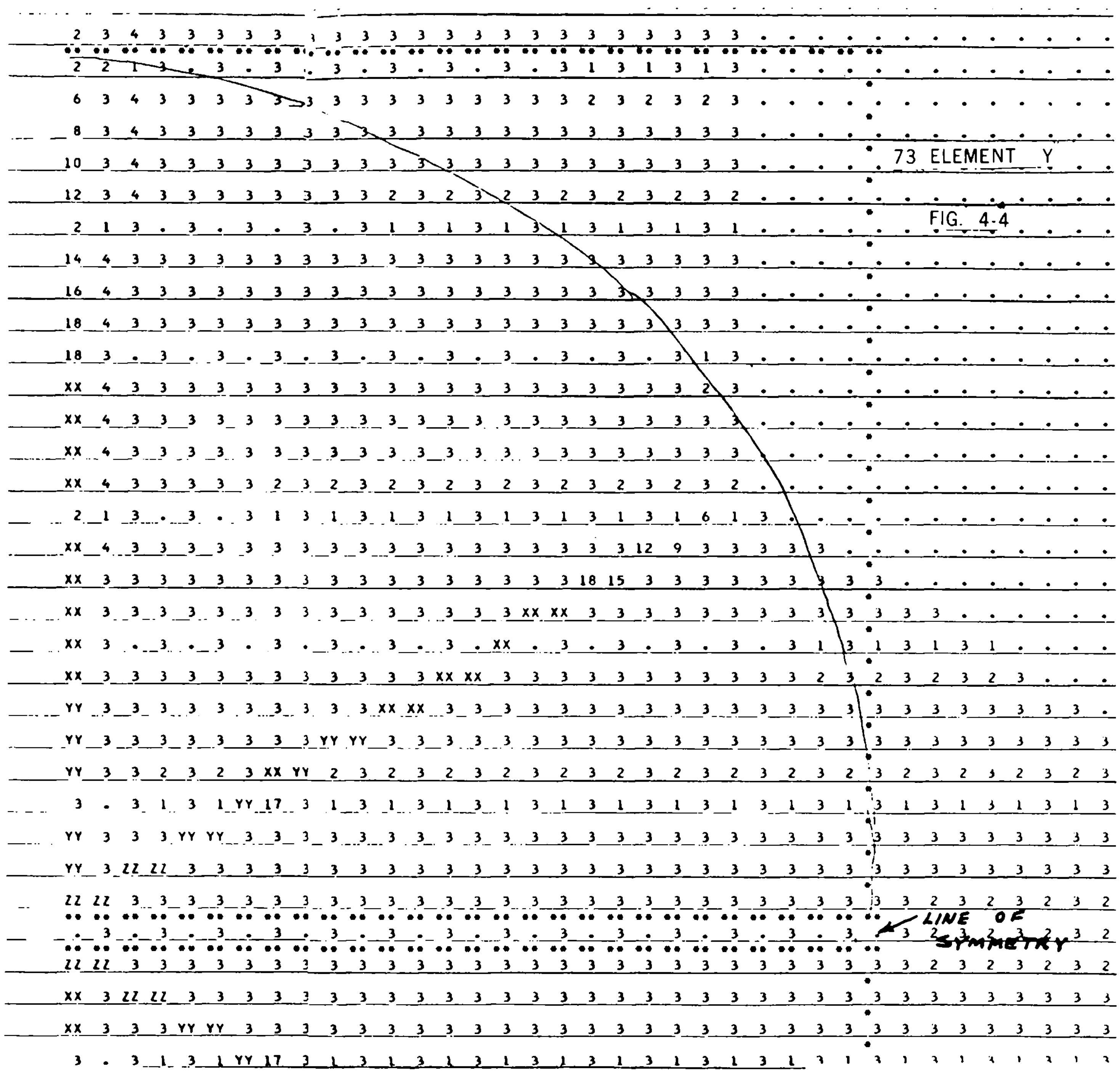


FIG. 4-4

plot, the numbers represent the number of times each Fourier component is measured. XX, YY, and ZZ represent large numbers of measurements. Dots represent sampling cells in which no measurements are obtained. The cell size is determined by the size of the sky area to be mapped; in this case it is 5 minutes of arc and the cell dimension is 688 wavelengths. It is seen that the sparsely-filled Tee array produces a transfer function that very closely approximates that of a continuously illuminated square aperture. Even though the arms of the Tee are quite sparsely occupied by antennas, only 7.5% of the sampling cells are missed. In the Wye array 15.5% are missed. However, these numbers are for a square transfer function. If a circular transfer function were used (by omitting the correlators for those vector spacings outside the circle of 18600 λ radius), the synthesized beam is circular in cross section, and the number of missing cells is only 6% for both the Tee and the Wye.

The signal-to-noise ratio of the correlator array must also be investigated. Blum (Ann. d'Ap. 22, 140) has shown that the sensitivity of a single antenna pair, with correlated outputs, is

$$\Delta T = \frac{\gamma}{\sqrt{2}} \frac{T}{\sqrt{\tau B}} \; ; \; \gamma \approx 1$$

where τ is the integration time, B is the bandwidth, T is the total system noise temperature, and ΔT is the thermal equivalent of the rms noise fluctuations. The correlator array antenna consists of a number of such correlated antenna pairs, or interferometers. If N antennas are available simultaneously, N(N-1)/2 combinations of two are possible, although it may not be necessary or desirable to supply a correlator for every possible combination.

It has been shown (B. G. Clark, Appendix II) that in a correlator array the unwanted components (noise) of the outputs of any two correlators are uncorrelated, even the correlators share a common antenna. Thus, as the noise outputs of all correlators are independent, each interferometer can be considered as a separate entity in calculating the sensitivity of the array.

For a point source, there is some point in the sky at which the fringe outputs of all baselines in an array are mutually in phase synchronism. This would occur at the center of the "main beam" if there are a sufficient number of baselines to give this term any validity. Thus, when the output records of all available baselines are superimposed, the fringe amplitudes add numerically, while the noise amplitudes, being uncorrelated from one record to the next, add in quadrature. The signal to noise ratio, therefore, varies as the square root of the number of baselines. This statement is valid regardless of whether or not any antenna is common to two or more baselines. It is also immaterial whether or not all baselines exist simultaneously. As the number of baselines varies apprεximately as the square of the number of antennas, the signal-to-noise ratio varies directly as the number of antennas.

One consequence of this statement is that all possible baselines in any array should be utilized, to achieve the maximum signal-to-noise ratio and to minimize the time required to acquire the necessary data.

The treatment can be generalized to include extended sources consisting either of groups of point sources or of one or more diffuse patches of emission. Suppose first that the extended source consists of M point sources. Assume also that the mth source is in the center of the synthesized "main beam" of the array. The various contributions add as follows:

$$P = MP_m + \sum_{p=1}^{m-1} \sum_{q=1}^{M} P_p \, e^{j\bar{w}_p \cdot \bar{x}_p}$$

where P is the total signal power, the sum of the outputs of all correlators,

M is the number of correlators (baselines)

m is the number of point sources

$\bar{x}_p$ is the vector distance from source m to source p

$\bar{w}_q$ is the vector spacing of the qth baseline

P_p is the single-correlator output due to the pth source when it is in a fringe maximum.

By inspection of the double sum it is seen that unless one of the point sources is at a location at which many of the baselines have fringe extrema (a grating sidelobe) most of the terms will be out of phase by varying degrees, so that the sum will be very small compared with the first term. Thus, only one source at a time influences the combined output of the correlators and the fringe visibility of an individual correlator's output does not affect the signal-to-noise ratio.

If the source is limited in extent its brightness distribution can be represented by a double Fourier series consisting of terms of discrete spatial frequency. The minimum distinguishable spacing between details of a brightness distribution is determined by the maximum spacing of the array used to scan the distribution. A finite-sized array cannot distinguish between a group of point sources, whose mutual spacings are below the resolution limit, and a diffuse distribution with details similarly below the limit. This is equivalent to the statement that any diffuse distribution can be simulated by a group of point sources suitably spaced. The arguments above concerning the response of the array to a distribution of point sources also apply, therefore, to a diffuse brightness distribution.

The array previously described in this chapter, consisting of 73 elements, with correlators interconnecting them into 2620 interferometer pairs (somewhat fewer if a circular array is assumed) involves a very great deal of electronic complexity. It is worth questioning whether such a high degree of complexity, and the attendant high costs, are justified by the rapidity with which such an array can gather astronomical data.

A 73 element array can yield a complete picture, to a resolution of 10", of a 5' diameter area in about one minute of observing time. Even with generous allowances of time for pointing the antennas and for maintenance, such an array could map tens of thousands of sources per year, far more than could be utilized by all the world's astronomers. Furthermore, the difficulty of maintaining in working order 73 antennas and receivers and 2620 correlators is frightening to contemplate. Preliminary cost studies indicate that both the initial capital cost and the operating costs of an array depend strongly on the number of antennas. Thus cost, operating, and information rate factors all indicate the desirability of reducing, at the expense of increased observing time, the number of antennas in the array, while still retaining the required performance characteristics.

The arrays illustrated in Figs. 4-3 and 4-4 show strong redundancy, in that most of the sampling cells in the transfer function are sampled two or three times. This suggests that the number of antennas could be reduced by increasing the spacing between them. The autocorrelation functions (transfer functions) of 70 element and 67 element Tee and Wye arrays were calculated in order to investigate this possibility. The result for both Tee andWye was that the degree of redundancy in general remained constant for those cells which were sampled, while the number of unsampled cells increased markedly. The results are shown in the following table:

Percent of Cells Missed

Type of array \ No. of antennas	73	70	67
Tee	7.5%	21%	31%
Wye	15.5%	20%	26%

Clearly the penalty paid in missed information outweighs the slight advantage in the reduced number of antennas. All these computations were for horizontal arrays at 30° latitude, observing the zenith. It is interesting to note that, while the 73-element Tee gathers more information than the 73-element symmetrical Wye, the latter does not deteriorate as rapidly as the former when antennas are removed.

It appears that the minimum number of antennas required for instantaneous synthesis is approximately 73, as predicted by the computations earlier in the chapter. There is, however, another method for reducing the number of antennas: the technique termed "supersynthesis" by the Cambridge group who made the first major applications to radio astronomy (Ryle, Nature 194, 517, 1962; Ryle, Elsmore, and Neville, Nature, 207, 1024, 1965). The method depends upon the fact that the projection at a fixed location on the celestial sphere of a given interferometer baseline rotates in direction and changes in length as the earth rotates. Over a 24-hour period the vector representing the projected baseline describes an ellipse in the transfer function plane. Even over a shorter period a single interferometer can give a large number of Fourier components of the sky brightness distribution, provided the individual antennas can track a point in the sky (B. Rowson, Mon. Not. R.A.S., 125, 177, 1963). The theory is developed by

Wade in Appendix III.

This technique, coupled with a multielement correlation array, appears to offer the most flexible, versatile, and economical solution to the problem of the seconds-of-arc radio telescope.

5. The Array Configuration

It was pointed out in Chapter 4 that a given array provides a much better coverage if it is made to track a source for a period of time than if it is used as a transit instrument. In the present chapter the performance of several different kinds of tracking array configurations will be examined. The discussion is based on the theory given in Appendix III.

It will be assumed throughout that the aim is to achieve a synthesized radiation pattern with a 10" half-power beamwidth, in a 5' field of view at 11.1 cm wavelength. It is also assumed that the latitude of the instrument is 30°N (the conclusions to be drawn are insensitive to latitude differences of a few degrees). We shall attempt to find what array plan gives the best coverage in the area $0 \leq u \leq 18600$, $0 \leq |v| \leq 18600$, at the most unfavorable declination (the celestial equator), for the fewest array stations. The limit of 18600 on u and $|v|$ follows from the resolution requirement and the assumption of linear weighting of the Fourier components in the inversion process. The 5' field of view corresponds to $\Delta u = \Delta v = 688$. The required number of sampling points is 1513. The differences between these values and those of the previous chapter result from adoption of different weighting functions in the two discussions.

Several general remarks are in order before proceeding to a consideration of specific arrays. First, all of the preliminary reconnaissance of the capabilities of different array models was made for operation at 0° declination, since this provides the most severe test of their relative merits (because of the degeneracy of the F.T. plane elliptical tracks). Second, the more southerly declinations offer special problems because of the foreshortening of the array and the limited tracking range allowed by the horizon. These difficulties have not yet been considered in detail.

Finally, two points quickly became evident in the early stages of the computer study: (1) the smallest number of elements capable of giving a good F.T. plane coverage (without moving the elements), for any configuration, is about 37; (2) relatively little is added to the F.T. plane coverage by increasing the tracking range beyond 2 hours from the meridian. Therefore, all of the comparisons between the various kinds of array plan are based on the performance when:

(a) $\delta = 0°$,

(b) Number of elements = 37,

(c) Tracking range is from 2^h E to 2^h W (4 hours total),

(d) $\lambda = 11.1$ cm,

(e) Station latitude = 30° N.

Section 5A briefly describes the computer program. Sections 5B - 5E discuss the following kinds of array:

(a) Hollow circle,

(b) Tee (T-shape),

(c) Wye (Y-shape),

(d) Random.

Concluding remarks and general comments are given in Section 5F.

A. The Computer Program

The study of the array performance was done with the aid of a computer. The program requires the following data:

(a) Latitude,

(b) Source declination,

(c) Operating wavelength,

(d) Limiting hour angle of the tracking range (assumed to be centered on the meridian),

(e) Array element locations on an east-west by north-south cartesian grid.

The computer calculates the values of u and v corresponding to each possible element pair for each sidereal minute in the tracking range, the integration time count for the appropriate F.T. plane cell is incremented by 1. Upon completion of this process for all antenna pairs, the following outputs are printed:

(a) Latitude, declination, wavelength, assigned model number, limiting hour angle;

(b) Locations of the array elements, on an (x,y) grid;

(c) Number of elements;

(d) The number of F.T. plane cells in $0 \leq u \leq 18600$, $0 \leq |v| \leq 18600$ which are

  (1) not sampled,

  (2) sampled for 1 to 20 minutes, separately for each number of minutes,

  (3) sampled for 21 to 40 minutes,

  (4) sampled for 41 to 60 minutes,

  (5) sampled for more than 60 minutes;

(e) A 37 by 73 grid, showing the amount of integration time in each F. T. plane cell in the range $0 \leq u \leq 25000$, $0 \leq |v| \leq 25000$. Lines of asterisks bracket the u-axis and mark the lines $u = 18600$, $v = 18600$, $v = -18600$.

(f) A tabulation of the distribution of fringe rates (needed for the RCA contract design study);

(g) An (x, y) grid showing the locations of the array elements.

B. The Hollow Circle

A hollow circle 26300 λ (2920 m) in diameter, with the elements evenly spaced about its periphery, gives a generally good F.T. plane coverage (Fig. 5-1). The sampling time distribution for the 1513 cells in which we wish to obtain data is:

not sampled	5.2%
sampled 1^m - 20^m	3.5
" 21^m - 40^m	16.2
" 41^m - 60^m	23.2
$> 60^m$	51.9

Most of the unsampled points lie on or near the u- and v-axes. They tend to cluster; it would be preferable to have them isolated and widely scattered.

C. The Tee

The performance of a tee array depends strongly on its orientation on the ground. We have examined 37-element tees (12 on each arm plus one at the junction of the arms) with the middle arm at four different azimuths: N, N30°E, N45°E, N60°E. These are illustrated by Figs. 2-5, respectively. The distribution of integration time in the 1513 cells is shown for each orientation by the following tabulation:

	Azimuth of Middle Arm			
	N	N30°E	N45°E	N60°E
not sampled	54.6%	18.3%	37.5%	17.6%
sampled 1^m - 20^m	0.0	6.8	1.1	6.3
sampled 21^m - 40^m	0.8	16.7	7.5	16.6
sampled 41^m - 60^m	2.6	24.8	9.0	24.1
sampled $> 60^m$	42.0	33.4	44.9	35.3

