NATIONAL RADIO ASTRONOMY OBSERVATORY
Charlottesville, Virginia

August 9, 1973

VLA COMPUTER MEMORANDUM #105

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EPHERMERIS ROUTINES FOR THE VLA - SPECIFICATION CONSIDERATIONS

I have previously set forth the algorithms appropriate for calculating the local oscillator phases in real time in VLA Computer Memorandum #103. These algorithms assumed the availability of the source positions, their derivatives, the equation of time and its derivative, and the baseline at the time of observation. This memo concerns itself with deriving these quantities for the case of a distant object which is stationary in an inertial coordinate system. Planetary and solar ephermeris routines will not be considered here.

The objective of these routines will be to place the VLA observations on the system of Newcomb's elements with high precision, enabling comparison of observations widely separated in time by, at most, using a simple relationship between Newcomb's coordinate system and a true inertial coordinate system (this will include periodic as well as secular terms, but they will be small).

The goal of the reduction is to place obsevations on Newcomb's system with an internal consistency of 0.002 seconds of arc. This represents a phase error of approximately 4° at 21 km baseline and 2 cm wavelength, and, I believe, places as safely beyond the accuracy of measurement we might achieve, even for relative measurements and periodogram analysis. To achieve this accuracy goal, I propose to include all terms in the periodic expansion of aberration and nutation larger than 0.0002 seconds. The sums of the remaining terms form one limit on the accuracy of the reduction. A second will be given by various systematic terms too small and too complex to be worth calculating. Chief among these are the use of the conventional diurnal aberration formula instead of a proper lagged baseline treatment (equivalent to a $0''.0009$ position error) and the use of a linear interpolation in the diurnal aberration ($0''.0007$ for a 1 hr observation). Also omitted are several terms in the description of latitude and time of the same order, because of their extreme uncertainty.

The treatment given below differs from the conventional, Besselian star constant, treatment in several respects, most conspicuously in the explicit calculation of the earths motion in latitude ($0''.0012$).

The program will first decide the time to which the reduction will be referenced. This will be done by averaging the present time and the scheduled end time (a length of one hour will be used if no end is scheduled). These times will be averaged to get a mean time for the observation, T, in fraction of a day, IAT. The complete time will be expressed in Julian centuries after January 0.5, 1900. That is, for this and the following century, the complete time is, for time T, day D, and year Y.

$$T_x = ((Y-1900) * 365 + \text{INTEGER}((Y-1901)/4) + D + T - 0.5)/36525.$$

I. Source Positions

A. Precession

Source positions will be referred to the mean equator and equinox of a given tropical date. (The difference between the tropical and Bessellian year may be neglected in this context, amounting to less than a second.)

The formulae for the precession constants from the Explanatory Supplement (p. 30) must be modified to convert from tropical centuries to Julian centuries. The epoch T_E is assumed to be in tropical centuries after 1900. Therefore,

$$\Delta T = T_x - T_E - 2.136 \times 10^{-5} T_x - 8.60 \times 10^{-6}$$

In seconds of arc,

$$\zeta_o = (2304.250 + 1.396 T_E) \Delta T + .302(\Delta T)^2 + 0.018 (\Delta T)^3$$

$$z = \zeta_o + 0.791 (\Delta T)^2$$

$$\theta = (2004.682 - 0.853 T_E) \Delta T - 0.426(\Delta T)^2 - 0.042(\Delta T)^3$$

The precession matrix is

$$X_{11} = \cos \zeta_o \cos\theta \cos z - \sin \zeta_o \sin z$$

$$X_{21} = -\sin \zeta_o \cos\theta \cos z - \cos \zeta_o \sin z$$

$$X_{31} = -\sin\theta \cos z$$

$$X_{12} = \cos \zeta_o \cos\theta \sin z + \sin \zeta_o \cos z$$

$$X_{22} = -\sin \zeta_o \cos\theta \sin z + \cos \zeta_o \cos z$$

$$X_{32} = -\sin\theta \sin z$$

$$X_{13} = \cos \zeta_o \sin\theta$$

$$X_{23} = -\sin\ \zeta_o \sin\theta$$

$$X_{33} = \cos\theta$$

The retangular coordinates of the given epoch are, in terms of the given coordinates α_E and δ_E

$$x_1^E = \cos\alpha_E \cos\delta_E$$

$$x_2^E = \sin\alpha_E \cos\delta_E$$

$$x_3^E = \sin\delta_E$$

The mean cordinates at the time of observation are

$$x_j^M = \sum_i X_{ij}\ X_i^E$$

B. Aberation

The correction for aberation is applied in the inertial coordinate system of the mean equator and equinox of the center of the observation. The equations used are those of Newcomb for the longitude and radius of the sun - the heliocentric location of the earth is r, $L_\odot + 180°$. The velocity of the earth-moon barycenter will first be calculated, and then the motion of the solar system barycenter and of the earth about the earth-moon barycenter added.

