NATIONAL RADIO ASTRONOMY OBSERVATORY

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VLA COMPUTER MEMORANDUM #104

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# VLA TELESCOPE POINTING ANALYSIS

## I. Introduction

It seems profitable to insistute a slightly more automatic system than is employed at the Green Bank interferometer to analyze pointing corrections. There, pointing corrections are updated on a rather ad hoc basis. The observer, or rather the interferometer friend, evaluates how much pointing data is available, and on this basis decides how many parameters of the fitting formula to fit to the data in hand, preserving the remainder of the parameters at their previous values.

The alt-az version of the fitting process is given below. The pointing correction variables are $\Delta Z$, the zenith distance correction, and $\Delta T = \Delta A \sin Z$, the correction in azimuth in angle units. The Einstein summation convention will be used below.

A matrix is constructed, called the observation matrix, describing the circumstances of the observations.

$$M_{ij} = f_{in} f_{jn} + g_{in} g_{jn}$$

where the $n^{th}$ observation is made at zenith distance $Z_n$ and azimuth $A_n$, and $f_{in}$ and $g_{in}$ describe the dependence of the pointing on the $i^{th}$ parameter:

$$\Delta Z = f_i (Z,A) X_i$$

$$\Delta T = g_i (A,A) X_i$$

and $f_{in}$ is $f_i$ (Z,A) evaluated at $Z_n$, $A_n$. A vector, called the results vector, is also generated from the measured pointing errors $\Delta Z_n$, $\Delta T_n$.

$$R_i = \Delta Z_n f_{in} + \Delta T_n g_{in}$$

We may also accumulate the square error sum, a scalar

$$S = \Delta Z_n \Delta Z_n + \Delta T_n \Delta T_n$$

when the above sums are evaluated, we solve for the least squares estimate of the parameter vector $X_i$ by solving the linear equation set

$$M_{ij} X_i = R_j$$

And we can also estimate the variance V by

$$V = (S - R_i X_i)/(N-M)$$

where N is the number of observations and M the number of parameters. The above discussion will remain the basis of pointing analysis.

An automatic procedure which would efficiently use all of the observations would be to have somebody tell the computer whenever any of the parameters might have changed. That is, when an antenna is moved, the computer could be informed that the two azimuth axis pointing parameters and the azimuth indexing might change. Replacing a feed might change the feed location parameters, etc. The computer would then extend the M matrix by one row and one column, and solve for both the new and old values of the affected parameter, using all data ever taken on the antenna. However, the prospect of having an indefinitely expanding matrix, and the memory for indefinitely old (and therefore indefinitely untrustworthy) data are both rather unattractive features. The following procedure eliminates these annoyances at little cost in statistical efficiency.

First, the infinitely growing matrix is eliminated by solving for the old value of the affected variable once and for all when a change is declared. That is, if a discontinuity is declared in $X_m$, we solve the usual equation set

$$M_{ij} X_i = R_j$$

using all data to that time, and then correct $R_j$ for the discontinued parameter

$$R'_j = R_j - X_m M_{jm}$$

(The summation over m is carried out over only the discontinued parameters.) Then room is made for the new parameters by setting

$$M'_{im} = M'_{mi} = R'_m = 0$$

The infinite memory may be eliminated by multiplying the matrix M and vector R by a constant less than one each time pointing data is added to the system.

The pointing constants derived this way are assumed to apply to an antenna. They will be added to another set of constants assumed to be associated with a station. The station constants will be two tilt errors and an azimuth index point. These parameters will be initially supplied by survey. It will be some years before we have enough data to properly separate antenna and station errors from the observations, and "sufficient unto each day is the evil thereof".

## II. Programming

This approach gives rise to the following preliminary program specifications.

### A. Program PAEST

Locates matrix $M_{ij}$, vector $R_i$ for the antenna requested. Reads observation descriptions, and generates new observation matrix, results vector, and variance.

$$M_{ij} = f_{in} f_{jn} + g_{in} g_{jn}$$

$$R'_i = f_{in} \Delta Z_n' + g_{in} \Delta T_n'$$

where $\Delta Z_n'$, $\Delta T_n'$ are the observations corrected for the station errors,

$$\Delta Z_n' = \Delta Z_n - s_1 \cos A - S_2 \sin A$$

$$\Delta T_n' = \Delta T_n - s_1 \sin A \cos Z - S_2 \cos A \cos Z - S_3$$

$$S = \Delta Z_n' \Delta Z_n' + \Delta T_n' \Delta T_n'$$

The new data are combined with the old

$$M''_{ij} = M_{ij} + M'_{ij}$$

$$R_i'' = R_i + R_i'$$

(M" and R" are passed for the use of other programs)

The number of degrees of freedom, NF, is estimated by counting the number of zeros on the diagonal of M and subtracting the number of zeros on the diagonal of M".