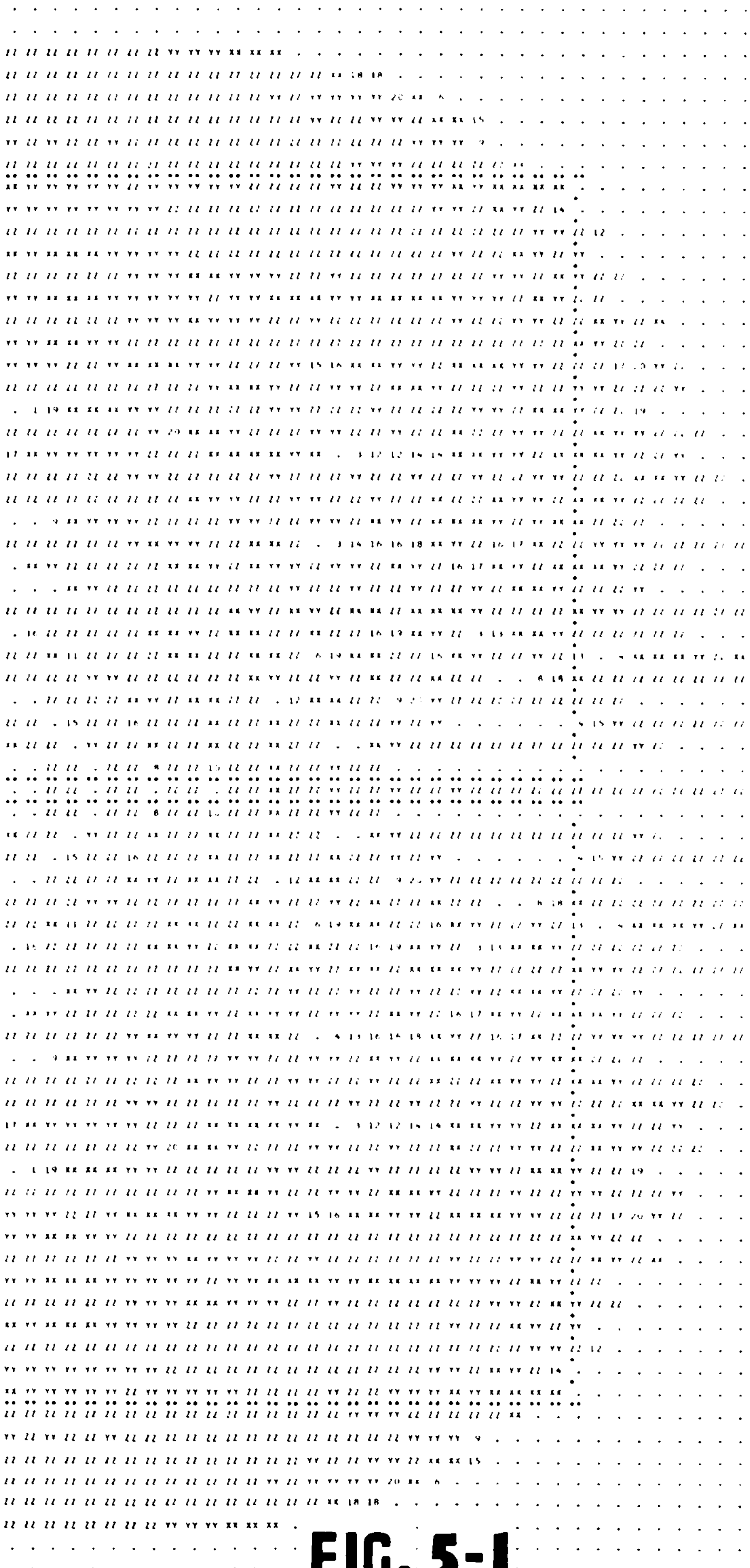


FIG. 5-1

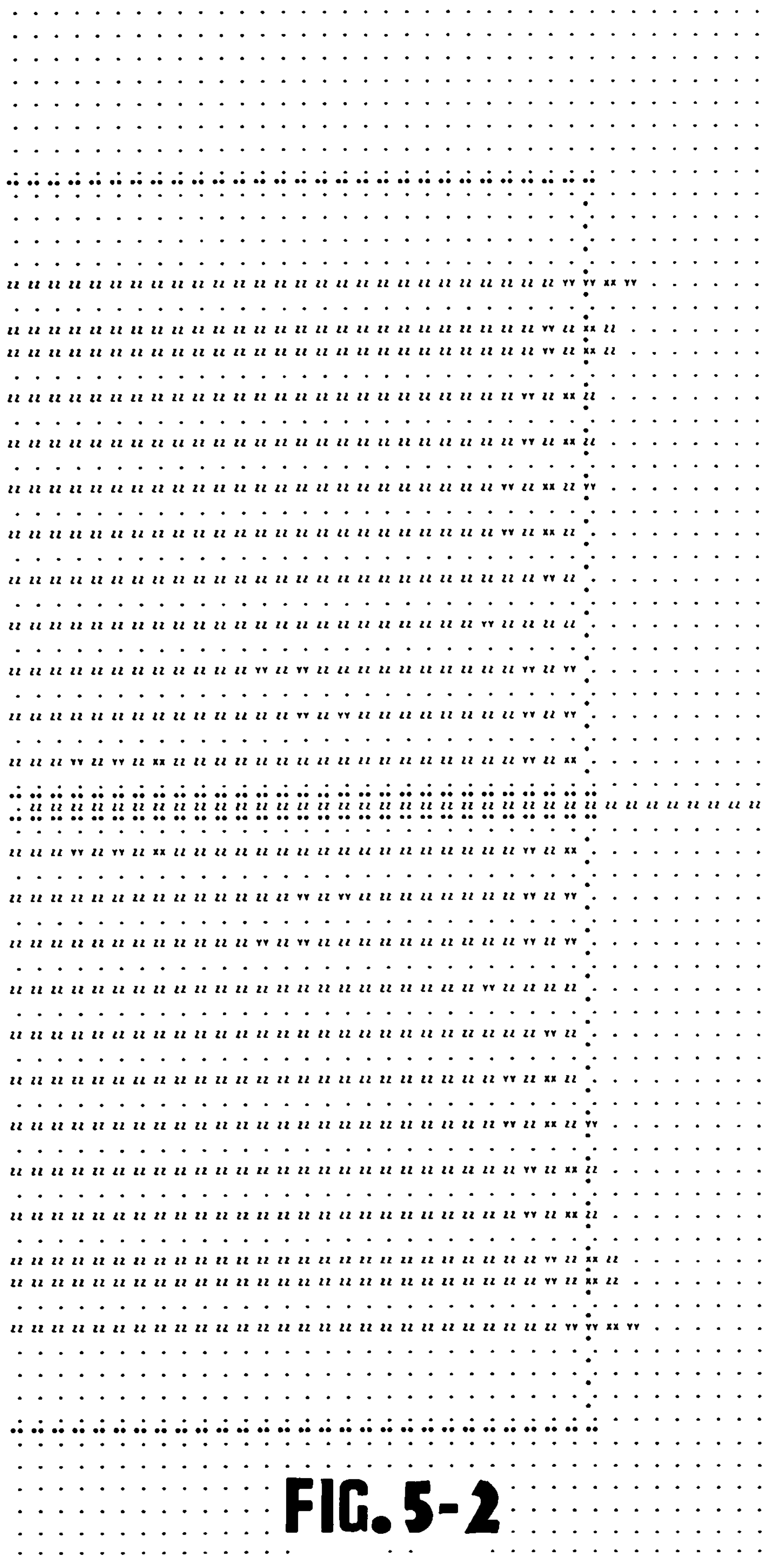


FIG. 5-2

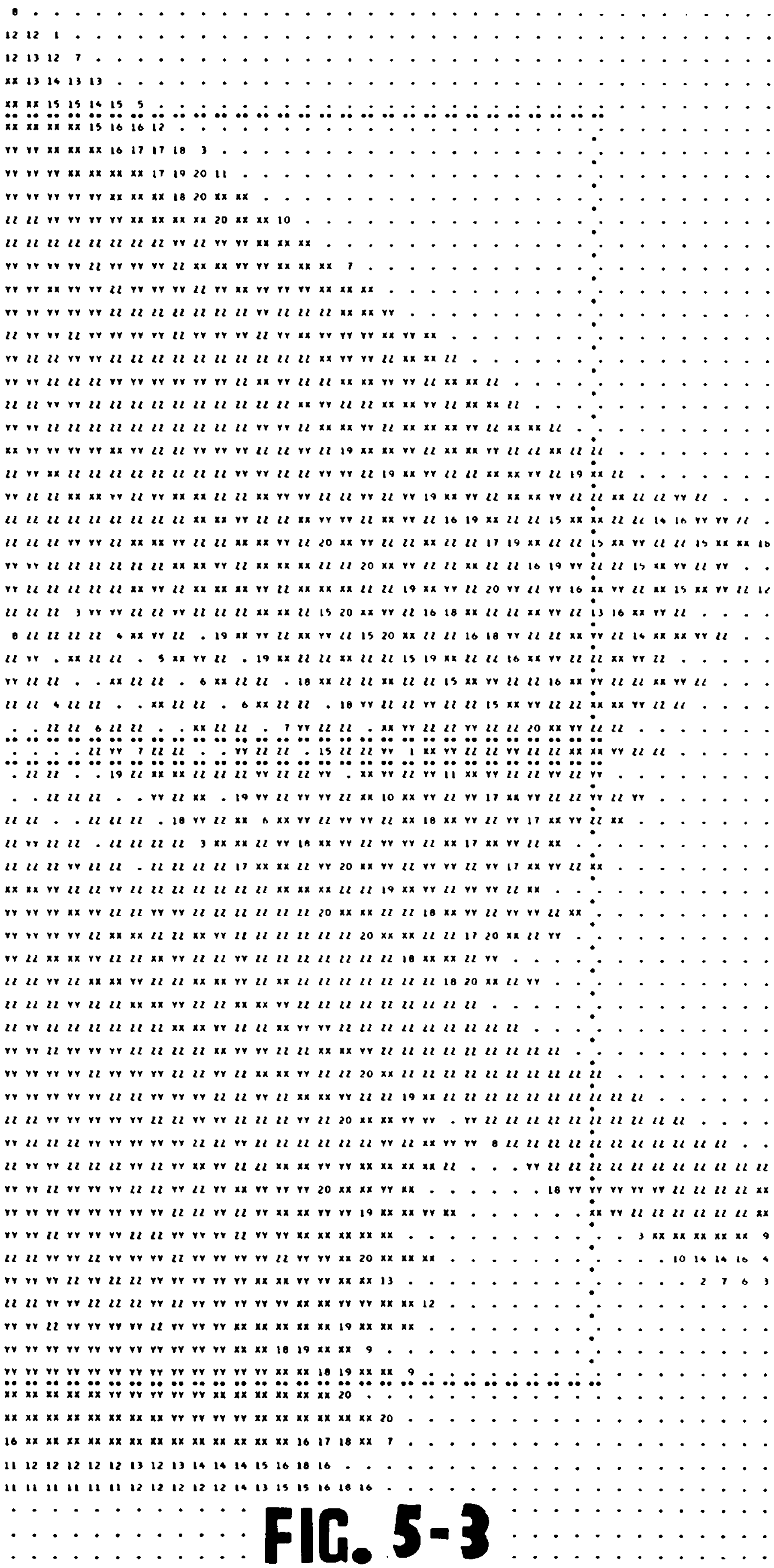


FIG. 5-3

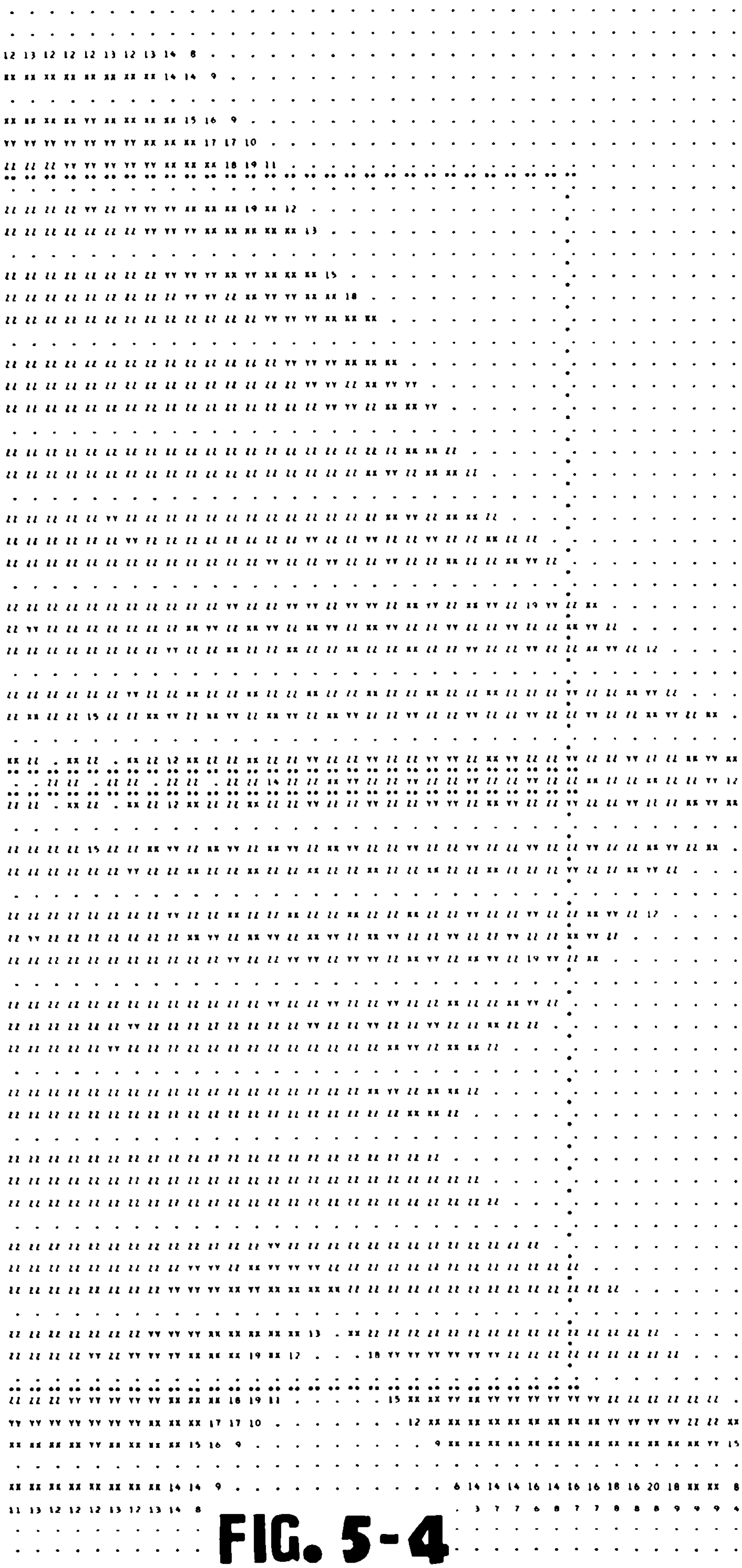


FIG. 5-4

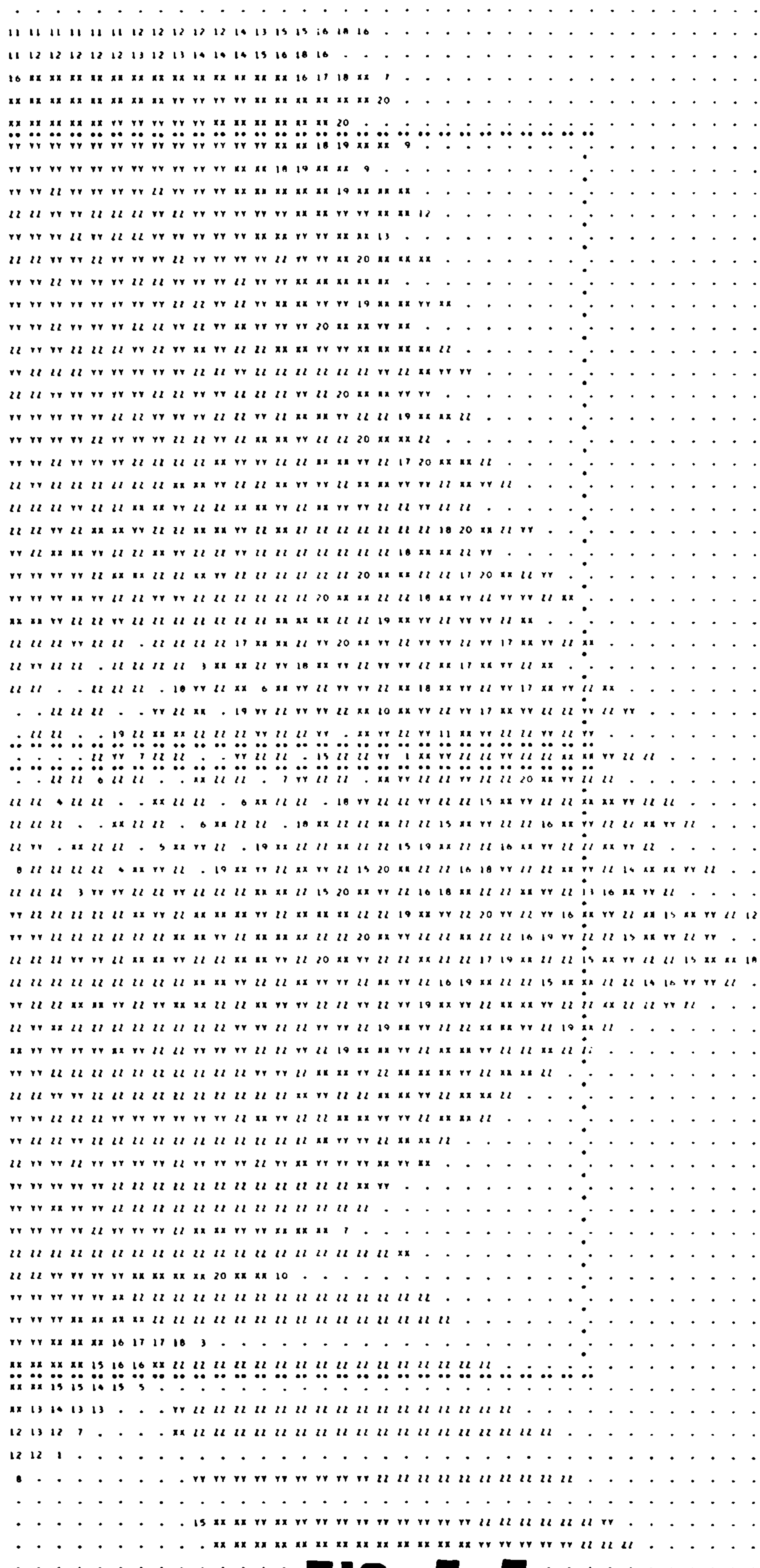


FIG. 5-5

None of these performs well at the equator. At higher declinations all of them would do better. The N and N45°E versions give a badly defective coverage because of the relatively small number of different N-S baseline components. The N30°E and N60°E versions perform similarly, as one would expect. They both fail to sample the outer corners of the 18600 by 37200 rectangle.

The N version (Fig. 5-2) becomes attractive if one considers moving the elements on the N-S arm 86 m toward the array center and making a second observation. This fills in the alternate lines which are left blank in Fig. 2. Furthermore, this array has a feature which is unique among the configurations considered in the present report: the foreshortening of the array for declinations removed from the zenith can be compensated simply by moving the 13 elements on the N-S arm away from the array center by an amount proportional to the secant of the minimum zenith distance. This means that the same beam shape and north-south resolution and field of view can be maintained at all declinations. This advantage is bought at the price of using the array in two configurations when observing near the equator. A further point, however, is that there are no unsampled cells within the sampled portion of the F.T. plane. This also is unique.

D. The Wye

Consider a Y-shaped array with three identical arms separated by 120° in azimuth. Each arm is 18600 λ (2064 m) in length. We have calculated its performance for two orientations: one arm north-south (Fig. 5-6), and one arm east-west (Fig. 5-7). The NS version gives a good F.T. plane coverage comparable with that of the hollow circle, while the EW version is quite poor. The integration time distribution in the 1513 cells is:

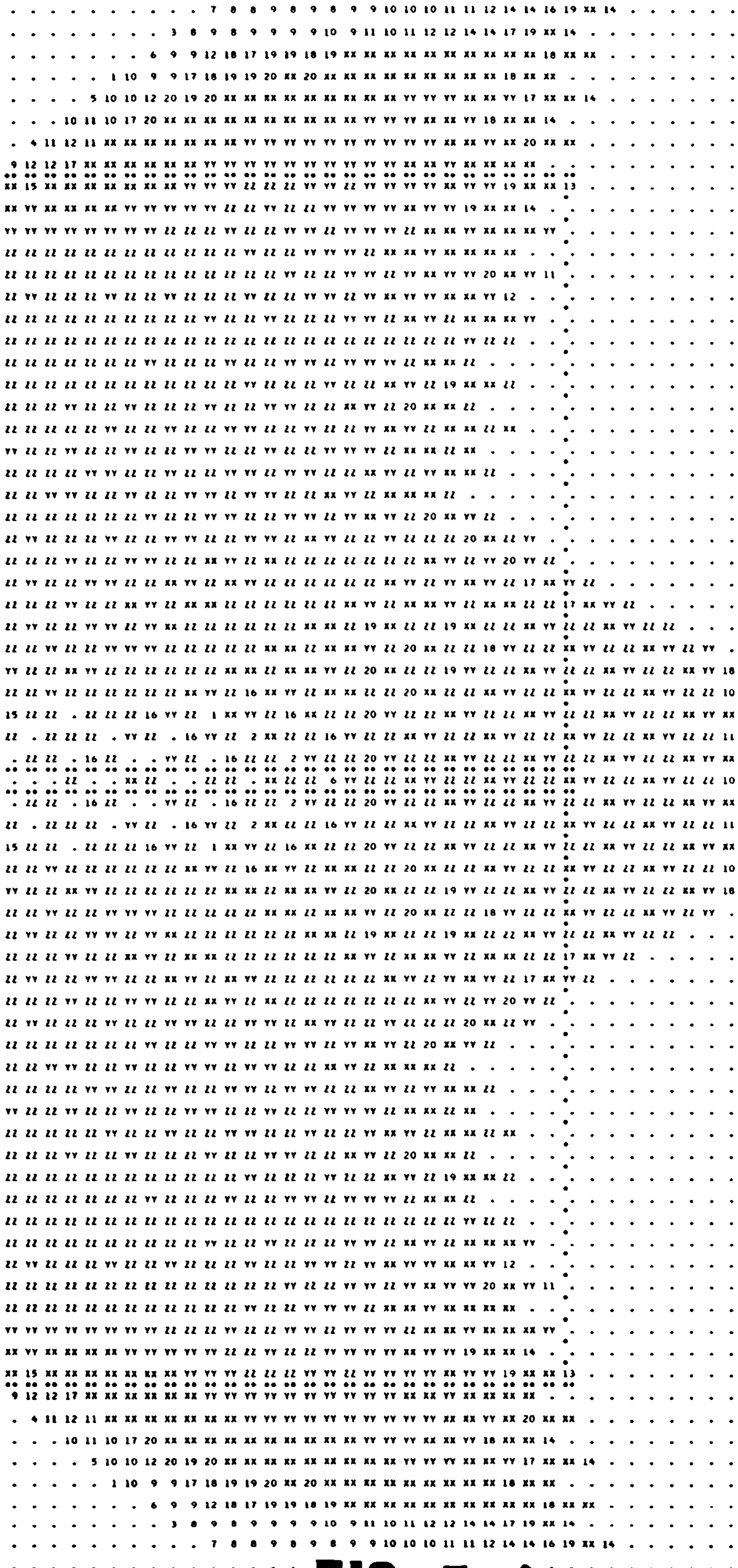


FIG. 5-6

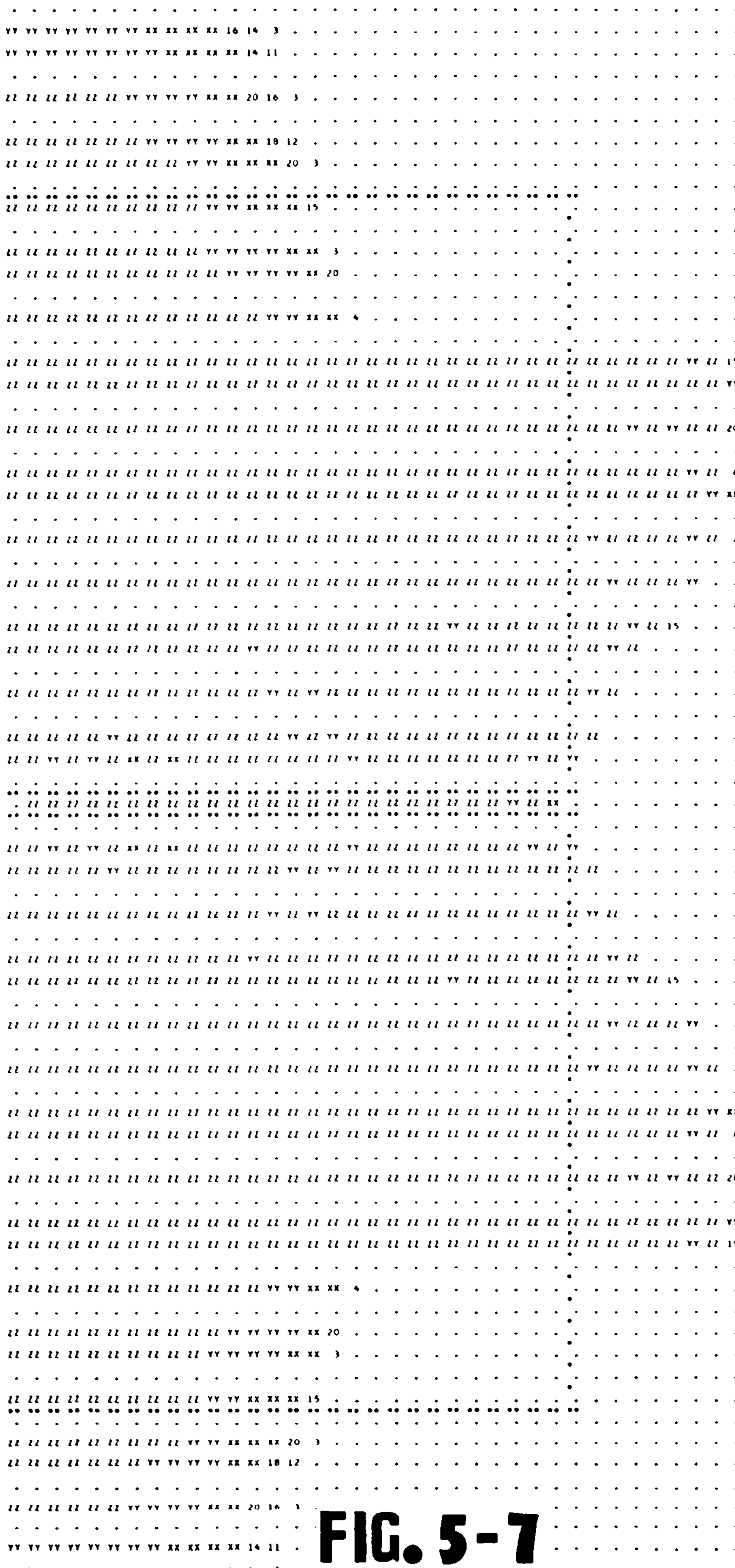


FIG. 5-7

	Orientation	
	NS	EW
Not sampled	6.4%	45.8%
Sampled 1^m - 20^m	4.3	0.4
Sampled 21^m - 40^m	15.1	1.4
Sampled 41^m - 60^m	24.7	3.1
Sampled > 60^m	49.5	49.3

The EW version clearly deserves no further consideration. The NS version, however, is interesting. Most of the unsampled points lie in two areas around $u = 18600$, $|v| = 11000$. These areas are continuous and amount to real holes in the synthesized aperture. They could easily be moved outside the 18600 by 37200 rectangle by increasing the arm lengths to about 2300 m. The remaining unsampled areas are scattered about the u-axis in the vicinity of the origin.

The integration is very heavy in most of the cells in the NS version. Therefore we investigated the effect of thinning the array by removing selected elements. In most cases this led to disaster. Removal of every second element on the SE and SW arms had a ruinous effect (Fig. 5-8). On the other hand, taking alternate elements away from the N arm had little effect on the over-all F.T. plane coverage (Fig. 5-9). The most noticeable result is the appearance of two long, narrow unsampled strips radiating from the origin. Fig. 5-10 shows the result of an unfortunate attempt to be clever. Each arm is an Arsac 0146 array, with the central element in common. The 0146 array performs nicely in one dimension, but the concept obviously cannot stand generalization. In this case, over 80% of the desired F.T. plane cells are left unsampled.