The equations given by Newcomb for the equation of the center were converted to rectangular form, and differentiated. (This was done, rather than using the rectangular form of the equation of the center, because Newcomb has included in his terms perturbations depending only on earth's mean anomaly.)

A term was then subtracted corresponding to the rotation of the vernal equinox at a rate of 1.281 degrees per century to reduce the rate from mean ecliptic coordinates to an inertial coordinate system (giving the standard aberation constant 20.4960). Newcomb's perturbations were then added (it was necessary to include only three terms each from Venus and Jupiter, one from Mars, and one long period term). For purposes of calculating the velocities due to perturbations, a circular orbit was assumed - the cross terms with the equation of the center are always negligible. The velocities were converted to angles of aberation using the constant of aberation of 20''4960 at unit distance. Secular terms amounting to 0''00005 or less after a century were neglected entirely and the others, which amount to less than 0''0005 after a century, were included with their value for the year 2000. This includes substituting the 2000 AD value of the longitude of perihelion in all terms smaller than 0''015. The result of the calculation is displayed in table I. The true rectangular ecliptic velocities are given by

$$V_1 = \frac{dx}{dt} = \Sigma\ C_i \cos\ (Argc_i + \omega_i\ T_x)$$

$$V_2 = \frac{dy}{dt} = \Sigma\ S_i \cos\ (Args_i + \omega_i\ T_x)$$

$$V_3 = \frac{dz}{dt} = A \cos\ (Argz + \omega_i\ T_x)$$

TABLE I

ω degrees/ Julian Cent	Argc degrees	C 10^{-4} arcsec	Args degrees	S 10^{-4} arcsec	Origen
36000.76953	189.69638	204960	99.69638	204960	☉
71999.819	188.173	3423	98.173	3424	2☉
1.719	11.221	3429	281.221	3429	E
36000.8	14.8	35	197.8	22	☉
107998.9	186.6	64	96.6	64	3 ☉
143997.9	185.1	22	95.1	22	4 ☉
58519.2	133	4	343	3	♀
81037.7	35	6	26	10	3⊕ - 2 ♀
45037.5	337	2	26	2	4⊕ - 3 ♀
69718.9	258	3	242	1	3⊕ - 2 ♂
68965.3	235	10	142	4	2⊕ - ♃
29928.1	257	2	164	2	⊕ - 2 ♃
35980.6	61	3	151	3	⊕ - L
36021.0	138	3	48	3	⊕ + L
3036.3	145.1	86	235.1	86	♃
1223.5	176.6	19	266.6	19	♄
429.9	153.	2	243	2	♅
219.9	355	2	85	2	♆
6071	13	4	103	4	2 ♃
481267.9	180.4	87	270.4	87	☾
958466.7	117	5	207	5	2 ☾
	Argz	A			
483202.0	191	8			☾

The terms in L in Table I are the long period perturbations in longitude induced by the combined action of Venus and Mars. The term labled E reduces true positions to astrometric positions calculated by an older method of computing aberation. The convention of the American ephermeris is followed in performing this reduction in mean coordinates, although the IAU resolution also allows the more logical interpretation of doing it in 1900 coordinates (substituting a frequency of 1.280 degrees/century). The difference is about 0".0026 per century. The velocity from the above calculation may be transformed from ecliptic coordinates to equatorial by multiplication by the rotation matrix M.

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & \sin\varepsilon \\ 0 & -\sin\varepsilon & \cos\varepsilon \end{pmatrix}$$

where $\varepsilon = 23^\circ\!.4458 - 0.0130\ T_x$ in degrees.

$$V_j^e = \sum_i M_{ij}\ V_i$$

The aberated source vector is now given by an antihermetian tensor product of v_i^e with x_i^m. Neglecting higher order terms,

$$x_j^A = (1 - \sum_i v_i^e\, x_i^M - \frac{1}{2} \sum_i (v_i^e)^2 - \frac{1}{4} (\sum_i v_i^e\, v_i^M)^2)(X_j^M + v_j^e)$$

This rotates the source vector through an angle $\sin^{-1} \frac{v}{c}$.