Zeros on the diagonal of M" are replaced by ones, to prevent singularities.

A linear equation solver is used to solve for $X_i$ from the equation set

$$M''_{ij} X_i = R''_j$$

The variance V is calculated from

$$V = (S + X_i X_j M''_{ij} - 2X_i R'_i)/(N-NF)$$

$$RMS = SQRT(V)$$

The program displays $X_i$, RMS, and passes X for use by following routines.

B. PADISP

Locates vector X passed from PAEST, reads observed pointing error, and displays residuals

$$r_Z = \Delta Z_n{}' - f_{in} X_i$$

$$r_A = \Delta T_n{}' - g_{in} X_i$$

C. PADISC

Locates matrix M, vector R for antenna requested.

Replaces zeros on the diagonal of M by ones.

Calls linear equation solver to solve for X

$$M_{ij} X_i = R_j$$

Constructs a matrix $D_{ij}$ which is zero off diagonal and one on the diagonal except in the positions of the specified discontinued variables, where the diagonal element is zero.

Adjusts the results vector R

$$R'_j = D_{ij} R_i - M_{mi} X_m D_{ij} + M_{mi} D_{mk} D_{ij} X_k$$

Clears M

$$M'_{ij} = M_{km} D_{ik} D_{mj}$$

Replaces M,R by M',R' on external storage.

(Only the diagonal of $D_{ij}$ need be actually formed, and it may be more convenient to make it a logical variable than an arithmetic one.)

D. PAFIN

This program locates matrix M", vector R" passed by previous execution of PAEST. Multiplies both by 0.9, and places them on external storage on the location of M,R for this antenna.

Locates X passed from PAEST, passes it to synchronous system for immediate use.

## III. Definitions, Formats, Formulae

### A. Correction formulae

$$\Delta Z' = f_i \; X_i$$

$$\Delta T' = g_i \; X_i$$

$$\Delta Z = \Delta Z' + S_1 \cos A - S_2 \sin A$$

$$\Delta T = \Delta T' + S_1 \sin A \cos Z + S_2 \cos A \cos Z + S_3$$

$$\Delta A = \Delta T/\sin Z$$

Axis tilt in the meridian

$$f_1 = \cos A$$

$$g_1 = -\sin A \cos Z$$

Axis tilt in the prime vertical

$$f_2 = \sin A$$

$$g_2 = \cos A \cos Z$$

Azimuth indexing

$$f_3 = 0$$

$$g_3 = \sin Z$$

Axis colination error

$$f_4 = 0$$

$$g_4 = \cos Z$$

Secondary support structure sag

$$f_5 = \sin Z$$

$$g_5 = 0$$

Elevation indexing/Colination error

$f_6 = 1$

$f_7 = 0$

$g_6 = 0$

$g_7 = 1$

L Band feed location

$f_8 = 1$ if at L band, 0 otherwise

$f_9 = 0$

$g_8 = 0$

$g_9 = 1$ if at L band, 0 otherwise

KU Band feed location
Parameters 10, 11
K Band feed location
Parameters 12, 13
Dicroic system location
Parameters 14, 15
Spare feed locations
Parameters 16-19.

Two points should be noted. First, all feed locations are referenced to the C band feed location. If this feed gets bumped there will be Hell to pay. We are at the mercy of either this feed location, or of the elevation indexing/secondary reflector location. I believe that this feed location is probably the more solid. The alternate formulation requires setting $f_6$ and $g_7$ to zero when not at C band.

Second, contrary to custom, refraction is not included as a free parameter. Refraction will be computed by the synchronous system, using the computed current refractivity.

B. Formats.

An observation description will contain:

1. Antenna number (range 1-28), station number (three digit number - arm, configuration, serial from center)

2. Source name and position information - Name (8 characters), modifier (2 digits), RA, Dec, HA, A(true), Z (true)

3. Either $\Delta A$ or $\Delta Z$, and an indication as to which.

Note that since a single observation contains only $\Delta Z$ or $\Delta T$ but not both, either $g_j$ or $f_j$ must be set equal to zero for the single observation.

All programs will reject data from an antenna not specified in the initial call, and will handle only one antenna at a time.

PADISP will display all the quantities above on a line printer except that $\Delta T$ and $\Delta Z$ will be put in different columns rather than in the same field as is done in the input record.

## IV. 360 Implementation

This program will be very useful early in the life of the VLA; probably before the assynchronous system is operating. It seems reasonable to implement this program on the 360. It should be written in FORTRAN for ease in later conversion. Input will be cards. M and R will be stored on a direct access disk file, but S (Station errors) will be stored on a sequential disk file. The eventual storage required for M and R is 380 numbers per telescope, 42560 bytes in all, seven tracks of 2314 storage. At this small quantity if seems unnecessary to complicate things by storing the matrix in triangular form rather than square.

X will be passed to the synchronous system as cards.