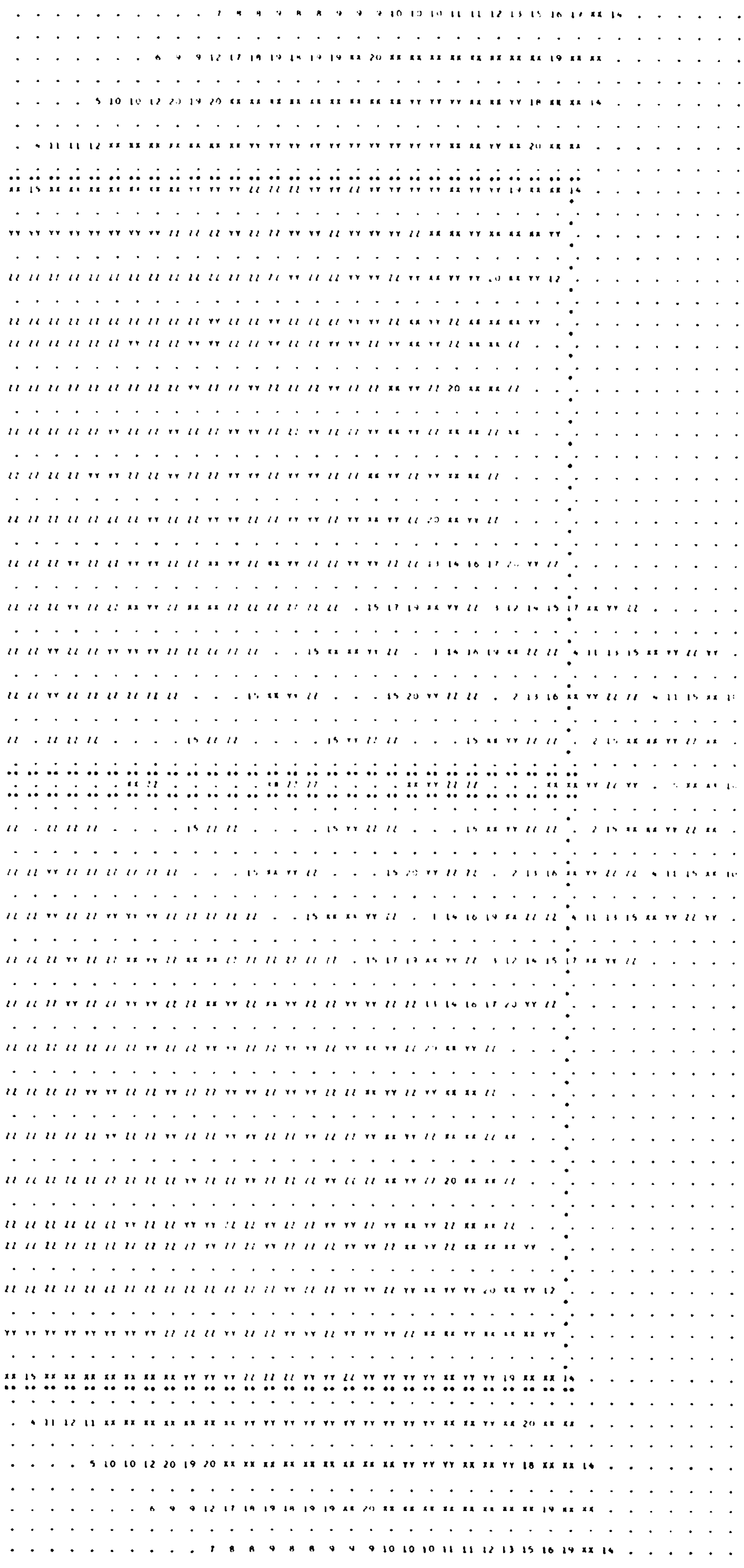


FIG. 5-8

FIG. 5-9

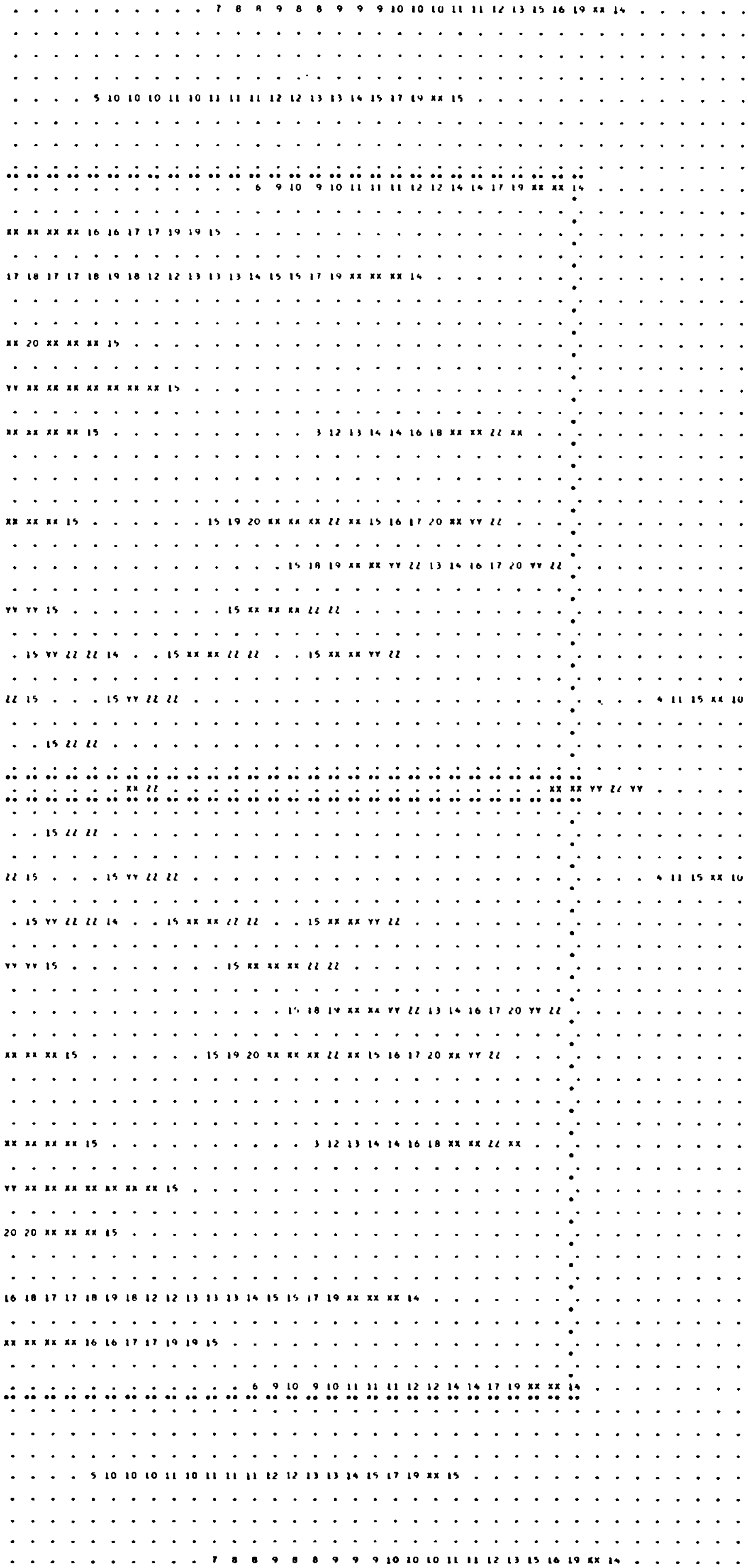


FIG. 5-10

E. Random Arrays

Three random arrays were examined. In each case, the 37 elements were scattered at random over a square 18600 λ (2064 m) on a side. The F.T. plane performances of these configurations are shown in Figs. 11-13. They are poor in every case - typically half of the desired sampling points are left untouched. These arrays have nothing to recommend them, and we shall not consider them further.

F. Conclusions

Three of the arrays we have considered are attractive from the standpoint of the quality of the F.T. plane coverage they afford. The hollow circle (Fig. 5-1), while it performs well, cannot easily be extended. The tee of Fig. 5-2 is excellent if we permit elements of one arm to be moved, for the reasons discussed earlier. The wye of Fig. 5-6 is good also, and would require no moves. Since we wish to be able to extend the array easily, it appears that our choice must lie between the latter two configurations.

Let us consider how critically the array performance depends on source declination and the hour angle tracking range. We have calculated the F.T. plane coverage given by the wye of Fig. 5-6 for a number of declinations and limiting hour angles. Table 5-1 gives the number of unsampled cells in the 18600 by 37200 rectangle for each, while Table 5-2 gives the number of cells within the same area which are sampled for more than one hour.

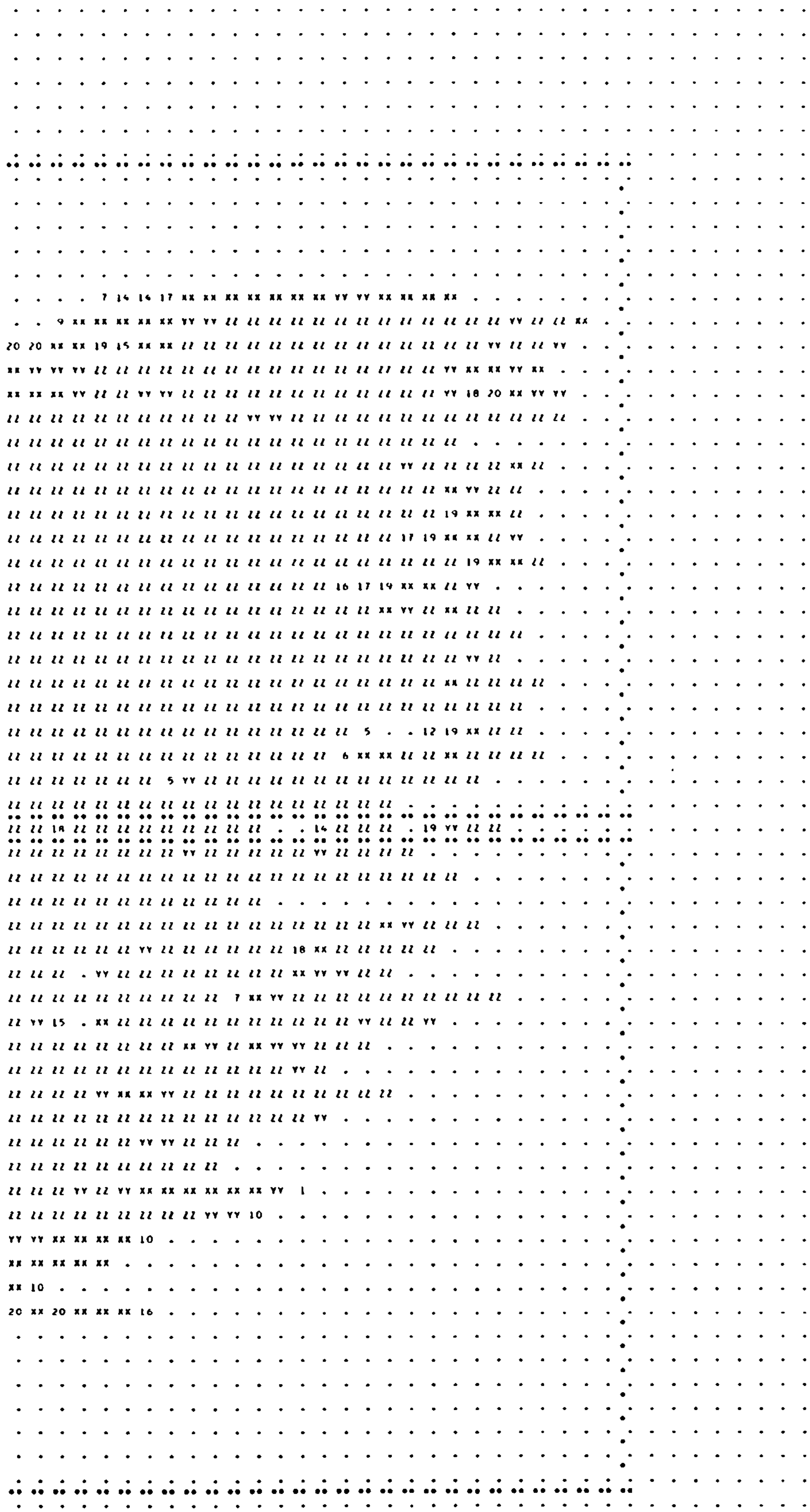


FIG. 5-11

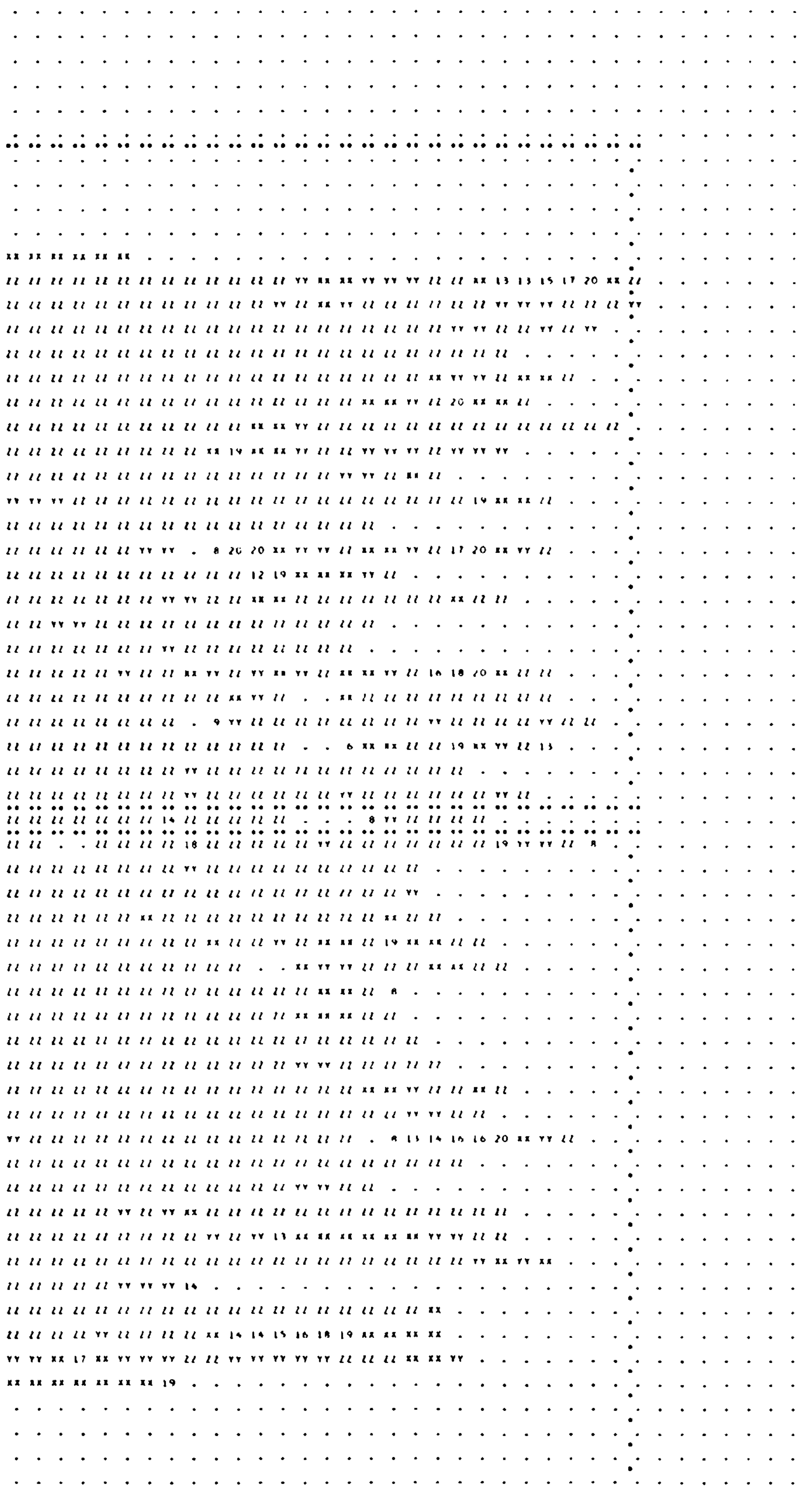


FIG. 5-12

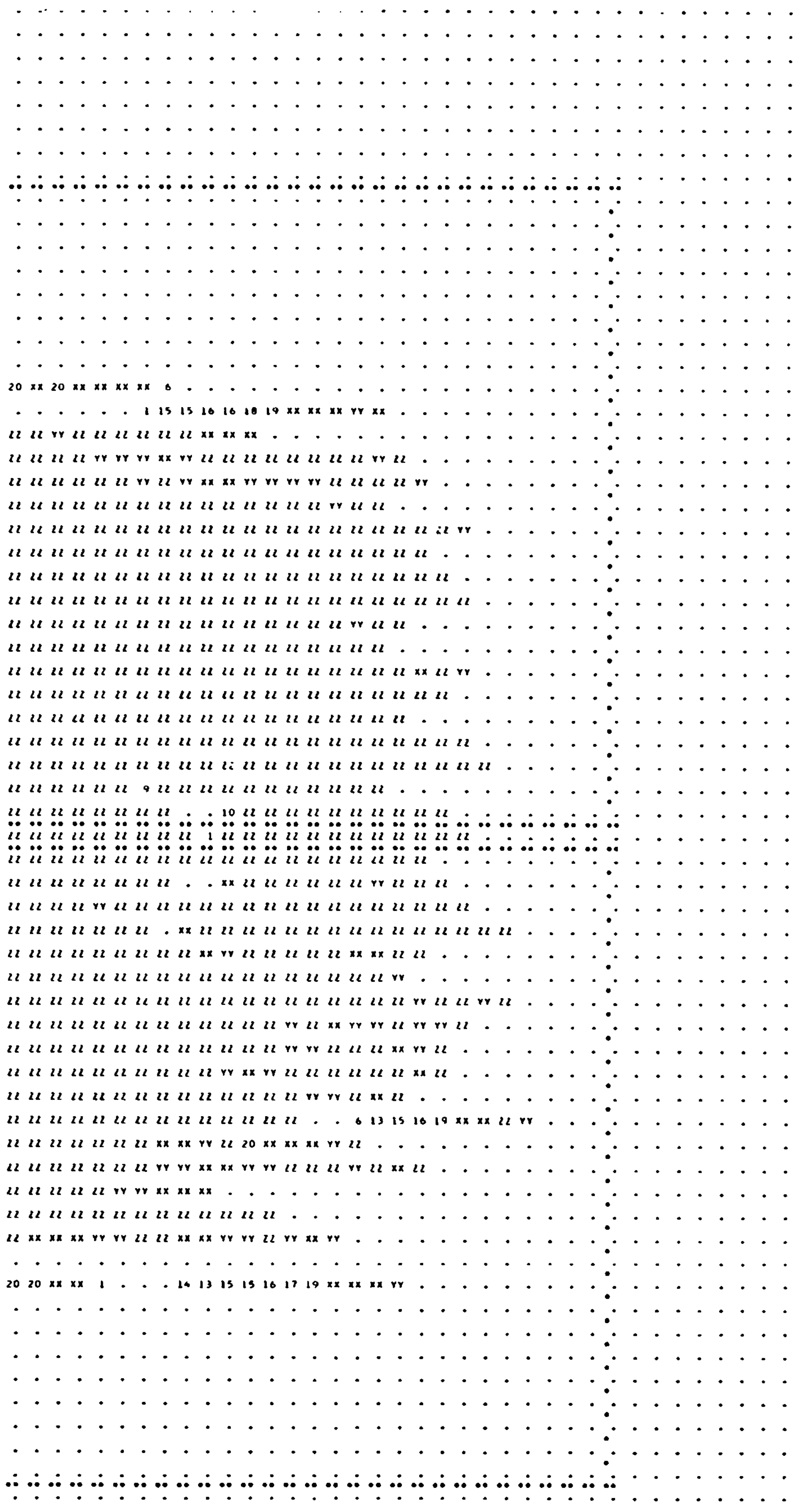


FIG. 5-13

Table 5-1

HA / δ	$\pm 1^h$	$\pm 2^h$	$\pm 3^h$	$\pm 4^h$
-30°	533	461	398	360
-15°	215	100	64	54
0°	284	98	75	73
15°	261	84	46	40
30°	187	46	15	7
45°	145	25	11	9
60°	119	22	15	10
75°	146	40	7	5

Table 5-2

HA / δ	$\pm 1^h$	$\pm 2^h$	$\pm 3^h$	$\pm 4^h$
-30°	371	881	1007	1079
-15°	250	940	1205	1297
0°	211	764	1220	1362
15°	146	725	1178	1345
30°	176	754	1222	1399
45°	161	759	1257	1415
60°	200	892	1290	1416
75°	221	930	1120	1264

The large number of unsampled cells at -30° is due mainly to the foreshortening of the array because of the large zenith angles involved. The tables clearly show the effect noted earlier - little additional F.T. plane coverage is achieved by increasing the hour angle range beyond $\pm 2^h$; the effect is rather to increase the integration time for the cells which are sampled. The F.T. plane coverage improves slightly with increasing declination, although there is no great variation except where foreshortening is important.

A point which is made by the figures is that the integration time in most cells is very large, on the order of an hour or more for about half of them for most configurations.

6. Array Sensitivity and Dish Size

The philosophy behind the computation of array sensitivity was discussed in Chapter 4, where it was shown that it is sufficient to investigate the sensitivity to a point source, and that the signal-to-noise ratio is proportional to the number of antennas in the array, under the assumption of identical antennas. It should be recalled that it is not necessary that all antennas exist simultaneously; the number to be used is the total number of antenna stations occupied during the set of observations of a given source. More precisely the number N is determined from

$$N^2 - N = 2\nu$$

in which ν is the number of different baselines in the synthesized array which are instrumented with correlators.

As important as the minimum detectable flux density is the lower limit on usable flux density, set by confusion. As the flux density decreases, the number of detectable sources per steradian increases, until at some value of flux density the probability of detecting "blends" of sources, too close together to be resolved by the telescope, increases above the tolerable limit. Von Hoerner (Pub. NRAO 1, 19, 1961) has analyzed the confusion problem for a pencil-beam antenna, using the Cambridge 3C Survey as a model for the variation of source numbers with flux density. Although surveys with larger numbers of sources per steradian are now becoming available, it does not appear that the relationship between source numbers and flux density is substantially altered from the one based on the 3C Survey. In the present case, however, it is necessary to consider the possibility that sources in the grating lobes, as well as in the main beam, may contribute to confusion.

In order to investigate the performance of the array in the observation of extended sources of low surface brightness, the concept of the "filling factor" has been applied to convert from the array's performance on a discrete source to its performance on a diffuse source which fills the main beam.

The assumptions made are detailed below, and the results tabulated:

1. Consider an array of antennas which yields a uniformly filled square grid in the transfer function plane of spacing ℓ, to maximum spacing L. There are then $2(\frac{L}{\ell})^2 = \nu$ Fourier components. The array resolution is proportional to $(L)^{-1}$ and grating lobes occur at intervals proportional to $(\ell)^{-1}$. The diameter of individual antennas is d meters. In the follow-in calculations we assume:

a) no redundancy in the transfer function plane;

b) uniform weighting and equal integration time per Fourier component;

c) 11 cm wavelength;

d) $T_R = 100°K$, $B = 10^7$ c/s, $\tau = 10$ sec.