C. Nutation

The formulae for nutation are given in the Explanatory Supplement. It is, however, convenient to re-express them in the format of Table I. This is done in Table II, with main terms in Table IIA and Secular terms in Table IIB and IIC.

$$\Delta\psi = \Sigma\, s_i \sin(Arg_i + \omega_i T_x)$$
$$+ T_x \Sigma\, t_i \sin(Argd_i + \omega_{di} T_x)$$
$$+ T_x^2 \Sigma\, v_i \sin(Arge_i + \omega_{ei} T_x)$$
$$\Delta\varepsilon = \Sigma\, c_i \cos(Arg_i + \omega_i T_x) +$$
$$T_x \Sigma\, d_i \cos(Arg_i^d + \omega_{di} T_x)$$
$$+ T_x^2 \Sigma\, e_i \cos(Arge_i + \omega_{ei} T_x)$$

TABLE II

A. Invaniant Terms

ω	Arg	S	C
1934.1420	100.8188	172327	92100
3868.276	201.638	-2088	-904
10072.1	49.5	45	-24
12006.	150	-10	0
1937.	303	-4	2
8138.	309	-3	2
4067.	53	2	0
72001.539	199.393	-12729	5522
35999.05	358.48	1261	0
108000.59	197.87	-497	216
36002.49	200.92	214	-93
73935.7	300.2	124	-66
63863.5	250.7	45	0
75869.8	41.0	-21	0
71998.	357	16	0
34065.	258	-15	8
144000.	196	-15	7
37933.	99	10	5
65798	352	5	3
37937	302	-5	3
61929	150	4	-2
109935	299	3	-2
31932	305	-3	0
962535.762	180.871	-2037	884
477198.87	296.10	675	0
964469.90	281.69	-342	183
1439734.63	116.97	-261	113
413335.4	45.4	149	0
485336.9	244.8	114	-50
890534.2	341.5	60	0
475264.7	195.3	58	-31
479133.0	36.9	+57	30
1375871.1	226.3	-52	22
1441668.8	217.8	-44	23
1853070.0	162.3	-32	14
954397.7	232.2	28	0
549200.4	135.5	26	-11
1916933.5	53.1	-26	11

TABLE II
(Continued)

ω	Arg	S	C
966404.0	22.5	25	0
487271.	346	19	-10
411401	305	14	-7
415269.	146	13	7
1377805	327	-9	5
377336	47	7	0
998535	179	7	-3
1367733	278	6	0
888600	241	-6	3
926537	182	-6	3
2330269	98	-6	3
1026399	72	6	-2
892468	82	5	3
1855004	263	-5	3
551135	236	5	-3
445267	351	-4	0
854545	343	+4	0
1441200	298	4	0
489205	86	-4	0
1918867	154	-4	2
1443603	319	3	
513198	295	-3	
1403736	118	-3	
956332	333	2	
403263	356	2	
952464	131	2	
1339872	228	-2	
1817071	164	-2	
473331	94	-2	
1475734	115	2	
2394132	349	-2	

TABLE II
(Continued)

B. Secular Terms

ω	Argd	t	d
1934.1	100.8	173.7	9.1
72002	199	-1.3	-2.9
35999	358	-3.1	0
108001.	198	1.2	-0.6

C. Secular Terms in T^2

ω	Arge	u	e
1934	191	-6.2	3.3

The nutution in longitude, $\Delta\psi$, should be preserved, as it will be used in computing sidereal time. Otherwise, the rotation matrix

$$M = \begin{pmatrix} 1 & -\Delta\psi \cos\varepsilon & -\Delta\psi \sin\varepsilon \\ \Delta\psi\cos\varepsilon & 1 & -\Delta\varepsilon \\ \Delta\psi\sin\varepsilon & \Delta\varepsilon & 1 \end{pmatrix}$$

may be formed and multiplied by the aberated position vector to produce the geocentric apparent source vector

$$X_j^G = \sum_i M_{ij} X_i^A$$

D. Diurnal Aberation

To a reasonable approximation, the sidereal time of observation is

$$T_s = T + 100.002\ T_x - 0.022 \text{ turns.}$$

The array center latitude is 34°04'43".5, elevation 2130 m (6990 ft), giving geocentric latitude and radius