2. The number, per steradian, of resolvable sources N_R. Von Hoerner's (Pub. NRAO 1, 19, 1961) formulation is followed. The total solid angle of grating lobes within the primary beam is proportional to $(\ell/Ld)^2 = \frac{2}{\nu d^2}$.

Then

$$N_R = .59\ \nu d^2. \qquad (\nu d^2 \leq 2L^2)$$

This gives about 75 beam areas per source, where beam area is now the total area of all grating lobes within the primary beam. Table 6-1 tabulates N_R vs ν and d.

3. The number, per steradian, of visible sources N_V. Again following von Hoerner, and taking $N(S) \propto S^{-1.5}$, we get

$$N_V = 3.54 \times 10^{-6} \; (\nu B)^{3/4} \; d^3 \; T^{-3/2}$$

$$N_V = 6.74 \times 10^{-3} \; \nu^{3/4} \; d^3$$

This is given in Table 6-1.

4. Minimum detectable flux density (for sig/noise = 5).

$$S_{min} = 5 \times \frac{2kT_A}{A_{eff}}$$

$$T_A = \nu^{1/2} \; (B\tau)^{-1/2} \; (T_R)$$

$$A_{eff} = \nu \times 1/2 \times \pi/4 \; d^2$$

$$\therefore S_{min} = 3.51 \times 10^{-22} \; (\nu B\tau)^{-1/2} \; T_R \; d^{-2}$$

$$S_{min} = 3.51 \times 10^{-24} \; \nu^{-1/2} \; d^{-2}$$

This is given in Table 6-1.

5. Minimum detectable brightness temperature (sig/noise = 5). The antenna temperature measured on a source of brightness temperature T_B which fills the synthesized beam is

$$T_A = T_B \times \text{filling factor},$$

where

filling factor = filled area/unfilled area.

In our case the "filled area" is proportional to νd^2 and the unfilled area to $(L)^2$. Also

$$T_A = 5 \times \nu \; T_R \; (B\tau)^{-1/2}$$

Therefore

$$T_{B_{min}} = 5 \; T_R \; (\nu B\tau)^{-1/2} \; L^2 \; d^{-2}.$$

Alternately,

$$T_{B_{min}} = \frac{S_{min} \, A'_{eff}}{2k}$$

where $A'_{eff} = .5(\pi/r)\ (L)^2$ and S_{min} is given in section (4). Then

$$T_B = 5\ T_R\ (\nu B\tau)^{-1/2}\ L^2\ d^{-2}.$$

$$T_B = .05\ \nu^{-1/2}\ L^2\ d^{-2}.$$

This is tabulated in Table 6-2.

The adopted detectable flux density goal is 2×10^{-28} Joule/m^2, as discussed in Chapter 2. Examining Table 6-1, it is seen that the minimum antenna diameter is determined as a function of ν, the number of Fourier components needed to synthesize the given field of view to the given resolution, and the lower flux density limit. For a 5' field of view and 10" resolution, 1300 components are needed, approximately (taking into account the necessary tapering of the transfer function). Using 10^3 for convenience, it is seen that the specified flux density, S_{min}, is achieved with 25 meter antenna diameter.

At this flux density, approximately 19,000 sources can be detected per steradian, while the number of resolvable sources is 370,000. Thus, the array is strongly sensitivity limited under the assumptions made here.

The assumed values of bandwidth and receiver noise are realistic, but it is worthwhile to examine in detail the assumption of 10 seconds integration time. This value was chosen to permit the evaluation of the array when operating in the meridian transit mode. As discussed in Chapter 4, this type of operation would require the simultaneous use of 73 antennas. It is not likely that this situation will be realized in practice, as the "supersynthesis" mode of operation seems more desirable in several respects. This mode was investigated in Chapter 5 where it was shown that for well over half of the sampling cells in the transfer function, the available integration time is more than one hour. Thus, it would be conservative to assume a time-constant of 30 minutes, or 1800 seconds, in place of the 10 seconds used in Tables 6-1 and 6-2. In this case, the minimum detectable flux density should be divided by the square root of 180, or approximately 13. Then the desired flux density could be achieved with antennas only about 8 meters in diameter. Alternatively, using 25 meter antennas, it would be possible to detect sources of flux density as low as 10^{-29} Joule/m^2. In any case, the instrument will still be sensitivity limited.

This analysis of sensitivity and confusion limitations has been based upon the assumption of a static, Euclidean universe. Although this is undoubtedly naive, it is nonetheless conservative. The use of more appropriate cosmological models would only result in fewer sources per steradian at a given flux density level, and would indicate even more strongly that the array will inevitably be sensitivity limited.

The sensitivity of the instrument for measurements of surface brightness of objects which fill the "beam" can be crudely estimated, with the aid of Table 6-2, for various different cases:

1) Extragalactic sources generally have surface brightness temperatures of more than 10^5 °K. Thus, even for one second resolution and 1000 Fourier components ($L = 3 \times 10^4$, $\nu = 10^3$ in Table 6-2) an array of only 10 meter dishes can detect temperatures of the order of 1% of those expected, in 30 minutes integration time.

2) Normal galaxies have brightness temperatures of the order of one degree. Dishes of at least 15 meter diameter would be desirable to detect

these with 30" resolution. 30 meter dishes would be required to study the brighter ones with 10" resolution.

3) HII regions have brightness temperatures ranging down from a few thousand degrees. Dishes of 15 or 20 meter diameter would allow detection of a number of them, even at 1" resolution.

4) Hydrogen line requirements are the most difficult. To achieve sensitivity of a few degrees, with 1 kc/s bandwidth, 30 min. integration time, and 30" resolution would require dish diameters greater than 40 meters.

Thus, it is apparent that there is no practical, optimum size for the individual antennas in the array based on astronomical requirements. For observations of extragalactic sources, very small antennas would suffice. For observations of spectral lines or extended objects, the largest possible antennas are desirable. The larger the antennas are, the more sensitive is the array and the less troublesome are the grating lobes. Evidently the only limitation on antenna size will be a budgetary one.

Table 6-1

	$N_R \times 10^{-3}$			$N_V \times 10^{-3}$			$S_{min} \times 10^{26}$		
d(meters) \ ν	10^2	10^3	10^4	10^2	10^3	10^4	10^2	10^3	10^4
5	1.5	15	150	.027	.149	.84	1.41	.45	.14
10	6	59	590	.21	1.20	6.57	.35	.11	.035
15	13	130	1300	.72	4.04	22.5	.15	.05	.015
20	24	240	2400	1.70	9.55	53.7	.085	.028	.008
25	37	370	3700	3.31	18.70	105	.057	.018	.006
30	53	530	5300	5.73	32.4	182	.040	.013	.004
35	73	730	7300	9.10	51.1	290	.028	.008	.003
40	94	940	9400	13.50	76.9	456	.021	.007	.002

$N_R = 0.59\ nd^2$ = number of resolvable sources per steradian

$N_V = 6.74 \times 10^{-3}\ \nu^{3/4}\ d^3$ = number of detectable sources per steradian

$S_{min} = 351 \times 10^{-26}\ \nu^{-1/2}\ d^{-2}$ = minimum detectable flux density

$B = 10^7$ c/s

$\tau = 10$ sec

$T_R = 100°K$

$\lambda = 11$ cm.

Table 6-2

$T_{B(min)}$ (Degrees Kelvin)

L (meter)	3 x 10^2			10^3			3 x 10^3			10^4			3 x 10^4		
d(meters) \ ν	10^2	10^3	10^4	10^2	10^3	10^4	10^2	10^3	10^4	10^2	10^3	10^4	10^2	10^3	10^4
5	20	6.4	2.0	200	64	20	2000	640	200	2.0×10^4	6400	2000	2×10^5	6.4×10^4	2×10^4
10	5.0	1.6	*	50	16	5.0	500	160	50	5000	1600	500	5×10^4	1.6×10^4	5000
15	2.1	.71	*	21	7.1	2.1	210	71	21	2100	710	210	2.1×10^4	7100	2100
20	1.2	*	*	12	3.9	*	120	39	12	1200	390	120	1.2×10^4	3900	1200
25	.9	*	*	9	2.9	*	90	29	9	900	290	90	9000	2900	900
30	.57	*	*	5.7	1.7	*	57	17	5.7	570	170	57	5700	1700	570
35	.40	*	*	4.0	1.2	*	40	12	4.0	400	120	40	4000	1200	400
40	.32	*	*	3.2	1.0	*	32	10	3.2	320	100	32	3200	1000	320

*ℓ < d = physically not possible for "instant" synthesis. If achieved by tracking or moving dishes, it is equivalent to increasing τ.

$T_{B(min)} = .05\ \nu^{-1/2} L^2 d^{-2}$ = minimum detectable brightness temperature.

7. A Suggested Array Configuration

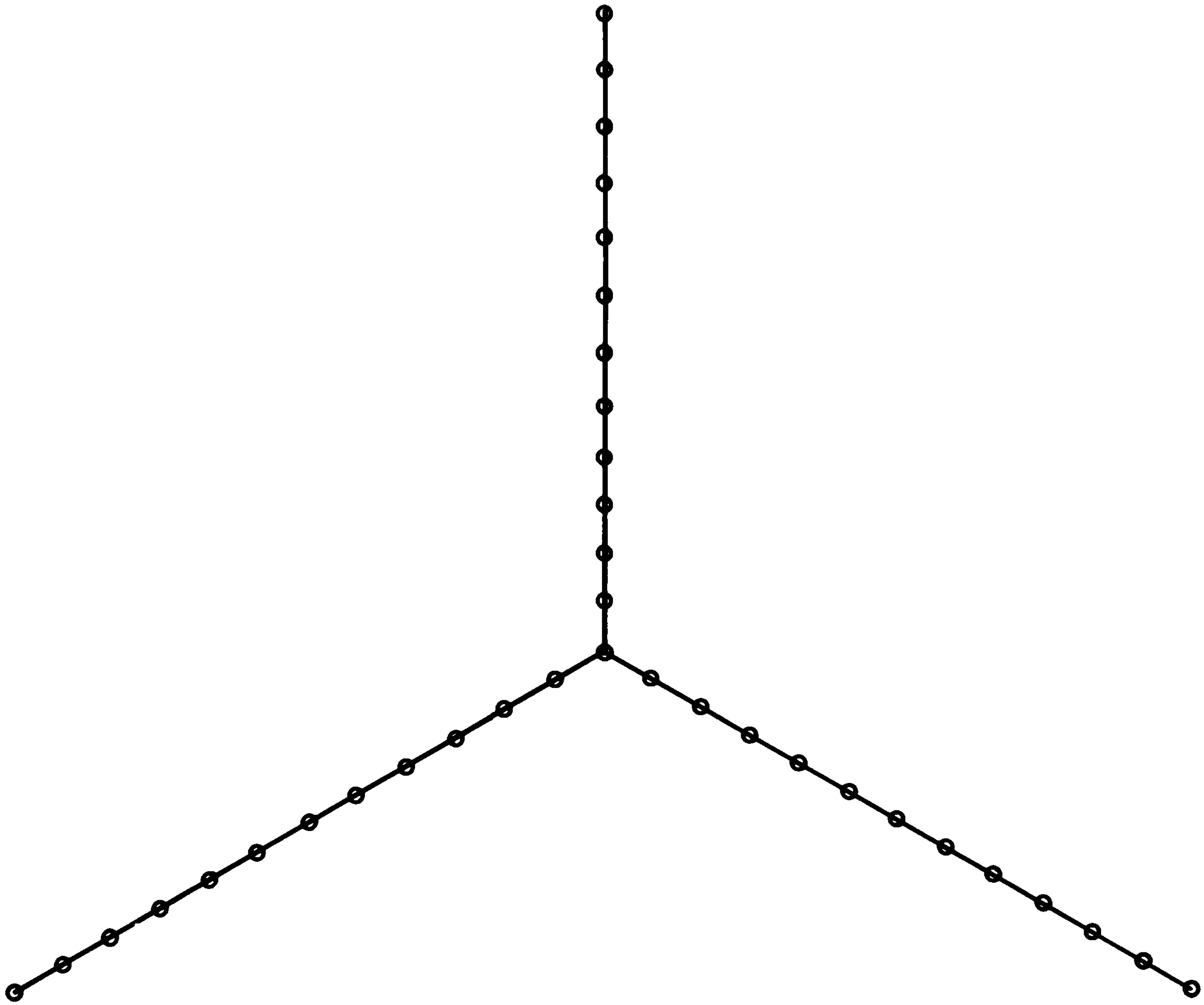
In Chapter 5 the performances of several possible array configurations were analyzed under various conditions with respect to the degree of completeness of the transfer function. Only a few of the results of the complete study can be presented here. The Tee, Wye, and Circle configurations can all be made to give good transfer-function coverage, as can certain other configurations. In addition, there are some other requirements, such as expandability of the array for higher resolution or longer wavelength, and ease of construction required to permit movement of the individual antennas. These latter two requirements militate against configurations involving curves, particularly closed curves.

The quantitative array-design procedure has assumed the requirements of 10" resolution and 5' field of view, and all trial array configurations have been evaluated in terms of these requirements. It is certain, however, that occasions will arise when a larger field of view (larger grating-lobe spacings) is required, or when a greater sensitivity is needed for observing low-brightness, extended sources. Both occasions call for closer-than-standard spacings of the antennas in the synthesized array. It may also occur that higher resolution is needed, calling for larger spacings of the available antennas. In fact, it is anticipated that resolutions as small as 1" may be achieved under special circumstances. This flexibility in array performance requires that the antennas be movable over the ground, probably on railway tracks. It also requires that the railway system be expandable to achieve the fine resolution discussed above. Clearly this rules out the circular array, and any other array involving closed curves.

The Y-array, however, meets all of the requirements. It is superior in transfer-function coverage to the Tee, especially in the vicinity of the celestial equator. It is expandable merely by extending the railway tracks along straight lines. Extensive computer investigation shows that under most conditions the Y-array gives better transfer function coverage, with fewer elements, in a shorter observing time, than any other array studied. The best compromise among length of hour-angle track, number of elements, and coverage of the necessary area of the transfer function, appears to be obtained with a 37-element, symmetrical Y-array, with one arm north-and-south (Figure 7-1).

With such an array it is possible to synthesize a 5'-diameter area of sky to 10" resolution in 4 hours of observing time. By moving the dishes in such a way that ultimately, twice as many positions on each arm of the Y are occupied, it is possible to extend the field of view to 10'. This, of course, requires a substantially greater observing time. In fact, it is possible to increase the field of view to that of the individual antennas in the array by making observations at an even larger number of positions. Extension of the baselines to the necessary lengths to achieve 1" resolution merely requires that the railway tracks be extended along the arms of the Y. At a wavelength of 11 cm, each arm must be a least 22 km long to achieve 1" resolution. Allowance for amplitude tapering for sidelobe reduction requires arms 25 km long. Of course, a very large amount of observing time is required to gather all of the necessary information to synthesize a 5' area to a resolution of 1", given only 37 antennas. However, many problems requiring 1" resolution would not require full synthesis of a 5' area.

An array of this type can be expanded, as scientific requirements and availability of funds dictate. A feasible procedure, for example, is to construct the Y-shaped system of tracks out to a radius of 2.5 km, initially. Then the 37 antennas should be constructed as quickly as possible, together



THE 37 – ELEMENT Y – ARRAY

FIG. 7–1

with the necessary computing facilities and the receiving equipment to permit operation with dual polarization on a wavelength of 11 cm. Observing stations would be provided along the track at the proper interval to permit observations in the standard mode: 5' field of view and 10" resolution. In addition, the necessary antenna stations will be provided to permit the synthesis of a 1' beam, substantially free of grating lobes and having a filling factor 36 times greater than that of the standard mode. This low-resolution mode will be useful for investigation of extended sources of low brightness, and, having no grating lobes, for preliminary surveys of sources to be mapped in greater detail in the standard mode.

At such time as can be justified by the progress of observations with 10" resolution, and as funds become available, the arms of the Y can be extended to improve the resolution beyond 10", by extending the track and using the same antennas or by adding additional antennas at fixed stations, or some combination of these possibilities. In choosing a site, the requirement for 25 km arms will be considered. In no event should a site be chosen where it is impossible to place at least a few antennas at sufficient distances from the central area to achieve 1" resolution.

The array as envisioned will be an instrument of very great versatility. In addition to operation in the standard mode and in the low resolution mode, it is adaptable to a number of other modes for the rapid determination of certain source parameters. The following list tabulates some of the possibilities:

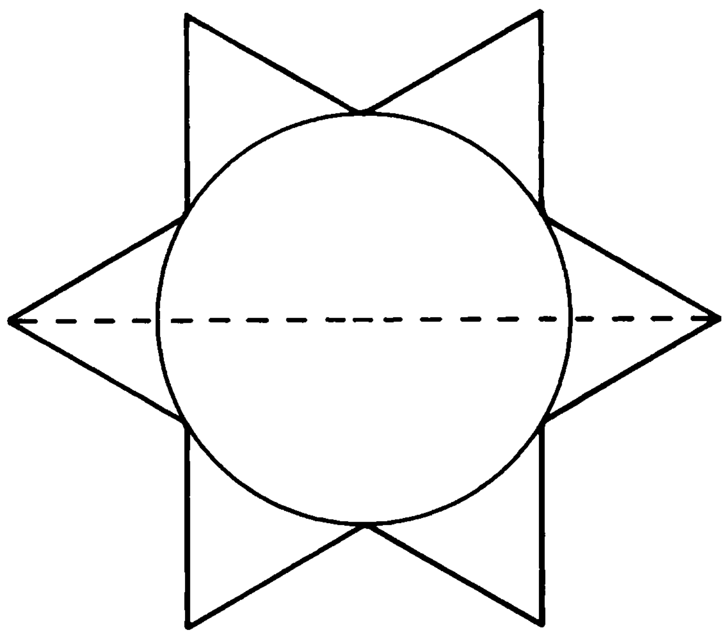
(1) Standard Mode: Ten seconds resolution, five minute field-of-view. For routine mapping of discrete sources.

(2) Low-Resolution Mode: One minute resolution, field of view limited by primary antenna beamwidth (say, 15 minutes). For mapping of extended sources with low surface

brightness. High filling factor and high sensitivity for low-brightness areas.

(3) Fan-Beam Mode: The transfer function of a Y-shaped array is a six-pointed star (Fig. 7-2). In synthesizing a circular antenna beam, only those Fourier components corresponding with a circle inscribed in the central area are utilized. For measuring source diameters or positions with twice the resolution (half the angle) of the circular-beam mode, the Fourier components corresponding with the points of the star can be utilized. For example, if those components falling along the E-W dotted line in Fig. 7-2 are used, a fan beam can be synthesized whose narrower beamwidth is half that of the "standard mode." These data can be collected simultaneously with those for the standard mode.

(4) Transit Mode: For position measurements on sources for which no danger of confusion exists and for rough, preliminary investigations of extended sources or of selected regions of the sky the array can be operated as a transit instrument, on the celestial meridian or elsewhere. In this mode the antennas are held fixed and the sources drift through the common primary beam. Data can be collected very rapidly, but the sensitivity is greatly reduced over the standard mode because of the reduced integration-time. If the antennas are moved together in the "low-resolution"



TRANSFER FUNCTION
OF A Y—ARRAY

FIG. 7-2

configuration (2) and the array operated in the transit mode, the grating lobes will be about 30' apart, so that they will be substantially reduced by the primary antenna pattern. In this case the transit mode represents an attractive type of operation for rapidly surveying a strip of sky up to, say, 20 minutes in declination.

(5) Total Power Mode: It has been suggested that many or all antennas might be connected together to a single detector, as in the conventional total-power array. Because of the method of signal processing used in the correlator array, which permits use of a relatively simple method of introducing the necessary time delays into the i-f amplification system, the array is essentially incompatible with "total-power" operation. As the correlator-array type of operation is superior in many respects, it is not anticipated that the necessity for "total power" operation will ever arise.

8. The Feasibility of the Suggested Array

Several problems may be anticipated in the construction and operation of an array capable of 10" resolution or better, the most important of which are as follows:

(1) Stability of the atmosphere. The possibility exists that the inhomogeneous structure of the atmosphere might introduce phase fluctuations which would seriously reduce the coherence between the downcoming waves reaching each antenna. This could reduce the effective collecting area of the array, and could add to the total "noise" in the system.

(2) Stability of the mechanical structure. Mechanical dissimilarities between the antennas of a given baseline might result in unknown changes in the baseline parameters, giving rise to erroneous data.

(3) Phase stability of the electronic system. Various possible sources of phase instability exist in the receiving system, the most critical of which is the transmission system for distributing local oscillator signals from the central phase reference to the mixers at each of the antennas.

(4) Delay tracking. The total time delay of signal transmission from the plane wave-front of the downcoming wave, through the receiving system, to the correlator, must be the same for all antennas in the array. These delays must be varied in synchronism with the apparent motion of the source in the sky.

In addition to these major problems, a number of less crucial ones exist. However, it has been adequately demonstrated that all these problems can be solved. In principle the proposed array is merely an assemblage of interferometers capable of tracking a source for several hours. If an interferometer can be made to operate properly with baselines equivalent in length to the longest baselines of the array, the stability of the atmosphere, the mechanical structure, and the electronic system can be considered adequate, and the delay tracking system clearly has suitable characteristics. Such operation has been demonstrated by several groups.

The existing instruments whose experience bears upon the design of a large array are the one mile Cambridge radio telescope and the long baseline interferometers at the Royal Radar Establishment, NRAO, and Cal Tech. The Cambridge supersynthesis instrument perhaps bears most directly on the problem, since it appears to have most if not all of the capabilities of the proposed array except the ability to supply a great deal of information very fast, and a slightly lower resolving power. The maps published by Ryle (Nature, 205, 1259, 1965; Nature 207, 1024, 1965, Ryle, Elsmore, and Neville) are observations made with the equivalent resolution of a 23" pencil beam, at 21 cm, and nicely demonstrate that the ability to combine the various correlator outputs to produce a contour diagram of the brightness distribution can be achieved. The sensitivity of correlator arrays is also strikingly demonstrated in these articles. With antennas of 60 foot diameter and receiver temperatures of 450° K at 74 cm, sources as weak as 0.02 flux unit appear to be visible on the map of a selected region of the sky.

Ryle et al. have apparently calibrated the baseline parameters of their instrument by means of an accurate survey by conventional surveying methods. The difficulty of locating several tens of points to an accuracy of half a centimeter over a region three kilometers across is probably greater than that of using observations of point sources to calibrate the instrument, since this method is probably the most convenient to calibrate the absolute phases of the instrument at a given instant. The absolute phases could, however, be calibrated by the technique of Swarup and Yang (IEEE Transactions on Antennas and Propagation 9, 75, 1961) and the baseline parameters by survey, if, indeed, the effects of the atmosphere may be ignored.

The NRAO interferometer also has demonstrated most of the capabilities of the proposed array except the ability to supply information fast. Operating at 11 cm wavelength, with a resolution of 8", it is essentially a prototype of one of the interferometer pairs that would comprise the array.

The Malvern (Adgie, Nature 204, 1023, 1964) and NRAO (Wade, Clark, and Hogg, Ap. J. 142, 406, 1965) interferometers have demonstrated that positions may be determined to an accuracy approaching 1". Adgie, observing with a 750 meter baseline, has, in producing positions good to about 1".5, calibrated his absolute phases to an accuracy of 1 cm. This was done at a wavelength of 50 cm, so that the phase deviations, measured in degrees, were very small indeed. The system of phasing employed by Adgie needs very little extension to work for a very large array. Similarly, the NRAO results indicate that a 20,000 wavelength baseline at 10 cm may be phase calibrated to $\lambda/8$ or better with present techniques.

The criticism which may be brought to bear on both of these position measurements is that the positions are derived from observations extending over hours or days, and that the various observed phases have been averaged to determine the calibration. Observations at NRAO have indicated that the rms deviation of the phases from the mean are of the order of 30°, within the $\lambda/10$ specification. These phase wanders occur with periods of a few minutes, so that the average of an hour generally agrees well with the mean for the day. If these phase wanders cannot be eliminated, the performance of the array may be worse for those weak sources for which we must average the complex visibility than for those for which the phase may be determined and averaged. However, it may be expected that these phase wanders are generated by the equipment, and may be eliminated by careful design, rather than being a fundamental limitation on the instrument. The calibration problem for a large array is discussed in greater detail in Appendix IV.

9. Antenna and Track Requirements for VLA

General Considerations:

The VLA system is planned as a full correlation array, i.e., all available Fourier components are individually recorded ($\frac{1}{2}$ (n-1)n components). Each interferometer pair will use superheterodyne receivers with equal response in signal and image bands. This offers no significant advantage in signal to noise ratio when low noise preamplifiers are used, but it greatly simplifies phase calibration of the system since the group delays in the IF cables needed for tracking sources do not affect the phase calibration of the interference fringes.

The array will be controlled by a central computer which also will perform the initial data reduction in real time.

In the following paragraphs the main features of the planned system are described.

Antenna and Track Requirements for the VLA

1. Antenna

The individual antennas should be parabolic reflectors with as large a diameter as economically feasible. An equatorial mount (with sky coverage limited to $\pm$ 4 hours) would be most convenient, but all types such as x-y or elevation-azimuth mounts should be considered, and might be preferred if it means a significant reduction in antenna cost.

A Cassegrain system is preferred over a prime focus arrangement because it gives (1) lower noise and (2) much easier maintenance of the electronics equipment. Present available designs for 25 m telescopes (Blaw Knox 85-ft antenna, for example) permits installation of a cabin of adequate size immediately behind the reflector vertex, which is needed for a Cassegrain system. If a Cassegrain antenna is selected, however, a somewhat lower F/D than now used on the NRAO interferometer should be considered for optimum performance.

Problems to be solved:

1) Reflector size

2) Type of mount

Sky coverage

Scanning (tracking) problems with x-y and elevation-azimuth mounts (cone of avoidance)

3) Cassegrain - prime focus

Near-field Cassegrain -- Far-field Cassegrain

Optimum F/D

4) Two frequency feed

Parallel or orthogonal polarization?

Single or dual polarization at each frequency?

5) Isolation required between two frequencies.

6) For either Cassegrain or prime focus feed - feed must have same illumination at both frequencies.

7) If Cassegrain - near field or far field system? See Hogg and Semplak - BTL Journal.

8) Feed system must be rotatable.

9) Must be designed for low-noise (T_a) operation. Loss?

10) Polarization resolution required - 3' arc?

11) Waveguide input to paramps?

12) Plane polarization required.

Track Considerations

In what follows, we assume a Wye array with individual arms of length 2.4 Km and with up to 37 antennas (12 per arm and one central). The arm length is selected as 2.4 Km merely for convenience in specifying the spacing between individual elements. Once a configuration and array size is specified all the following figures can be easily scaled.

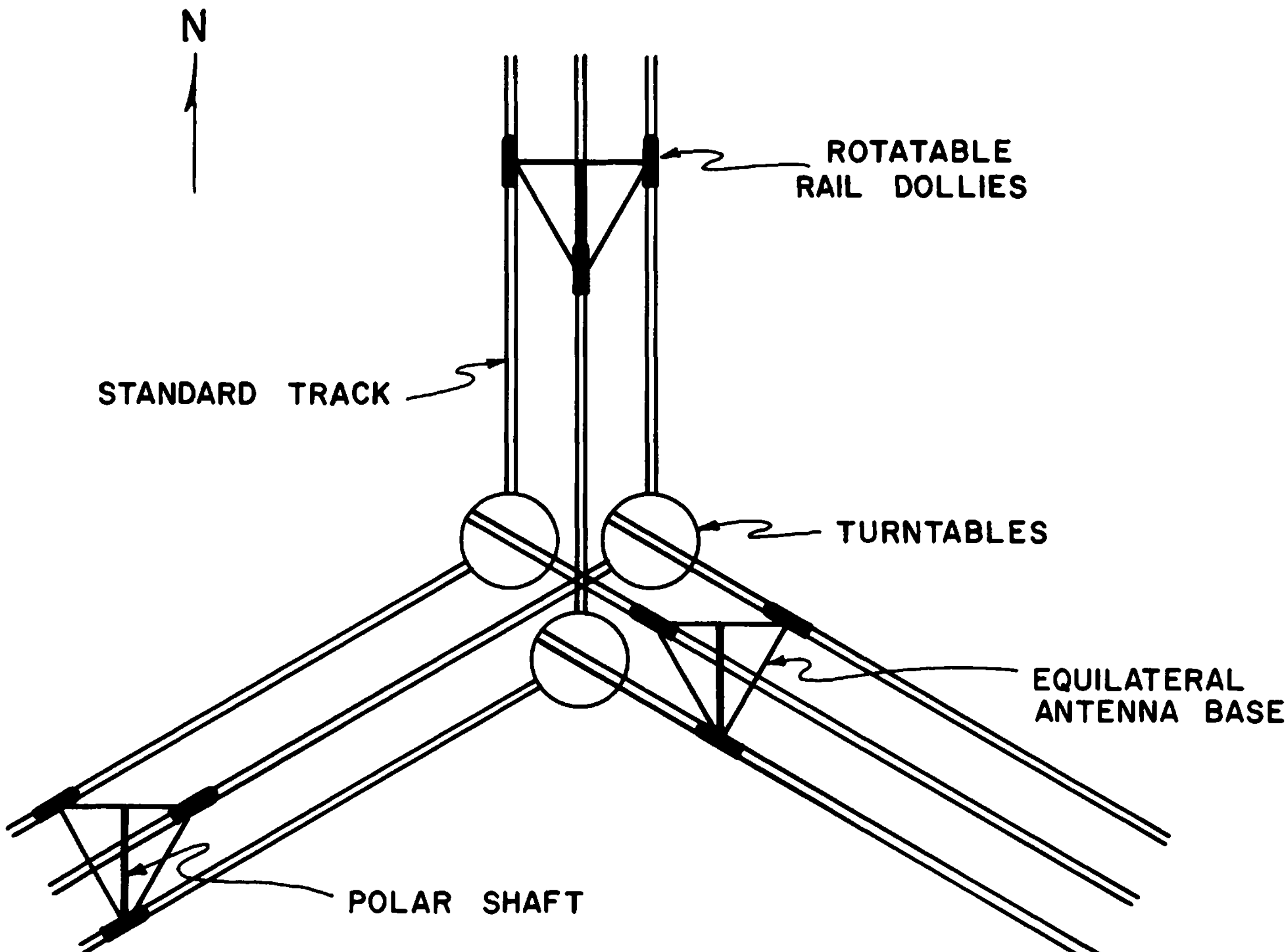
Certain assumptions regarding sky coverage, element size and geographic latitude will also be made for the purpose of illustration. These must also be allowed for once the final array location and specifications are set.

Assumed Array Specifications

Configuration	:	North-South "Wye" uniform spacing
Length of arms	:	2400 meters
Element Size	:	25 m diameter
Mount Type	:	Equatorial
Mobility	:	All elements movable on track
Observing locations	:	Combination of observing stations and continuously adjustable spacing
Type of track	:	For each arm - 3 dual rail tracks
Features	:	a.) All antennas identical (i.e. can occupy any arm
		b.) All antennas can occupy the same arm simultaenously (for high resolution fan-beam operation).

All antennas will be movable on railroad tracks along straight lines in a Y configuration. By using a three-legged supporting frame for the antenna, all antennas may work on all three branches of the Y patterned track by rotating the three trucks on turntables as shown in Figure 9-1. Heavy gauge standard railroad track can be used, with standard railroad trucks and turntables.

It would be desirable to have the track rigid enough to permit operation of the antennas at any point along the tracks. This may be very expensive, however, and Figure 9-2 and Table 9-1 show an alternate solution where only



CENTRAL TRACK INTERSECTION
FOR
Y-ARRAY

FIG. 9-1

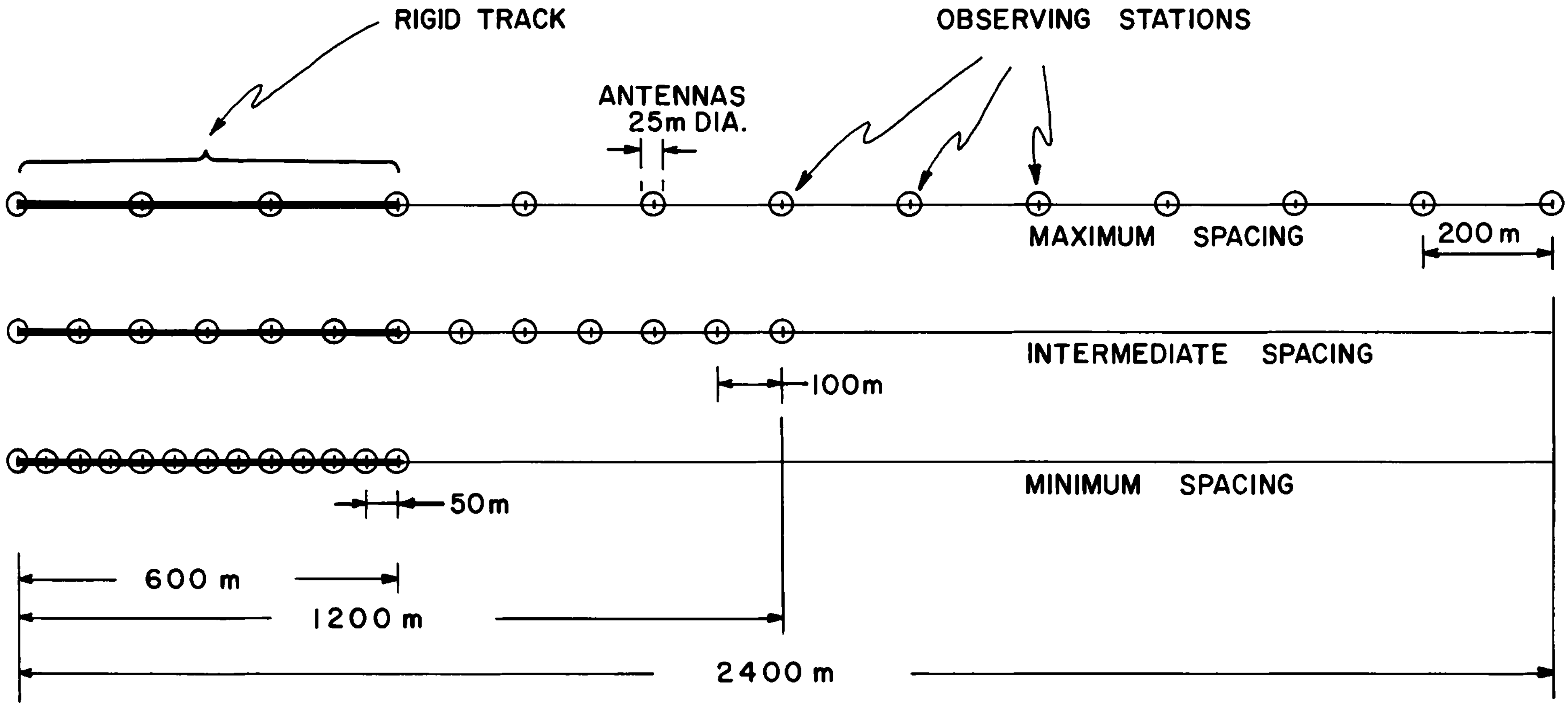


DIAGRAM SHOWING PROPOSED
TRACK & STATION LAYOUT FOR
ONE ARM OF 10 Sec. ARRAY.

FIG. 9--2

the inner 600 m track is made that rigid. A 10" resolution array will then require 1800 m of rigid track and 37 fixed stations.

Problems to be solved:

1) Array configuration Y or T
2) Railroad tracks or road
3) Should all antennas be able to occupy all positions
4) Number and locations of observing stations
5) Part of track rigid enough to permit observations without stations
6) Length of arms.

Table 9-1

RESOLUTION (Sec. Arc)	SPACING (Meters)	ARM LENGTH (Kilometers)	NO. OF STNS. (Cumulative)
40"	50	0.6	Continuously Variable
20"	100	1.2	6
10"	200	2.4	12
5"	400	4.8	18
2.5"	800	9.6	24
1.25"	1600	19.2	30

Total of 1800 meters of rigid track + 37 stations (piers) will suffice for 10" array.

For 1" array, add 54 stations.

10. The Electronic System

The basic system of the VLA is the same as that of the NRAO interferometer, which, in turn, has been derived from the Cal Tech interferometer system as described by Read (Ap. J. 138, 1, 1963). A simplified block diagram is shown in Figure 10-1. The signal is amplified and converted into an intermediate frequency band by a conventional superheterodyne receiver connected to each antenna. Both sidebands (the "signal" and the "image") are utilized, and as Read has shown, this feature ensures that the operation of the system is insensitive to phase shifts in the i-f equipment. The heterodyne signals for the various mixers must be fixed in phase with respect to one another; considerable effort is needed to achieve this.

The "delay" systems shown in Fig. 10-1 are necessary to provide that all signals experience approximately equal transit-times in travelling from the source to their respective correlators. If this is not so, the effective system bandwidth will depend upon the position of the source with respect to the array, and the operation of the array will be unsatisfactory. No method is known of producing the necessary variable time-delays at the intermediate frequency without introducing incidental phase shifts. These would render the system useless for aperture synthesis if it were not for the double sideband mode of operation.

The i-f output from each pair of antennas is multiplied in a correlator and integrated in a circuit with a time-constant short with respect to the fringe period (the time-period of the Fourier component represented by the baseline in question). The output of the correlator is a quasi-sinusoidal function of time, contaminated by noise, of course. This output can be recorded graphically or digitally, or delivered to an on-line computer for further processing.

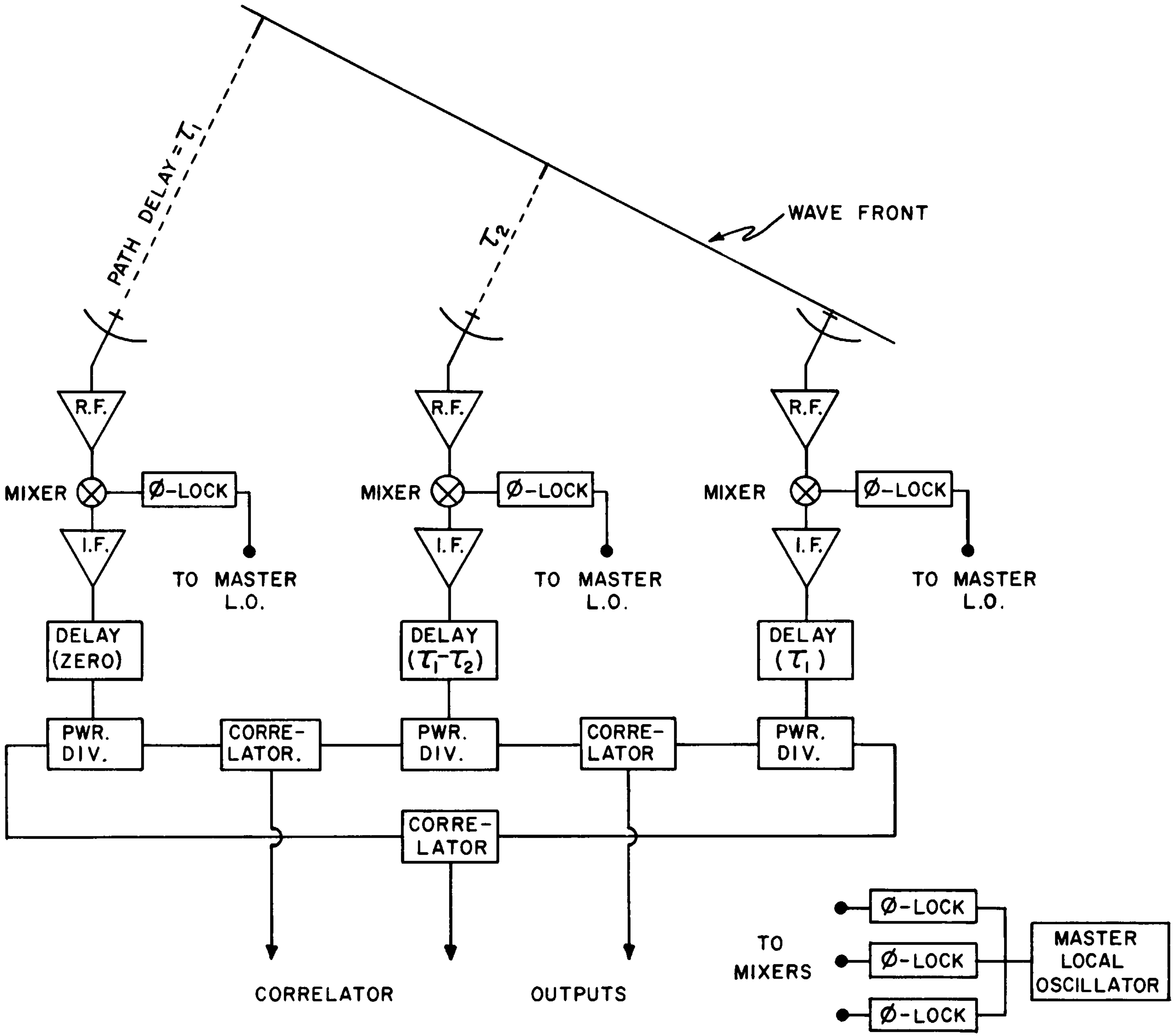

PATH DELAY = τ_1
τ_2
WAVE FRONT
R.F.
R.F.
R.F.
MIXER
MIXER
MIXER
Ø-LOCK
Ø-LOCK
Ø-LOCK
I.F.
I.F.
I.F.
TO MASTER L.O.
TO MASTER L.O.
TO MASTER L.O.
DELAY (ZERO)
DELAY $(\tau_1 - \tau_2)$
DELAY (τ_1)
PWR. DIV.
CORRE-LATOR.
PWR. DIV.
CORRE-LATOR
PWR. DIV.
CORRE-LATOR
CORRELATOR
OUTPUTS
TO MIXERS
Ø-LOCK
Ø-LOCK
Ø-LOCK
MASTER LOCAL OSCILLATOR

FIG. 10-1

Some of the functions briefly described above are accomplished only at the expense of considerable complexity in equipment. The most critical of these are:

1) Low noise preamplification.

2) Supply of fixed-phase local-oscillator signal to each receiver.

3) Provision of proper time delays and delay variation.

The methods of accomplishing these functions in the NRAO interferometer are shown in the block diagram of Fig. 10-2. Clearly such an interferometer is not a simple device; however, this is an experimental installation and there are many ways in which it can be simplified in the light of experience.