$$\phi' = 33.^\circ88633, \quad r = 6373.61 \text{ Km.}$$

This results in a constant of diurnal aberation 0".2655 and velocity components

$$V_1^d = -0.2655 \sin T_s$$

$$V_2^d = 0.2655 \cos T_s$$

$$V_3^d = 0$$

resulting in the final apparent source vector

$$X_j = (1 - \sum_i V_i^d X_i^G)\ (X_j^G + V_j^d)$$

At this point, we may profitably solve for the source position in explicit terms

$$\sin\delta = x_3$$

$$\cos \delta = (1 - X_3^2)^{1/2}$$

$$\cos \alpha = X_1/\cos \delta$$

$$\sin \alpha = X_2/\cos \delta,$$

preserving the trig functions for later use.

II. The derivatives of the source position

A. Precession

To sufficient accuracy

$$\left(\frac{d\alpha}{dt}\right)_\rho = m + n \sin \alpha \tan \delta$$
$$= .132 + .055 \sin \alpha \tan \delta \text{ seconds of arc/day}$$

$$\left(\frac{d\delta}{dt}\right)_\rho = n \cos \alpha$$
$$= .055 \cos \alpha \quad \text{seconds of arc/day}$$

B. Aberation

All but the first (circular orbit) term above may be omitted.

$$\frac{dV_{x\odot}}{dt} \simeq .352 \cos L_m$$

$$\frac{dV_{y\odot}}{dt} \simeq .352 \sin L_m$$

When the solar mean longitude

$$L_m = 279.7 + 36000.8\, T_x.$$

Due to aberation, therefore

$$\left(\frac{d\delta}{dt}\right)_A = .352 \cos L_m \cos \alpha \sin \delta + .323 \sin L_m \sin \alpha \sin \delta$$
$$+ .140 \sin L_m \cos \delta$$

and

$$\cos \delta \left(\frac{d\alpha}{dt}\right)_A = .323 \cos \alpha \sin L_m - .352 \sin \alpha \cos L_m$$

C. Nutation

The derivative of $\Delta\psi$ and $\Delta\varepsilon$ can conveniently be computed at the same time as the quantities themselves:

$$\frac{d\Delta\psi}{dt} = \Sigma\ \omega_i\ s_i \cos\ (\text{Arg}_i + \omega_i\ T_x)\ \cdot 4.78 \times 10^{-7}$$

$$\frac{d\Delta\varepsilon}{dt} = -\Sigma\ \omega_i\ c_i \sin\ (\text{Arg}_i + \omega_i\ T_x)\ \cdot 4.78 \times 10^{-7}$$

where the derivatives are in 10^{-4} arcsec/day, s and c in 10^{-4} arcsec, and ω in degrees per century.

The derivatives of position due to nutation are

$$\cos\delta\ \left(\frac{d\alpha}{dt}\right)_N = (\cos\delta\cos\varepsilon + \sin\varepsilon\sin\alpha\sin\delta)\frac{d\Delta\phi}{dt} - \cos\alpha\sin\delta\ \frac{d\Delta\varepsilon}{dt}$$

$$\left(\frac{d\delta}{dt}\right)_N = \sin\varepsilon\cos\alpha\ \frac{d\Delta\psi}{dt} + \sin\alpha\ \frac{d\Delta\varepsilon}{dt}$$

D. Diurnal aberation

$$\frac{dv_1}{dt} = -\ 1.673\cos T_s$$

$$\frac{dv_2}{dt} = 1.673\sin T_s$$

whence

$$\cos\delta\ \left(\frac{d\alpha}{dt}\right)_{DA} = 1.673\ (\cos\alpha\sin T_s - \sin\alpha\cos T_s)$$

$$\left(\frac{d\delta}{dt}\right)_{DA} = -\ 1.673\sin\delta\ (\cos\alpha\cos T_s + \sin\alpha\sin T_s)$$