The detailed discussion of the electronic system for the VLA will be divided into three parts:

I. Equipment proportional to the number of antennas, such as input amplifiers, mixers, IF amplifiers, delay lines, etc.

II. Equipment proportional to the square of the number of antennas, such as the correlators, integrators, etc., and

III. The local oscillator system.

I. Equipment proportional to N

a) Input amplifiers. The most important single parameter of the input amplifier is its effective noise temperature. Usually, state of the art noise temperatures can be combined with the very high reliability which is needed for the VLA. Among the different types of low noise amplifiers available today, it seems possible to achieve about 100°K noise temperature at 10 cm with the required reliability using uncooled parametric amplifiers. One may safely assume that, a few years from now, reliable uncooled parametric amplifiers with a noise temperature of 50°K or better at both 10 cm and 6 cm will be available. One less reliable component in present

INTERFEROMETER SYSTEM BLOCK DIAGRAM

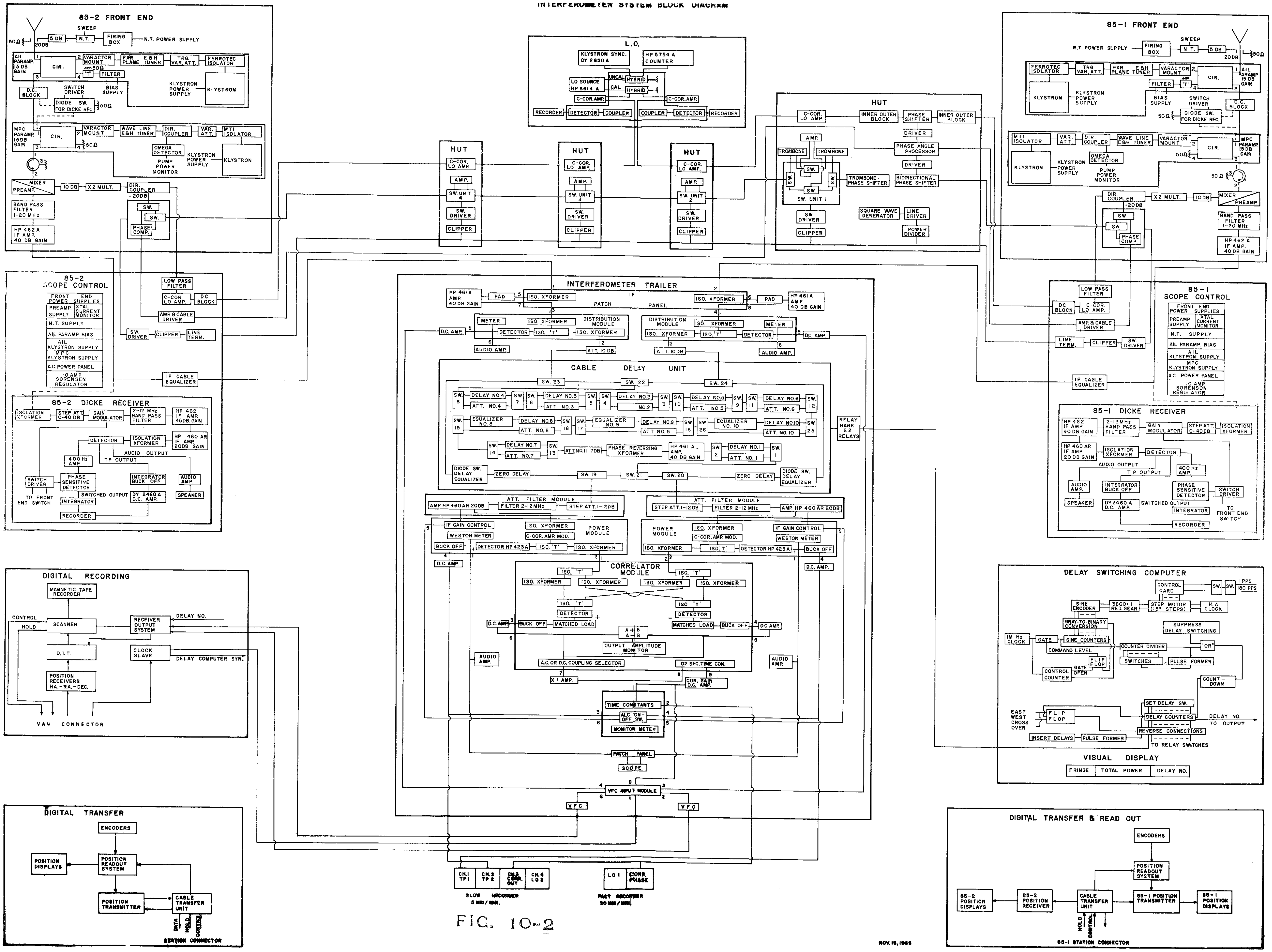


FIG. 10-2

NOV. 15, 1965

parametric amplifiers is the pump klystron. In this field rapid progress is currently being made, and by the time the construction of the VLA amplifiers should start, reliable solid-state pump sources may be available.

For the particular system planned for the VLA, where both signal and image frequencies are used, degenerate parametric amplifiers should be considered. Lower usable noise temperatures can be achieved, and since the pump frequency is twice the signal frequency, the pump circuits can be made much simpler and more reliable. In the VLA system the local oscillator frequency lies in the center of the reception band, and the pump power can be derived directly from the local oscillator signal by frequency doubling.

The low noise input amplifiers should meet the following basic requirements:

1) The noise temperature should be 50°K, or less, if possible. Cooling the amplifier to achieve low noise temperature should be avoided.

2) The amplifier gain should be at least 25 dB in order to make the mixer noise contribution negligible. To achieve this gain a two stage amplifier will be needed; both stages should be pumped with a common pump source. Alternatively a tunnel diode amplifier could be used as the second stage.

3) The gain stability of the input amplifier should be better than 2% per hour. A major cause of gain variations in a parametric amplifier is instability in the pump power and frequency. The pump power level should be stabilized; the pump frequency can be (in the case of the degenerate amplifier) locked to the stable local oscillator frequency.

4) The phase stability is a critical requirement. With a phase locked local oscillator system, the final phase stability of the VLA will be determined mainly by phase variations in the amplifiers ahead of

the mixer. Again, a phase locked pump (for the degenerate amplifier) and power level stabilization will result in good phase stability. Long term phase variations within $\pm$ 1° are feasible and acceptable.

5) Good phase linearity over the amplifier passband is required. A phase linearity of $\pm$ 3° over a 10% bandwidth is acceptable and readily obtained.

Problems that need further study:

1) Are parametric amplifiers the right choice for the input amplifier?

2) Is it practical to cool the amplifiers for optimum noise performance?

3) Will 50°K radiometer noise temperature be adequate?

4) Is the parametric amplifier stable enough?

5) Which is best for the VLA: Non-degenerate or degenerate amplifier?

6) Is the parametric amplifier reliable enough for use in a multi-element array?

Mixer - IF Preamps

The general characteristics of the mixer-preamplifier system required are well known and present no problems in design. The following considerations and characteristics are important in the final design.

1. Reliability, stability, and simplicity of design - all are achieved by solid-state circuitry presently available. The cost-performance figures on this particular item are very good presently.
2. Mixer conversion efficiency and amplifier noise figure. Present-day specifications very adequate.
3. Bandwidth - 10-20 MHz. - Easily obtained.
4. Phase linearity across pass-band. Specifications required here are easily met.

5. Balanced-mixer system required. This is an area where we have experienced trouble due to noise side-bands in the present local oscillator system. Thus, the important consideration is balancing the mixer for maximum suppression of the local oscillator noise side-bands rather than for a) minimum noise figure or b) maximum conversion efficiency.
6. Phase stability. The stability (both long-term and short-term) of the path length, delay, or phase shift through the amplifier must be good.
7. Gain and Gain Stability. The amplifier should have high enough gain and output power capability to drive the line to the central control and distribution terminal.
8. IF Filter for Band-limiting. Sharp cut-off band-pass characteristics are desirable in order that interfering signals close to the IF pass-band are reduced to a minimum. This has been a problem with the NRAO interferometer.
9. Amplifiers with integrated power supplies are very desirable. Amplifiers which have long D.C. power cables running up the antenna are quite susceptible to interference. However, integrated power supplies do require highly stabilized and regulated A.C. power.

b) The IF delay system. The delay system for the VLA will consist of one set of delay lines for each IF output of each antenna in the array. Then the IF signal from each antenna of the array will be delayed by such an amount that each antenna is effectively "translated" to some common reference plane whose normal is along the line of sight to the source under observation. The most efficient method for doing this would be to have the reference plane intersect the center point of the array. This would require both "negative" and positive time delays for all antennas. This can be accomplished by having the delays switch about some "bias" delay which would be 1/2 the maximum delay between any two elements.

The important considerations to be studied in the design of the VLA delay system are:

1. Delay switching control system. Minimize the complexity of the computer program required to perform the delay switching.
2. Selection of the type of delay lines. For a large array a cable delay system becomes unwieldy and costly. The major problems encountered in other types of delay lines are
   a) The large percentage bandwidth of the proposed IF signal (B = 10 MHz, f_o = 7 MHz.
   b) The smallest incremental delay required (of the order of 10 nanoseconds).

   The present NRAO interferometer successfully uses coaxial cables to achieve the necessary delays. Other types of delay such as lumped constant lines are presently being tested as long delays.
3. Equalization networks. For very long delays, operating with signals of moderately high bandwidths, undesirable characteristics such as high loss, dispersion, phase non linearity, etc., must be compensated for. Where possible this compensation or equalization should be accomplished by means of passive networks.

After being properly delayed, the IF output from each antenna will be correlated with the outputs from the (n-1) other antennas. Thus the output power level must be sufficient to drive (n-1) correlators. This requires output amplifiers (solid state) capable of delivering several watts of IF output for a 37 element array.

Problems that need further study:

1) What type of delay:

Cables?

Ferrite loaded cables?

Ultrasonic?

Lumped constants?

2) Should delays be time-shared? What savings can be expected?

3) Gain and bandpass equalization problems for different delay combinations

c) IF post-amplifiers and correlator drivers. The requirements for the IF post-amplifiers at the central terminal are in general the same as for the IF preamplifiers insofar as band-width, gain and noise figure stability, phase stability and linearity are concerned. Some special considerations are

1. Automatic Level Control - Due to the multitude of small variations in the total system gain (or loss) which are brought about by gain variations, delay switching and so forth, a final automatic level control is desirable. This should be done on the last active element prior to the correlators, which will be the final IF amplifier at the central terminal. The ALC should control the level to better than 0.1 dB (~2%), in order to insure a satisfactory operation of the correlators.

II. Equipment proportional to N^2

The only units that must be present in large quantities (666 units for a 37 antenna single frequency array in which all antenna combinations are used) are the correlators and preintegrating networks. Fortunately, these are very simple units, well adapted for mass production as integrated circuits. The use of integrated circuits also keeps the size of the units down to a reasonable level.

An important consideration in the design of the correlators and their drivers is the isolation needed through these circuits in order to prevent spurious correlations in the outputs.

Problems that need further study:

1) Optimize correlator circuit design.

2) Integrated circuit technique should be investigated.

3) High isolation between correlator inputs needed.

4) Required drive power should be minimized.

III. The Local Oscillator

In addition to supplying local oscillator power to all mixers, the local oscillator signal must have a fixed stable phase relation at the mixers in all antennas. In order to accomplish this a phase-locked local oscillator system is needed.

The requirements for the local oscillator system are the following:

1. Furnish sufficient drive level at the mixers of each antenna (approximately 2-5 mw).
2. Phase lock the local oscillator input at each mixer to some common reference. Phase equality for all mixers must be held to better than 5 electrical degrees.
3. Provide a frequency pure signal - (No multiple outputs or spurious side-bands). Also the noise side-bands should be low (more than 70 dB below L.O. signal). In addition, the short term stability (periods smaller than the phase lock loop time constants) should

be good.

In the basic phase-locked local oscillator system the local oscillator output at each antenna is phase locked to a master oscillator at a central location. This could be achieved by two different methods:

(1) Using a separate phase locked local oscillator line between the master oscillator and the individual antennas. This system would give the same phase error in all antennas.

(2) A "serial" system where the element farthest from the central location is phase locked to its neighbor and so on into the master oscillator at the central location. This system would give a larger phase error for the remote antennas, but would require far less cable.

The experience with the NRAO interferometer has shown that a very small part of the phase variations in the local oscillator system is caused by phase changes in the buried cables, since its temperature variations are extremely small, and it might be possible to utilize a buried trunk line local oscillator feed system. Each element would have a phase locked tap line connected to the phase stable trunk. Since there are reasons to assume that the very small phase variations that occur in the buried trunk line are evenly distributed, a total phase shift monitoring device in the trunk line could be used for small phase corrections of the observations.

For a 10" resolution array the arm length would be 2.5 km and a cable system as described above would be both technically and economically feasible. The present NRAO interferometer which has dimensions of this order uses a cable system. For a 1" resolution array which requires a arm length of 25 km, however, a radio link system may have to be used. A phase locked radio link system for transmission of phase stable local oscillator signals as well as the IF signal information is presently being developed.

11. Data Processing, Monitor and Control Systems

For a correlation array consisting of 37 elements, there will be 666 correlator outputs if all possible pairs of elements are utilized. For further processing of these correlator outputs, a high speed digital computer is proposed. This first stage reduction, providing the amplitude and phase averaged over the appropriate time interval for each correlator output, constitutes the main function of the computer system. In addition, it must control and monitor the antennas and electronics systems. The magnitude of these tasks suggested a detailed study of the computer requirements with specific costing information for an optimized system. Some of the results of this study will now be summarized.

The array parameters which were selected for the study are as follows:

1) "WYE" configurations with 1" arc resolution at 10 cm wavelength;

2) 37 elements - all pairs correlated;

3) Single frequency operation.

The functional requirements for the computer system will not be summarized.

A. Real time acquisition of the correlator outputs. The outputs of the correlators are first smoothed by an appropriate R.C. filter whose output is then sampled by an analog-to-digital converter and multiplexer unit. Each correlator will have at its output a noisy quasi-sine wave of frequency ranging from 0 to some maximum value which is determined by the source position and the separation and orientation of the two elements forming the pair for the correlator under consideration. The maximum fringe frequency encountered among all correlators will be approximately 15 cps. There will be a distribution of maximum fringe frequencies among the correlators which corresponds directly to the distribution of maximum u values for the tracks in the u-v plane. It is therefore proposed that the correlators be arranged in groups according to their maximum fringe frequencies. Each group will then have a fixed sampling rate consistent with the maximum fringe frequency to be encountered. All correlators in the group will have the same output time constant which will be set according to the sampling rate. Organization of the correlator outputs in this manner will remain fixed for a given array configuration.

B. Control functions. The computer must be used to perform all of the repetitive control functions necessary in the operation of the array throughout a full 6 to 8 hours source track. The most important of these functions will be:

1. Pointing the telescopes to the source position. This will include making periodic pointing corrections for individual telescope tracking errors.
2. Delay line switching. Delay lines must be switched into the IF line for each element so as to maintain approximately zero relative delay between the outputs of each array element.
3. Calibration. Periodic calibrations of the electronics must be made during observations for use in data reduction. The electronic components used in the calibration will be controlled by the computer.
4. Polarization position angle of the antenna feed horn will be under programmed control.
5. Operating point control. The variety of sources which may be investigated with this array will require different settings for the electronics operating points. These settings will be under computer control.

C. Monitoring functions. For adequate operation and maintenance of the array, real time monitoring of a variety of functions must be performed with the aid of the computer system.

1. Encoder outputs. This is actually a part of the control functions pertaining to telescope positioning and polarization position angle control. However, the equipment and organization of the data must be integrated with that of the other monitoring functions.
2. Continuous monitoring of electronic components in order to establish rate of degradation of system sensitivity.
3. Detection and location of failures in any part of the system.

D. First reduction and storage. The computer must determine the fringe parameters (amplitude and phase) describing the output of each correlator for each minute of sidereal time. This is to be accomplished by making a least-squares fit of a sine function to the array of data points from the analog to digital converter. The results of this processing will be stored in a readily accessible storage device along with:

1. Source identifications
2. Constants and correstions used in the reduction
3. Time and date
4. Frequency and polarization
5. Calibration data.

E. Final reduction (Fourier inversion) and output. This may take one of the following forms, dependent on the results of the study of computer requirements.

1. Preliminary reduction to facilitate later final reductions on an independent computer.
2. Complete Fourier inversion on a limited amount of source observations

or on calibration sources.

3. Full Fourier inversion on all data.

The computer design which provides the capability to perform to above functions must also have two other features which will be important in the efficient and economical operation of the array.

1. Reliability. The computer must have a reliability which far exceeds that of the electronics systems which comprise the array. This appears to be easily met with a small degree of redundancy in the computer peripheral equipment and output devices.
2. Growth capability. The design of the computer should allow for a natural growth of computing and control capacity along with the natural growth of the array during construction phase. This seems to indicate a modular design consisting of several duplicate computer systems, each handling a portion of the array.

The data processing, monitor and control systems are major items of the array. RCA is nearing completion of a fairly detailed study of the computer requirements. When the study is completed (in late December) a separate report will be distributed, covering the data processing and computer aspects of the array. The study has already shown that the computer requirements are readily within present state-of-the-art, and can be met at reasonable cost.

As an example of some of the computer requirements, a description of requirements for an on-line computer for use with the interferometer is included as Appendix V.

12. The Site

The performance of any radio telescope depends in large part upon the properties of the site. Traditionally radio telescopes have been built at locations as remote as possible from sources of man-made interference, such as cities, airports, major highways, radio transmitters, etc. The detrimental effects of a poor site have been demonstrated again and again and are so well known that they do not merit extended discussion.

The few existing quantitative data on the noise characteristics of cities and highways do not appear to permit the establishment of definite criteria for the necessary isolation of the proposed array from a city of given population. Experience has shown that the broad-band noise from cities and highways is attenuated rapidly with distance; isolation of a few miles from a city of, say, 50,000 population appears to give adequate protection. It is important to be conservative, however, as cities tend to grow, and as any particular city may be atypically noisy. Pending actual measurements on specific sites, the following distance criteria are adopted for screening possible sites:

Minimum distance from a city of over 50,000 population . . . . 40 miles

Minimum distance from a city of over 10,000 population . . . . 15 miles

No settlement over 3000 population within . . . . 5 miles

While isolation from man-made noise is essential, it is also important that the instrument be situated within commuting distance of a reasonably populous center with good schools and other cultural facilities. Satisfactory operation of an electronic instrument with the contemplated complexity and sophistication demands the services of a number of electronic engineers and technicians of the highest skill and training. Such personnel are in chronic short supply; it is important not to add extreme isolation and lack of educational facilities to the usual difficulties of recruiting and retaining the electronic staff. Thus, the following criterion is adopted:

> "Not more than 75 miles from a city of 25,000 to 40,000 population; preferably one with an accredited college or university, and accessible to the site by high-speed highway."

It must be recognized, of course, that all the criteria mentioned here are for guidance only It will probably be impossible to satisfy them all exactly, and a site which fails in one or two respects may be so superior in others as to make it advisable to relax the criteria in which it is deficient.

For reasons of accessibility, cost, and political control it is considered necessary that the instrument be located in the United States. At the same time, it is important that the maximum sky coverage be achieved. This requires that the site be at the lowest possible latitude. A good selection of sites meeting all the aforementioned criteria lies below 35° north latitude in the continental USA; thus, this has been adopted as a desirable upper limit to the latitude of the site.

A roughly circular area approximately 2 1/2 miles in diameter is needed for the array with 10" resolution at 11 cm wavelength. Only a very small percentage of the area will actually be occupied by the array, its control buildings, dormitories, shops, etc.; the remainder can be used for grazing, forestry, crops, or recreation. This area would permit $3''.7$ resolution at 3 cm wavelength.

Undoubtedly it will become desirable to extend the array to higher resolutions than 10" at 11 cm wavelength; in fact, the ability to expand is one of the major criteria in the design. Thus, it is important at the outset to acquire a sufficiently large site to permit substantial expansion. The suggested size of the central area is 5 miles in diameter. Furthermore, there will be occasions to make observations of the diameters of sources as small as 1" of arc. This would require the ability to place isolated antennas at several points outside the central area, up to distances of 25 miles. In order to simplify the transmission of data and of local-oscillator signals, it is desirable that all sites be intervisible without the use of extremely high towers.

The geological substructure, of course, must be suitable for the stable support of moderately heavy, concentrated loads. The central area should be free of excessive relief. Suggested criteria are: no relief greater than 300 feet in the central area; slopes in the central area should not exceed 1%.

In the matter of climate, it is again necessary to rely on the accumulated experience of radio astronomers, rather than upon quantitative experiment or theory. Much speculation has occurred concerning the effect of atmospheric inhomogeneities on the performance of long-baseline correlator arrays. To date, as discussed in Chapter 2, no evidence is available to show that the atmosphere causes any discernable deterioration in array performance. There is evidence, however, to show that the performance of a single paraboloidal reflector can be affected by atmospheric moisture at wavelengths as long as 11 cm. Thick clouds and precipitation can have appreciable optical depth and can thus increase the unwanted component of the antenna temperature. This effect can be particularly serious at 3 cm wavelength. Apparently differential phase shifts induced by the atmosphere have negligible effects on the efficiencies of paraboloidal reflector antennas at any of the wavelengths being considered Atmospheric effects are not likely to cause any serious deterioration of performance of the 10" array at 11 cm wavelength Whether one can extrapolate the meager existing data to 3 cm wavelength or 1" resolution is an open question. Further, it may prove desirable to observe with single antennas of the array, on special occasions, or to erect a large single dish for ambiguity resolution or preliminary mapping of an area of sky to be mapped in detail by the array. Such single antennas are well known to be vulnerable to water vapor clouds, especially at the shorter wavelengths In view of this, a high altitude and a dry climate, with a low incidence of cloudy weather, seems indicated for the site of the array. Fortunately, these criteria are easily compatible with those involving terrain and isolations

The site criteria established by the foregoing reasoning are summarized below:

<u>Site Specifications - VLA</u>

1. Below 35° N latitude -- United States Territory
2. Not closer than 40 miles to a city of over 50,000 population, not closer than 15 miles to a city of over 10,000 population; not closer than 5 miles to a settlement of over 3000 population.
3. A population center of 25,000 to 40,000 must be within 75 miles, accessible to the site by high-speed highway
4. Available area must be at least 5 miles in diameter and must have no relief greater than 300 feet in this area. In general, slopes in the area should not be greater than 1%.
5. It must be possible to place isolated antennas at several points outside the central area, up to distances of 25 miles.

6. Preferable that the necessary land be available from public ownership.

7. Geological substructure suitable for erection of heavy structures and for road building. Adequate water supply, adequate power supply.

8. Screening of possible sources of interference by two or more successive ranges of hills or mountains is very effective.

9. Other things being equal, a high-altitude site in a dry climate is probably to be preferred.

10. Areas of high winds, heavy snow loads, icing, and heavy rainfall should be avoided if possible.

Screening of possible sites to meet these specifications is now under way.

Appendix I

On the Minimum Redundancy in a Correlated Antenna Array

B. G. Clark
July 1965

Consider an array of antennas occupying points (not necessarily all points) on a square grid of unit spacing. Let us suppose that this array is capable of instantaneous synthesis to a maximum spacing of N, i.e., that every gridpoint in the square $-N \leq u \leq N$, $-N \leq v \leq N$ in the transfer function plane is occupied by at least one "interferometer." If there are k antennas present, situated at (X_n, Y_n) $n = 1,k$ then clearly $k(k - 1) \geq (2N + 1)^2 - 1$, as the number of points covered cannot exceed the number of antenna pairs. A further restriction may be derived as follows:

Let us, rather than correlating, simply add and square the outputs. This array has a beam

$$B(x,y) = \left| \sum_{n=1}^{k} e^{iX_n x + Y_n y} \right|^2 \tag{1}$$

$$= \sum_{m=1}^{k} \sum_{n=1}^{k} e^{i(X_n - X_m)x + (Y_n - Y_m)y} \tag{2}$$

since this latter form has all differences $(X_n - X_m, Y_n - Y_m)$, and since we supposed that the grid points in the square $-N \leq u \leq N$, $-N \leq v \leq N$ are a subset of these differences, we can write

$$B(x,y) = \sum_{\nu=-N}^{N} \sum_{\eta=-N}^{N} e^{i(\nu x + \eta y)} + \text{other terms} \tag{3}$$

We can even say how many other terms, since the expression (2) has k^2 terms and the sum in (3) has $(2N + 1)^2$ terms, there must be $k^2 - (2N + 1)^2$ of them.

$$B(x,y) = \sum_{\nu=-N}^{N} \sum_{\eta=-N}^{N} e^{i(\nu x + \eta y)} + \sum_{n=1}^{k^2-(2N+1)^2} e^{i(u_n x + v_n y)} \tag{4}$$

or evaluating the first sum

$$B(x,y) = \frac{\sin\left(N + \frac{1}{2}\right)x \, \sin\left(N + \frac{1}{2}\right)y}{\sin\left(\frac{x}{2}\right) \sin\left(\frac{y}{2}\right)} + \sum_{n=1}^{k^2-(2N+1)^2} e^{i(u_n x + v_n y)} \tag{5}$$

As B(x,y) is an absolute square, the sum in (5) must be real, and since each term is less than or equal to 1, the whole sum must be less than or equal to the number of terms

$$B(x,y) \leq \frac{\sin\left(N + \frac{1}{2}\right)x \, \sin\left(N + \frac{1}{2}\right)y}{\sin\left(\frac{x}{2}\right) \sin\left(\frac{y}{2}\right)} + k^2 - (2N + 1)^2 \tag{6}$$

since B(x,y) is an absolute square, $B(x,y) \geq 0$ for all x,y, so

$$0 \leq \frac{\sin\left(N + \frac{1}{2}\right)x \, \sin\left(N + \frac{1}{2}\right)y}{\sin\left(\frac{x}{2}\right) \sin\left(\frac{y}{2}\right)} + k^2 - (2N + 1)^2 \tag{7}$$

substituting $\left(N + \frac{1}{2}\right)x = \phi$ and taking the particular case $y = 0$

$$k^2 \geq (2N + 1)^2 \left(1 - \frac{\sin \phi}{(2N + 1) \sin \frac{\phi}{2N + 1}}\right) \tag{8}$$

in particular, for $\phi = \frac{3\pi}{2}$, and $N >> 1$

$$k^2 \geq (2N + 1)^2 \left(1 + \frac{1}{(2N + 1) \frac{3\pi}{2(2N + 1)}}\right) \tag{9}$$

$$k^2 \geq (2N + 1)^2 \left(1 + \frac{2}{3\pi}\right) \tag{10}$$

$$k \geq 2.202\ N + 1.1 \tag{11}$$

If we wish to insist on occupying only the inscribed circle, of radius N, then the exact formulation becomes very difficult, but to the highest order, it is clear that $(2N + 1)^2$ is to be replaced with πN^2 and $\frac{\sin \phi}{\phi}$ by $\Lambda_1(\phi)$ yielding

$$k^2 \geq \pi N^2\ (1 + .1323) \tag{12}$$

$$k \geq 1.886\ N \tag{13}$$

and the redundancy $\frac{k(k-1)}{\pi N^2 - 1} > 1.13$.

<u>Appendix II</u>

On the Correlation Between the Outputs
of Correlators in a Correlator Array

B. G. Clark
27 October 1965

Say one has N independent noise sources with voltages out $\mathrm{Re}\{V_n(t)\,e^{jwt}\}$, where ω is the signal frequency and $V_n(t)$ is a complex amplitude varying with a characteristic time of the reciprocal of the bandwidth. To these noises let us add a noiselike signal from a point source (so that it is identical in all elements of the interferometer), e(t). Then the output of the $(m,n)^{th}$ (let us say for convenience $n > m$) correlator is

$$\mathrm{Re}\ \{(V_n(t) + e(t))\ (V_m + e(t))^*\}$$

where * denotes the complex conjugate.

Let us add M of these correlator outputs together. The final output is

$$G(t) = \mathrm{Re}\left\{\sum_{j=1}^{M}\left[V_{n_j}V_{m_j}^* + e(t)\ (V_{n_j}^*(t) + V_{m_j}^*(t)) + e(t)\ e^*(t)\right]\right\}$$

The expectation value of this is the signal, $\underline{S}$. Since V_n and V_m are assumed independent $< V_nV_m > = 0 \quad n \neq m$

$$S = \langle G(t)\rangle = M\ \langle e(t)e^*(t)\rangle$$

The RMS noise N_o is given by

$$N_o^{\ 2} = <\ (G(t)\ -\ \langle G(t)\ \rangle)^2\ >$$

if all the noises are assumed circular, i.e., the probability that V_i has a phase between φ and $\varphi + d\varphi$ is independent of φ, it may be shown that

$$N_o^{\,2} = \frac{1}{2} < \left| \sum_{j=1}^{M} V_{n_j} V_{m_j}^{*} + e(t) \quad (V_{n_j}^{*} + V_{m_j}^{*}) + ee^{*} - < ee^{*} > \right|^{2} >$$

If we assume that the signal-to-noise ratio is not too high in the predetected signal, i.e., $< ee^* > \lll < V_n V_m^* >$, we may neglect the cross terms containing $< ee^* V_n V_m^* >$ in the expansion of $N_o^{\,2}$

$$N_o^{\,2} = \frac{1}{2} < \sum_{j=1}^{M} \sum_{k=1}^{M} \quad V_{n_j} V_{m_j}^{*} \, V_{n_k}^{*} \, V_{m_j} >$$

Since V_n are independent

$$< F(V_{m_1},\, V_{m_2}, \ldots V_{m_k}) \; G(V_n) > \; = \; < F(V_{m_1}, V_{m_2} \ldots V_{m_k}) > \, < G(V_n) >$$

if $m_1,\ m_2,\ \ldots\ m_k \neq n$.

This is sometimes used as a definition of independence, and follows obviously from the usual definition that the joint probability distribution is the product of the distributions. Therefore

$$< V_{m_1} V_{m_2}^{*} \; V_{m_3} V_k^{*} > \; = 0 \text{ if } m_1,\ m_2,\ m_3 \neq k$$

hence, since $n > m$

$$< V_{n_j} V_{m_j}^{*} \, V_{n_k}^{*} \, V_{m_k} > \; = \delta_{m_j m_k} \; \delta_{n_n n_k} \; < V_{n_j} V_{n_j}^{*} \, V_{m_j} V_{m_j}^{*} >$$

therefore

$$N_o^{\,2} = \sum_{j=1}^{M} \quad < V_{n_j} V_{n_j}^{*} > < V_{m_j} V_{m_j}^{*} >$$

if $< V_n V_n^* > \; = \; < VV^* >$ for all n, i.e., the noises are equal in power

$$N_o^2 = \frac{1}{2} M \; (< VV^* <)^2$$

$$S/N_o = \frac{< ee^* >}{< VV^* >} \sqrt{\frac{2}{M}}$$

if all correlators are added in, $S/N_o \approx \dfrac{< ee^* >}{< VV^* >} \quad \dfrac{2}{N}$

The terms in ee* neglected in the above analysis correspond to Hanbury Brown-Twiss interferometers operating between the various correlator pairs. Their effect is to make the effective noise at a given phasing equal to the receiver temperature plus the source temperature in the synthesized beam.

Appendix III

Aperture Synthesis with a Correlation Array

C. M. Wade

12 Oct. 1965

When using a correlation array, one observes the two-dimensional Fourier transform (F.T.) of a brightness distribution rather than the distribution itself. The highest spatial frequency observed determines the angular resolution of the mapping. Since it is not practicable to observe the F.T. continuously over the full spatial frequency range in both coordinates, we must think in terms of sampling at discrete points on the F.T. plane. In the present section, we discuss the considerations which underlie the design of an array which can be used effectively for aperture synthesis.

A. The Transfer Function and the Radiation Pattern.

The sampling in the F.T. plane is described conveniently by the *transfer function* $W(u, v)$. We assign it the following properties:

1. $W(u, v)$ is real.
2. $W(0, 0) = 1$. This defines the numerical scale.
3. $0 < W(u, v) \leq 1$ at the sampled points, and $W(u, v) = 0$ elsewhere.
4. $W(u, v) = W(-u, v) = W(u, -v) = W(-u, -v)$. This is an assumption of symmetry.

The value of $W(u, v)$ is simply the weight given to the corresponding component in the Fourier inversion.

The power radiation pattern of the synthesized aperture is the F.T. of the transfer function:

$$p(x, y) = \frac{1}{C} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(u, v)\, e^{-j2\pi(ux + vy)}\, du\, dv, \tag{5-1}$$

$$C = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(u, v)\, du\, dv. \tag{5-2}$$

When expressed in this way, p(x, y) is normalized so that

$$p(0, 0) = 1.$$

B. <u>The Rectangular Sampling Grid</u>.

Since W(u, v) differs from zero only at a finite number of discrete points, the above integrals can be replaced by finite series. We shall assume that these points (the sampling grid) form a uniformly-spaced rectangular grid, one point of which is at the origin (u = 0, v = 0). Then we can express the sampling function as a two-dimensional sequence,

$$W(u, v) = W_{k,\ell}\, \delta(u - k\Delta u)\, \delta(v - \ell\Delta v) \tag{5-3}$$

with

$$k = -K, \ldots, 0, \ldots, K$$

$$\ell = -L, \ldots, 0, \ldots, L.$$

The largest spatial frequencies along the two coordinate axes are

$$u_{max} = K\Delta u, \quad v_{max} = L\Delta v.$$

Figure 5-1 illustrates the arrangement.

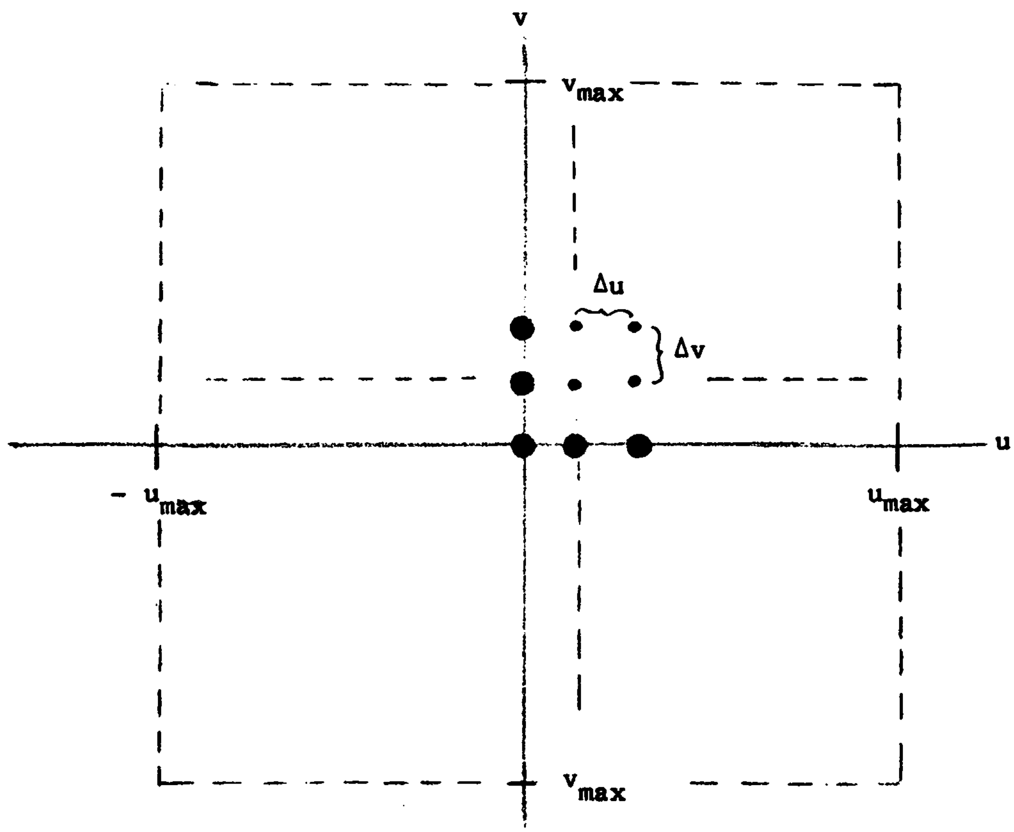


Fig. 5-1

Using the four properties assigned to W(u, v) in Section 5A above, and equation (5-3), we can express equations (5-1) and (5-2) as

$$p(x, y) = \frac{1}{C}\left\{1 + 2\sum_{k=1}^{K} W_{k,o}\cos 2\pi kx\Delta u + 2\sum_{\ell=1}^{L} W_{o,\ell}\cos 2\pi\ell y\Delta v\right.$$

$$\left. + 4\sum_{k=1}^{K}\sum_{\ell=1}^{L} W_{k,\ell}\cos 2\pi kx\Delta u\cos 2\pi\ell y\Delta v\right\} \qquad (5\text{-}4)$$

$$C = 1 + 2\sum_{k=1}^{K} W_{k,o} + 2\sum_{\ell=1}^{L} W_{o,\ell} + 4\sum_{k=1}^{K}\sum_{\ell=1}^{L} W_{k,\ell}. \qquad (5\text{-}5)$$

It is evident that the radiation pattern has the same kind of symmetry as the transfer function:

$$p(x,\ y) = p(-x,\ y) = p(x,\ -y) = p(-x,\ -y).$$

An important consequence of the rectangular sampling grid can be seen immediately from equation (5-4) - the radiation pattern is periodic, since

$$p\left(x \pm \frac{\mu}{\Delta u},\ y \pm \frac{\nu}{\Delta v}\right) = p(x,\ y),$$

where

$$\left.\begin{matrix}\mu \\ \nu\end{matrix}\right\} = 0,\ 1,\ 2,\ \ldots$$

Therefore the radiation pattern repeats at intervals

$$\Delta x = 1/\Delta u,\ \Delta y = 1/\Delta v$$

radians. This self-replicating property is exactly analogous to that of a two-dimensional diffraction grating, and for this reason we speak of the family of repeated patterns as "grating lobes". These considerations bear directly on the manner in which the transfer function is to be specified, since we must insure that the separation of the grating lobes is at least as great as the angular dimensions of the largest source we wish to map. If these are φ_{EW} (east-west) and φ_{NS} (north-south) radians, we clearly must have

$$\boxed{\Delta u \leq 1/\varphi_{EW},\ \Delta v \leq 1/\varphi_{NS}} \qquad (5\text{-}6)$$

The transfer function of any real array will have a further property which is of practical importance, although it is not included in the above discussion. We have assumed that the sampling occurs only at discrete geometrical points on the F.T. plane. Since the array elements must be of finite size, the sampling actually takes place over small areas centered on these points. The transfer function in the vicinity of a grid point is $W'(u - u_o, v - v_o)$, where (u, v_o) are the coordinates of the point, and $W'(u, v)$ is the transfer function of a single array element. $W'(u, v)$ is the F.T. of single-element radiation pattern $p'(x, y)$. Hence the true transfer function will be the convolution of $W(u, v)$ and $W'(u, v)$, and the actual radiation pattern of the synthesized aperture will be $p'(x, y) \cdot p(x, y)$. This bears on the present discussion only in the fact that the grating lobes are reduced in proportion to $p'(x, y)$. Thus those grating lobes which fall outside the main lobe of $p'(x, y)$ are of little importance.

Our advocacy of a rectangular sampling grid, when this leads inevitably to the unwanted grating lobes, requires justification. There are two strong points which favor the rectangular arrangement. First, if we vary the sampling interval over the F.T. plane, we in effect are assigning a greater importance to some parts of the F.T. than to others. This is arbitrary and unreasonable in the absence of <u>a priori</u> knowledge of the structure of the source being observed; it follows that no such arrangement could be equally well suited to all sources. The second point is that <u>any</u> discrete sampling scheme must have unwanted response analogous to the grating lobes. While their shapes and positions can be altered, they cannot be eliminated; and this would be done at the expense of the real conveniences which the rectangular scheme affords, particularly in the Fourier inversion.

C. Separable Transfer Functions.

If the transfer function is separable, i.e., if we can write

$$W_{k,\ell} = W_{k,o} \; W_{o,\ell} = W_k \; W_\ell ,$$

equations (5-4) and (5-5) reduce to

$$p(x, y) = \frac{1}{C} \left\{ 1 + 2 \sum_{k=1}^{K} W_k \cos 2\pi k x \Delta u \right\} \left\{ 1 + 2 \sum_{\ell=1}^{L} W_\ell \cos 2\pi \ell y \Delta v \right\}, \tag{5-7}$$

$$C = \left\{ 1 + 2 \sum_{k=1}^{K} W_k \right\} \left\{ 1 + 2 \sum_{\ell=1}^{L} W_\ell \right\}. \tag{5-8}$$

Therefore the radiation pattern is separable also:

$$p(x, y) = p(x, 0) \; p(0, y).$$

Two averages based on the transfer function will be needed in later sections of this proposal. These are the mean value $\overline{W}$ and the r.m.s. value $\rho = \sqrt{\overline{W^2}}$. If $W_{k,\ell}$ is separable, we have

$$\overline{W} = \frac{C}{(1 + 2K)\,(1 + 2L)} \tag{5-9}$$

where C is given by (5-8), and

$$\rho_W = \sqrt{\left(\frac{1 + 2 \sum_{k=1}^{K} W_k^2}{1 + 2K} \right) \left(\frac{1 + 2 \sum_{\ell=1}^{L} W_\ell^2}{1 + 2L} \right)} \tag{5-10}$$

At this point, let us consider two specific examples of separable transfer functions. The first assigns equal weights to all Fourier components and therefore corresponds to the so-called "principal solution". The second weights the Fourier components in inverse proportion to their spatial frequencies and is analogous to a uniformly-fed rectangular aperture.

I. Uniform Weighting

$$W_k = W_\ell = 1 \qquad \begin{cases} |k| \leq K \\ |\ell| \leq L \end{cases}$$

This gives

$$p(x,\ y) = \frac{\sin\ [(2K + 1)\ \pi x \Delta u]}{(2K + 1)\ \sin\ \pi x \Delta u} \cdot \frac{\sin\ [(2L + 1)\ \pi y \Delta v]}{(2L + 1)\ \sin\ \pi y \Delta v} \qquad (5\text{-}11)$$

and

$$\bar{W} = 1,$$

$$\rho_W = 1.$$

II. Linear Weighting

$$W_k = 1 - \frac{k}{K+1}, \quad k \leq K;$$

$$W = 1 - \frac{\ell}{L+1}, \quad \ell \leq L.$$

This gives

$$p(x,y) = \frac{\sin^2 [(K + 1)\ \pi x \Delta u]}{(K + 1)^2\ \sin^2\ \pi x \Delta u} \cdot \frac{\sin^2 [(L + 1)\ \pi y \Delta v]}{(L + 1)^2\ \sin^2\ \pi y \Delta v}, \qquad (5\text{-}12)$$

$$\bar{W} = \frac{(1 + K)\ \ (1 + L)}{(1 + 2K)\ (1 + 2L)}, \qquad (5\text{-}13)$$

$$\rho_W = \frac{1}{3}\left[\left\{1 + \frac{K + 2}{(K + 1)(2K + 1)}\right\}\left\{1 + \frac{L + 2}{(L + 1)(2L + 1)}\right\}\right]^{1/2} . \tag{5-14}$$

Since $u_{max} = K\Delta u$ and $v_{max} = L\Delta v$, we have in the limit as $(K, L) \to \infty$ and $(\Delta u, \Delta v) \to 0$, for constant u_{max} and v_{max},

$$p(x, y) = \left[\frac{\sin \pi x\, u_{max}}{\pi x\, u_{max}}\right]^2 \left[\frac{\sin \pi y\, v_{max}}{\pi y\, v_{max}}\right]^2 ,$$

$$\overline{W} = 1/4,$$

$$\rho_W = 1/3.$$

These are the familiar relations for a uniformly-fed rectangular aperture.

It should be noted that this transfer function gives a radiation pattern without negative sidelobes, since the right-hand side of (5-12) is an absolute square.

D. Beamwidth and Sidelobe Levels.

There is no general way to find the beamwidth and sidelobe levels of the synthesized radiation pattern directly from the transfer function. It is necessary to calculate the radiation pattern (in both dimensions) and then to determine these quantities from the result.

Since the half-power beamwidth for any given transfer function is always inversely proportional to the highest spatial frequency, we have

$$\beta_{EW} = \varkappa/u_{max}, \quad \beta_{NS} = \varkappa/v_{max},$$

where β_{EW} and β_{NS} are respectively the east-west and north-south beamwidths in radians, and $\varkappa$ is a constant of the order of unity whose exact value depends on the transfer function.

An alternative way to specify the width of the synthesized main lobe is to give the angular distance between the first zeros. This is not as satisfactory conceptually as the half-power beamwidth, since some radiation patterns have sidelobes between the main lobe and the first zeros. On the other hand, the beamwidths between first zeros can often be found analytically, since these are the smallest values of Θ_{EW} and Θ_{NS} for which

$$\sum_{k=1}^{K} W_k \cos \pi k \Theta_{EW} \Delta u = -1/2, \qquad (5\text{-}15)$$

$$\sum_{\ell=1}^{L} W_\ell \cos \pi \ell \Theta_{NS} \Delta v = -1/2, \qquad (5\text{-}16)$$

if the transfer function is separable.

Let us consider the two kinds of beamwidth for the two examples treated in Section 5C. We have found the values of Θ from (5-15) and (5-16), and the values of β from (5-11) and (5-12) under the assumption that M and N are large. We have:

I. Uniform Weighting

$$\Theta_{EW} = \frac{1}{(K + 1/2)\Delta u} \approx 1/u_{max}; \quad \beta_{EW} \approx 0.604/u_{max};$$

$$\Theta_{NS} = \frac{1}{(L + 1/2)\Delta v} \approx 1/v_{max}; \quad \beta_{NS} \approx 0.604/v_{max}.$$

II. Linear Weighting

$$\Theta_{EW} = \frac{2}{(K + 1)\Delta u} \approx 2/u_{max}; \ \beta_{EW} \approx 0.886/u_{max};$$

$$\Theta_{NS} = \frac{2}{(L + 1)\Delta v} \approx 2/v_{max}; \ \beta_{NS} \approx 0.886/v_{max}.$$

E. The Required Number of Sampling Points

The number of points on a rectangular sampling grid of the kind we are considering is

$$N_W = (1 + 2K)(1 + 2L) \tag{5-17}$$

We accomplish the Fourier inversion of the observational data by performing the double summation

$$B(x,y) = C' \sum_{k=-K}^{K} \sum_{\ell=-L}^{L} W_{k,\ell} V_{k,\ell} e^{-j2\pi(kx\Delta u + \ell y\Delta v)} \tag{5-18}$$

where $V_{k,\ell}$ is the complex visibility of the source at $u = k\Delta u$, $v = \ell\Delta v$, and C' is a constant which serves to express $B(x, y)$ in the appropriate units. Now the brightness distribution $B(x, y)$ is always real; hence its Fourier transform $\tilde{V}(u, v)$ is hermitian:

$$\tilde{V}(-u, -v) = \tilde{V}^*(u, v).$$

Therefore one needs to sample only half of the transform plane area covered by the transfer function. Each observed value of $\tilde{V}(u, v)$ will be used twice in (5-18), once directly and once as a complex conjugate. Then the number of grid points where we must obtain independent observations (including the origin, $u = 0$, $v = 0$) is

$$N_{obs} = 2KL + K + L + 1. \tag{5-19}$$

We can express this directly in terms of the resolution and field of view requirements. Since

$$K = u_{max}/\Delta u = \varkappa \left(\frac{\varphi_{EW}}{\beta_{EW}}\right),$$

$$L = v_{max}/\Delta v = \varkappa \left(\frac{\varphi_{NS}}{\beta_{NS}}\right),$$

we have

$$\boxed{N_{obs} = 2\varkappa^2 \left(\frac{\varphi_{EW}}{\beta_{EW}} \cdot \frac{\varphi_{NS}}{\beta_{NS}}\right) + \varkappa \left(\frac{\varphi_{EW}}{\beta_{EW}} + \frac{\varphi_{NS}}{\beta_{NS}}\right) + 1} \qquad (5\text{-}20)$$

If we require the same ratio of field of view to resolution in both coordinates (as we shall see, this probably will not be true), i.e., if

$$\frac{\varphi_{EW}}{\beta_{EW}} = \frac{\varphi_{NS}}{\beta_{NS}} = \frac{\varphi}{\beta},$$

then (5-20) reduces to

$$N_{obs} = 2\left(\frac{\varkappa\varphi}{\beta}\right)^2 + 2\left(\frac{\varkappa\varphi}{\beta}\right) + 1 .$$

F. A Correlation Array Using Sidereal Tracking.

When using a correlation array, one finds the complex coherence $\tilde{V}(u, v)$ between the signals received from each pair of antennas. If there are M antennas, M(M -1)/2 simultaneous pairs can be formed among them. Each pair can be treated as a two-element interferometer, and the array can be thought of as M(M - 1)/2 interferometers operating concurrently.