E. Source vector extropolated back to midnight.

The quantities desired by the observing program (see VLA memorandum #103 for definitions) are

$$DDEC = (\frac{d\delta}{dt})_P + (\frac{d\delta}{dt})_A + (\frac{d\delta}{dt})_N + (\frac{d\delta}{dt})_{DA}$$

$$DRA = (\frac{d\alpha}{dt})_P + (\frac{d\alpha}{dt})_A + (\frac{d\alpha}{dt})_N + (\frac{d\alpha}{dt})_{DA}$$

$$CRA = \cos\alpha + DRA * T * \sin\alpha$$

$$SRA = \sin\alpha - DRA * T * \cos\alpha$$

$$CDEC = \cos\delta + DDEC * T * \sin\delta$$

$$SDEC = \sin\delta - DDEC * T * \cos\delta$$

III. Time

The.fundamental time of the array is International Atomic Time. There will be manually entered the relationship between IAT and UT2. That is, these will be entered an epoch, T_t, and constants

$$c_o = UT2(T_t) - IAT(T_t)$$

and $$c_1 = \frac{dUTZ}{dIAT}\Big|_{t_t} -1$$

Then we shall assume

$$UT2 = IAT + C_o + (T_x - T_t)\, C_1$$

and

$$UT1 = UT2 - .0220 \sin T_A + .0120 \cos T_A$$
$$+ .0060 \sin 2\,T_A - .0070 \cos 2\,T_A$$
$$- .00065 \sin(2F + 2\Omega) - .00027 \sin(2F + \Omega)$$
$$- .00066 \sin \ell$$

Where the units of the coefficients are seconds of time. T_A is (as before) the fractional part of T_X * 100.

The first four terms in this formula are the conventional seasonal terms in UT2-UT0. The last three are caused by tidal effects on the earth's angular momentum, and are here placed between UT2 and UT1, despite the BIH viewpoint that they should be viewed as a fluctuation of UT2, i.e. between UT2 and UTC.

$$F = 11^\circ\!. + 483202^\circ \, T_x$$

$$\Omega = 259^\circ - 1934^\circ \, T_x$$

$$\ell = 296^\circ + 477199^\circ \, T_x$$

Greenwich mean sidereal time at time T is

$$GMST = UT1 + 8640184.5420 \, (T_x + C_o) + .0929 \, T_x^{\,2} + 6^h 38^m 45\overset{s}{.}8360$$

And the local mean sidereal time at the VLA site center reference meridian of 107°37'03".8 W

$$LMST = UT1 + 8640.84.5420 \, (T_x - C_o) + .0929 \, T_x^{\,2}$$
$$- 0^h 31^m 42\overset{s}{.}4173$$

$$LAST = LMST + \Delta\psi \cos \varepsilon$$

The rate of change of sidereal time is

$$\frac{d}{dt} LAST = \frac{d}{dt} LMST + \cos \varepsilon \, \frac{d\Delta\psi}{dt}$$

This last term is called DEEQ in VLA Computer Memo #103. The quantity DUT1 is, sufficiently closely, C_1.

Extrapolating the quantities back to midnight, as desired by the later programs, yields the desired quantities

$$EEQ = -\cos \varepsilon \Delta\psi - DEEQ * T$$

$$OUT1 = UT1 - T - T * DUT1$$

$$LSTM = LMST - T * 1.002737811908$$

IV. Baselines

A. Polar motion

The manual inputs to the computer are the equatorial rectangular components of the locations of the antennas relative to the conventional international origen. (The X axis points to longitude 107°37'03".8). These shall be B_i^f.

At some epoch T_p the coordinates of the pole and its derivatives will be entered - x_{po}, y_{po}, dx_p, dy_p. Then

$$x_p = x_{po} + (T_x - T_p)dx_p - .009 \sin (\mathrm{LMST} - 2\ell - 2\Omega + 14°) + .002 \sin (\mathrm{LMST} + 2\ell + 2\Omega)$$

$$y_p = y_{po} + (T_x - T)\, dy_p - .009 \cos (\mathrm{LMST} - 2\ell - 2\Omega + 14°) + .002 \cos (\mathrm{LMST} + 2\ell + 2\Omega)$$

where the last two terms are Federov's evaluation of the diurnal nutation of the axis of figure about the axis of rotation. (Note that this corresponds to a radius of the opening of the nutation cone of about 27 cm at the pole.) This motion of the pole corresponds to a rotation matrix, in the local equatorial coordinate system, of

$$X = \begin{pmatrix} 1 & -\eta y & x\cos\lambda + y\sin\lambda \\ \eta y & 1 & -x\sin\lambda + y\cos\lambda \\ -x\cos\lambda - y\sin & x\sin\lambda - y\cos\lambda & 1 \end{pmatrix}$$

where λ is the array longitude, and

$$\eta = \sec \phi_{\mathrm{Greenwich}} - 1 = 0.6056$$

which arises because of the definition of UT1 as the hour angle of the mean sum at Greenwich rather than at the prime meridian on the equator, as would be more logical.