Each antenna pair has associated with it a set of four constants $\{B_1, B_2, B_3, h\}$ (see the NRAO Report, "Geometrical Aspects of Interferometry", C. M. Wade and G. W. Swenson, December 1964). These constants depend on the local oscillator frequency and the length and orientation of the line joining the phase centers of the elements of the pair. B_1 and B_2 are respectively the components of the baseline parallel and perpendicular to the earth's rotational axis, in wavelengths; B_3 allows for the difference in electrical path length between the local oscillator and the two mixers; and h is the hour angle of the point where an extension of the line joining the antennas would strike the celestial sphere. The transform plane coordinates corresponding to a pair of antennas pointed at hour angle H and declination δ are

$$u = B_2 \sin (H-h), \tag{5-21}$$

$$v = B_1 \cos \delta - B_2 \sin \delta \cos (H-h). \tag{5-22}$$

These are the parametric equations of an ellipse, with H as the parameter. Therefore, if the antennas are made to track a radio source along its diurnal circle, the position sampled in the F.T. plane will describe an elliptical arc.

The elliptical track in the F.T. plane has the following properties:

1. Its major axis lies parallel to the u-axis.
2. Its center lies on the v-axis, at $u = 0$, $v = B_1 \cos \delta$.
3. Its major and minor semi-axes are B_2 and $B_2 \sin \delta$, respectively.
4. Its axial ratio is $\sin \delta$ and its eccentricity is $\cos \delta$.

It is clear that the nature of the transform plane coverage offered by any such track depends strongly on the declination of the source being observed. At the celestial equator, the ellipse degenerates to a straight line paralleling the u-axis, while at the pole it becomes a circle. Similarly, the F.T. coverage given by an array using sidereal tracking is a function of declination. This is an important point which must always be kept in mind when discussing a tracking array. Generally speaking, it is most difficult to obtain a good F.T. plane coverage for sources at low declination.

With an array of M antennas tracking a radio source, one obtains F.T. samples on $M(M - 1)/2$ elliptical tracks simultaneously. These ellipses will have numerous points of intersection and, unless care is taken with the array plan, there will be appreciable areas on the F.T. plane which are not sampled at all. An optimum array configuration will let one avoid such unsampled areas entirely while holding to a minimum the number of regions which are sampled by many tracks, and it will do so for all declinations within the range of the instrument. This problem will be treated in detail in Section 6.

The data sampling afforded by such an array is virtually continuous along the elliptical tracks. Prior to performing the Fourier inversion, one would organize the data in a suitable rectangular grid by determining the average $\tilde{V}(u, v)$ for each grid cell. The grid dimensions would be made small enough that the number of grating responses within the primary (single-element) main lobe is not excessive, yet not so small that there would be an appreciable number of unsampled cells. It generally should be possible to make Δu smaller than Δv because the major axes of the elliptical tracks lie parallel to the u-axis.

Appendix IV

Calibration of a Large, Many Element, Phased Array

B. G. Clark
18 October 1965

In order to make an antenna operate with high efficiency and narrow beamwidth, it is well known that its surface must be so set that the phase path to the receiver is identical to within 0.1 λ for all parts of the wave front. The same theorem, of course, applied to the phasing of an array, namely, that the phasing of each pair of antennas must be known to better than 0.1 revolution in order to avoid degrading the sensitivity and beamwidth of the synthesized beam.

An interesting difference between solid antennas and arrays is that in the case of an array, the pointing problem and the surface error problem are essentially identical, the problem of the absolute phasing of the elements of the array. In the case of the solid, steerable antenna, these two problems are separate, in that the surface of the antenna is set to the required accuracy and then the pointing mechanism is calibrated by observations of sources of known position. It is occasionally the case that the pointing mechanism of an antenna is not as well constructed as the surface itself, so that it becomes difficult to use the narrow beamwidth of the antenna because it cannot be pointed accurately at the source. This cannot be the case for a phased array. That a phased array works at all necessarily implies that positional accuracy of the order of 0.1 beamwidth can be obtained on a strong point source.

Thus the problem of phasing an array of 10" beamwidth includes the problem of obtaining 1" positional accuracy. Accuracy approaching this standard has been obtained by long baseline interferometry (Adgie, Nature, 204 1028, 1964 Wade, Clark, and Hogg, Ap. J. 142, 406, 1965).

There are three basic considerations having to do with the phasing of a large array. These are the establishment of a basic radio coordinate system with an angular accuracy greater than 1", the signal-to-noise problem for calibration, which is different from that for observing weak sources, and the effect of inhomogeneities in the atmosphere on the calibration. These will be treated separately below.

The coordinate systems used in radio astronomy have to date been tightly tied to the fundamental optical coordinate systems, and all measured deviations from these systems has been taken to be an instrumental position error effect. However, the fundamental optical coordinate systems probably have internal errors of several tenths of a second of arc, so that it may be profitable to set up a coordinate system more or less independent of the optical fundamental coordinate systems. This may be done from first principles, or may perhaps be tied to the mean of the optical determinations of the positions of radio objects. A fundamental coordinate system may be established, for instance, by an east-west interferometer looking at a circumpolar object. The length of the baseline may be determined nearly independently of errors in the assumed baseline orientation and of errors in source positions by observing the phase of a source near the equator as it rises and sets over the poles of the interferometer baseline. With the length of the baseline known, the phase of the circumpolar source at six hours, hour angle east and west gives the declination of the source, and the mean phase, compared with the mean phase of the equatorial source gives the declination of the pole of the baseline. It is not convenient to observe the ecliptic with a radio instrument, so the zero of right ascension may not be derived directly, but once it is assumed, relative right ascensions may be determined by the times of transit of sources across the central fringe of the interferometer. This system of absolute calibration may not be convenient in practice, but it illustrates that the determination of positions may be carried out for radio objects alone, without reference to optical positions.

If we suppose that we are looking at a point source of known position, and are going to use this source to calibrate the phases of the elements of the large array, we may inquire about the signal-to-noise ratio necessary to calibrate. Two separate cases may be considered. In the first case, the source is sufficiently strong that it may be seen in the output from each correlator, and the phase determined with an error less than a radian. There are only n-1 phases to be determined, since the phase is unaffected by operations done at IF, and there are only n receivers at RF. There are n-1 phases to be determined, because the zero of phase is indeterminate, and may arbitrarily be assigned. However, there are $\frac{n(n-1)}{2}$ quantities measured, the phase from each correlator. These may thus be combined to improve signal to noise ratio. Indeed, it may easily be shown that the best estimate of the phase of the correlator connecting the j^{th} and the k^{th} antenna is not the measured phase from this correlator, θ_{jk}, but $\frac{1}{n}\sum_{i=1}^{n}(\theta_{ji} + \theta_{ik})$. If the source is not sufficiently strong to be seen in the output of each correlator, it should still be possible to combine the $\frac{n(n-1)}{2}$ observations to yield the n-1 phases of the elements. However, the process now becomes nonlinear, and it does not seem possible to resolve the equations in any simple fashion. However, a qualitative argument may be applied as follows: If all of the phases are to be calibrated to an accuracy of $\lambda/10$, then the beam synthesized by adding the n correlator outputs containing a given phase must be detected with a signal to noise ratio of at least 4:1. Therefore, the weakest source usable for calibration is about $\sqrt{n}$ times stronger than the minimum detectable source. However, the redundancy available from the ability to detect the source fringes in the output from a single correlator would probably be a welcome check on the performance of the correlators and latter parts of the receiver system. This is not a very strong requirement. Experience with the NRAO interferometer indicates that a source may be reliably detected (4 standard deviations) at a flux level of

$$S_o = 500 \frac{T_r}{\sqrt{B\tau}\, D^2}$$

where T_r is in degrees, B in Mc/s, τ in minutes, and D, the diameter of the elements, in feet. The NRAO 85' telescopes could thus calibrate effectively on sources as weak as 0.3 flux units, with a one minute integration time. At ten centimeters there are several thousand sources this bright or brighter in the visible sky. Since at least 2% of the sources in the 3C list have diameters less than 1", it would appear that there are at least 50 and possible more than 100 sources in the sky suitable for calibration.

Assuming perfect engineering of the electronic systems, there is one source of phase instability which cannot be eliminated. This is the atmosphere. The behavior of the atmosphere at the decimeter wavelength range is not well known; however, we may make an estimate of its effect from the NBS measurements of the refractive index as quoted by Barton (Proc. IEEE 51, 626, 1963). The power spectrum of the refractive index fluctuations is given by $W(\nu) = 3 \times 10^{-18}\ \nu^{-2.5}$, for $\nu > 10^{-5}$. If we than make the most pessimistic assumptions, that the scale length in the horizontal direction is smaller than the size of the array, and that the scale length in the vertical direction is equal to the scale height of the atmosphere, 7 km, we find that the path length variation introduced by fluctuations with frequencies greater than ν is

$$\sqrt{\overline{\Delta\ell^2}} = 10^{-5}\ \nu^{-0.75}$$

where $\Delta\ell$ is given in meters. For 10 cm operation, one wishes to keep $\Delta\ell$ below one centimeter, which is met if the atmospheric phase fluctuations are cut off at a frequency of 10^{-4} c/s, i.e. if the element phase is calibrated every three hours. This requirement does not appear too stringent for the operation of a large array. However, for three centimeter operation this equation indicates that calibration every half hour might be necessary, which would impose serious restrictions on observing. These measurements are, of course, not applicable directly to the problem under discussion. I think it unlikely that the situation could be worse than outlined here, but it is in need of experimental checking.

All of the considerations above apply to the determination of the instantaneous phasing of an array. Since most of the designs studied require at least limited amounts of "super synthesis", i.e. allowing the rotation of the earth to change the effective baselines of the interferometers comprising the array, the array must be calibrated in terms of the baseline parameters which describe the change with time of the phases as well as with the absolute phases themselves. This problem is no great addition to the problem of calibration. One must merely determine the absolute phases of the elements at several different hour angles over the region of interest. Since they must already be calibrated over the declination range, this is no great addition. Although an unfavorable aspect of an interferometer baseline may mean that some of the baseline parameters are not well determined, it also necessarily means that this parameter is not of much importance in the construction of the synthesized beam. The baseline parameters may be regarded as a formalized interpolation formula, so that what matters is the accuracy of interpolation of the absolute phase, rather than the values of the parameters themselves. As long as calibrations have been done all around the edges of the region in which we are working, the accuracy of interpolation is going to be high.

In addition to this calibration on point sources, it may be possible also to do a partial calibration on small, bright sources at the same time as the synthesis of their brightness distributions is being carried out. In all of the designs considered, there are some locations in the u, v plane where two or more interferometer pairs come close together at various times during the synthesis. If these points are sufficiently close (the necessary closeness depends on the source size and intensity of the signal), and the signals are sufficiently intense in this region of the u, v plane that the phases are well determined from the observations with a single correlator, then equating the phases observed imposes a condition on the calibration of the array. Depending on the size and intensity of the source being investigated, there may be sufficient information available from this procedure to completely calibrate the array phases, or at least to provide internal checks on the phase behavior in the times between the observations of calibrators.

Appendix V

Interferometer On-Line Computer

W. C. Tyler
May 1965

Gentlemen:

The National Radio Astronomy Observatory is interested in acquiring an on-line computer for incorporation into an expanded version of a radio interferometer which is now operating here at Green Bank, West Virginia. The computer will be used both as a special purpose data processing and recording system and as a process control computer. The more detailed functional requirements and specifications will be outlined below.

The purpose of this letter is to solicite your interest in furnishing us with such a computer system and to request specifications and price quotations of items in your standard lines of equipment which you feel will satisfy the requirements set forth. No special development items should be required other than for the normal interfacing problems encountered during installation.

The on-line computer will be an integral part of a 3 or 4 element interferometric array. The array will consist of 85-foot diameter paraboloidal antennas as the basic elements, their associated electronic equipment, and a central control building which will house a variety of electronic equipment in addition to the computer. The antenna elements will be located at distances of up to 1 mile from the central control building and are connected electrically to the control building by cables and radio links.

The computer functions will generally fall into the following categories:

Data Reduction
Systems Control
Systems Calibration and Test
Systems Monitoring
Data Recording

The basic speed, cycle time, memory and word length requirements are set by the Data Reduction function. During normal operation or observing, the computer will be required to carry out the basic Data Reduction and Systems Control functions and record the reduced data on magnetic tape. In addition, a certain amount of system monitoring will be required. At periodic intervals, the normal observing routine will be interrupted for calibration and/or test routines which will be partially controlled by the computer. A small amount of data reduction will be required during this phase of operation, with certain results printed out using a low speed console typewriter-printer.

Normal operation of the array will ideally imply a 24 hour/day, 7 day/week schedule, with one 8 hour period per week for routine maintenance and repair.

An analysis of our present computing requirements for the existing 2 element interferometer, coupled with the design for the expanded system, has resulted in the following list of computer requirements and specifications. These specifications should not be considered restrictive; instead they are guidelines to assist in the adaption of data processing systems you may offer to the problem at hand.

SYSTEM SPECIFICATIONS

A. CENTRAL PROCESSING UNIT (CPU)

1. Word length	15 bits + parity or greater
2. Cycle time	5 µ sec or faster
3. Multiply time	40 µ sec average or faster
4. Memory	8,000 words, preferably expandable to 16,000 words. 16K required if no random access memory is provided.
5. Data rate	200 words per second

ADDITIONAL FEATURES REQUIRED:

1. External interrupt capability (at least 5 levels, preferably 8 or more)
2. Index registers - 2 or more
3. Overlap capability on analog and digital inputs reading, tape output and computing
4. Interval timers - preferably more than 1
5. Hardware multiply and divide
6. An assembly language

DESIRABLE FEATURES (NOT REQUIRED)

1. Four or more overlapped channels (analog inputs, digital input-output, tape units and random access memory)
2. Synchronization without interrupt
3. Storage protection
4. Double precision capability
5. Indirect addressing

FLOATING POINT HARDWARE IS NOT NECESSARY

B. INPUT-OUTPUT HARDWARE REQUIREMENTS

1. <u>Disk-File Random Access Memory</u>
   - Greater than 100,000 words in interchangeable disks
   - Operation should overlap CPU processing
   - Transmission rate not important

2. Magnetic Tape Units
   - Two units required
   - Vacuum column or similar quality
   - 7-track IBM compatible with ability to later convert to 9-track
   - Tape operation must overlap CPU operation
   - 556 BPI density (also quote for 800 BPI)
   - Unit must check what it writes and notify CPU of errors
   - Rapid backspace capability

3. Console Typewriter
   - To provide slow speed output
   - To provide data entry into CPU
   - Must be capable of forcing an interrupt

4. Card Reader
   - Rate equal to or greater than 60 cards per minute
   - Card punch feature not necessary, but separate quote might be included

C. INPUT-OUTPUT LINES

1. Digital Inputs (Voltage)
   - 384 lines (bits)
   - It is desirable to read these in an overlapped mode

2. Digital Inputs (Contact)
   - 140 lines (bits)

3. Digital Outputs (Contact)
   - 244 lines (bits)

4. Digital Outputs (Voltage)
   - 144 lines (bits)

5. Analog Inputs
   - 98 separate input lines required
   - Overlapped with CPU operation and tape write
   - 12 lines will require 13 bit resolution accuracy

We will match any reasonable voltage levels required for the input and output lines. There should exist the possibility for later expansion (up to 30%) for each type of input and output.

If you are interested, we would like to have the following basic information, in addition to any other consideration you might suggest to aid us in the determination of the final computer configuration:

1. Basic data on the central processing unit which you recommend
2. Input/Output equipment recommended
3. Peripheral interface equipment recommended (ADC, multiplexers, etc.)
4. Purchase price (breakdown, including detailed breakdown of I-O, interface equipment)
5. Rental price (detailed breakdown)
6. Monthly maintenance charge (if purchased)
7. Delivery schedule
8. Software available
9. Is each device a stock item?
10. Location of nearest service engineers
11. Type, frequency and duration required of any preventive maintenance

From the information you furnish, we should be able to approximately establish the functional characteristics and costs of various configurations of the computer system .

A response within two weeks would be appreciated. Following that time, meetings with representatives of your company can be arranged, if required. Please address replies to:

W. C. Tyler
National Radio Astronomy Observatory
Post Office Box 2
Green Bank, West Virginia

Sincerely yours,

W. C. Tyler

FUNCTIONAL DESCRIPTION OF

ON-LINE COMPUTER REQUIREMENTS

1. Basic Fringe Reduction — The computer accepts inputs from 12 correlators (4 antennas, 2 frequencies) after approximately 1/10 sec time constant smoothing. The inputs are analog, 13 bits. Analog to digital converters of the digital voltmeter type are employed and the sampling rate is 10 per sec. It is anticipated that the same basic least squares routine (Clark and Wade, April 1965) will be employed for the reduction. Additional inputs required in addition to those from the correlators are:

   a) Automatic level control voltage
   b) Time input
   c) Meteorological data
   d) Frequency
   e) Pre-set data
   f) ø-lock data

   The output data from the fringe reduction program will be recorded on magnetic tape.

2. Delay Switching — The computer, with the input (card) data, will calculate the number of delays to be inserted in each of the IF channels in order that the relative delays from the antennas be equalized. At periodic fringe intervals, the computer will open and close 1 bit contacts which will accomplish the actual switching of the appropriate delays for each IF channel.

3. Antenna Pointing — From data furnished on the input program cards, the computers will automatically point the antennas to the desired source position. In performing this function, the computer will use the antenna encoder outputs as error signals and will operate a series of 1 bit

contact closures which will set the direction and speed of the slew, scan and track motors for each of the individual antennas.

4. Antenna Corrections to Pointing — The telescope encoders give indicated antenna positions which can differ significantly from the true positions. From data obtained during calibration of these "pointing errors", coefficients for a 5th or 6th order polynominal least squares fit to the correction curves will be stored in computer memory. The computer will then periodically calculate the hour angle and declination corrections to the indicated (encoder) positions and operate the low speed contact enclosures to reduce the error to a value below some pre-set level.

5. Calibration and System Sensitivity Subroutine — Prior to and following observations on a given source, or on some regularly scheduled basis (not during observations), an input control card will instruct the computer to initiate a calibration sequence. Various control functions will be required of the computers:

a) A.L.C. off-on control
b) Noise tube control
c) IF step attenuator control
d) Delay switching control
e) RF swept frequency oscillator control
f) Parametric amplifier control
g) Antenna pointing control

From data accumulated (from IF total power output) during the calibration sequence, the computer will calculate various indicators of system sensitivity:

a) System noise figure

b) Paramp band pass

c) Delay band pass

d) A.L.C. control characteristic

6. Feed Polarization Control — From data furnished on an input program control card, the computer will operate contact closures which activate the rotating feed drive motor. The rotating feed encoder will present to the computer an indicated position which will be used as an error signal.

7. System Monitor — To insure operation of the system at an adequate level of sensitivity and to facilitate location of system malfunctions, a certain amount of monitoring of critical components and sub-systems is required. The most critical areas are:

a) Front-end box

b) Local oscillator distribution

c) Output signal levels and gain control settings

d) IF signal distribution

e) Phase-lock system.

The monitoring functions required for these specific areas will now be detailed.

A. Front-end box

1. Monitoring of the level of the parametric amplifier pump inputs. This function is of importance primarily from the point of view of preventative maintenance in that the slow deterioration of the pump klystron will signal a need for replacement in advance of total failure.

2. Mixer crystal current monitor — will provide data concerning the local oscillator level at the mixer and will qualitatively measure the degree of balance in the IF input stages.
3. Local oscillator level — monitor the LO amplifier output level presented to the multipliers.
4. Phase-lock monitor — will measure the LO reference levels presented to the phase comparator at each telescope.
5. Parametric amplifier cooling units — for cooled paramps only, this monitoring function is primarily used for checking the performance of the cooling units.
6. Front-end equipment box temperature — this monitor is required because of the sensitivity of various front-end electronics components to environmental temperature changes. An example is the frequency dependence of the paramp pump on temperature.

B. Local oscillator distribution

1. Cable pressure transducer — monitoring the cable pressure for both LO and phase-lock cables is important both from the operational and preventive maintenance standpoint.
2. Local oscillator line amplifier — monitor the level of the line amplifier outputs.
3. LO and phase lock line amplifier hut — monitor the temperature of the amplifier enclosures.
4. Master local oscillator — level monitor of the master local oscillator output.

C. Output signal levels and gain control settings

1. A.L.C. error voltage — indicator of system gain stability and noise figure.
2. Total power output — monitor A.L.C. function and operating point.
3. Step attenuator settings — check settings to assure proper operating point for A.L.C. input.
4. Correlator gain settings — since the level of the correlator outputs must be adjusted for some of the stronger sources, the gain settings should be checked against the observing program source cards to assure the proper level into the ADC unit in the computer.

D. IF signal distribution

A technique will be developed for testing the amount of isolation between the individual IF signal lines feeding the correlators. The degree to which this function can be accomplished by computer control is uncertain at this time.

E. Phase-lock system

Once the phase-lock system is installed and its operational characteristics are determined, monitoring requirements will be established and incorporated in the computer controlled monitoring routine.

Warren C. Tyler
May 25, 1965