B. Earth tides

The displacement of a point on the earth at latitude ϕ, longitude λ due to the tidal action of a body at location (α, δ) is

$$r = R_\oplus D \{(\frac{1}{2} - \frac{3}{2}\sin^2\phi)(\frac{2}{3} - 2\sin^2\delta) \quad \text{long period}$$
$$+ \sin 2\phi \cos 2\delta \cos(t_s - \alpha + \lambda_c - \lambda) \quad \text{diurnal}$$
$$+ \cos^2\phi \cos^2\delta \cos 2(t_s - \alpha + \lambda_c - \lambda)\} \quad \text{semidiurnal}$$

where t_s is the sidereal time at the array center longitude, λ_c. The constant $R_\oplus D$ is a constant depending on the elastic properties of the solid earth, and is about $\frac{2}{3}$ the value of a perfect fluid. $R_\oplus D$ is about 23 cm for the moon and 10 cm for the sun, yielding $D_☾ = 0\rlap{.}''0075$, $D_\odot = 0\rlap{.}''0034$.

$$\Delta\phi = D\{-\frac{3}{4}\sin 2\phi\,(\frac{2}{3} - 2\sin^2\delta)$$
$$+ \cos 2\phi \sin 2\delta \cos(t_s - \alpha + \lambda_c - \lambda)$$
$$- \frac{1}{2}\sin 2\phi \cos^2\delta \cos 2(t_s - \alpha + \lambda_c - \lambda)\}$$
$$\cos\phi\,\Delta\lambda = D\{\sin\phi \sin 2\delta \sin(t_s - \alpha + \lambda_c - \lambda)$$
$$+ \cos\phi \cos^2\delta \sin 2(t_s - \alpha + \lambda_c - \lambda)\}$$

If we expand these expressions about the array center, we may calculate the effects of these distortions of the solid earth on the baselines. These expressions are simplified if we assume the array is level in the final step and approximate

$$R_\oplus(\phi - \phi_c)\cos\phi_c \simeq B_z$$

$$R_\oplus(\phi - \phi_c)\sin\phi_c \simeq B_x$$

$$R_\oplus(\lambda - \lambda_c)\cos\phi_c \simeq B_y$$

The resulting expressions (neglecting time invariant terms) are

$$\Delta B_x = D \{- \frac{3}{2} B_x \sin^2 \delta$$
$$+ [\cos 2 \phi \cos 2 \delta B_z + 2 \sin 2 \phi \sin 2 \delta B_x] \cos(t_s-\alpha)$$
$$+ [\sin 2 \phi \cos 2 \delta + \sin \phi (2 \cos \phi - \sec \phi) \sin 2\delta] B_y \sin(t_s-\alpha)$$
$$- \cos^2 \delta B_x \cos 2 (t_s-\alpha) + 2 \cos^2 \delta B_y \sin 2 (t_s-\alpha)\}$$
$$\Delta B_y = D \{\sin 2 \delta \sin (t_s-\alpha) B_z$$
$$- \tan \phi \sin 2 \delta \cos (t_s-\alpha) B_y$$
$$+ \cos^2 \delta \sin 2 (t_s-\alpha) B_x$$
$$+ 2 \cos^2 \delta \cos 2 (t_s-\alpha) B_y\}$$
$$\Delta B_z = D \{2 \sin^2 \delta B_z$$
$$-2 [\cos 2 \phi \cos 2 \delta B_x + \sin 2 \phi \sin 2 \delta B_z] \cos (t_s-\alpha)$$
$$+ [2 \sin^2 \phi \cos 2 \delta + \cos 2 \phi \sin 2 \delta] B_y \sin (t_s-\alpha)$$
$$+ B_z \cos^2 \delta \sin 2 (t_s-\alpha)\}$$

The positions of the bodies are given, to sufficient accuracy, by

Moon

$$\alpha = 270^\circ\!.4 + 481267.9 \, T_x$$

$$\delta = 5^\circ\!.1 \sin (11^\circ + 483202^\circ\!. \, T_x) + 23^\circ\!.4 \sin \alpha$$

Sun

$$\alpha = 280^\circ\!.1 + 36000^\circ\!.8 \, T_x$$

$$\delta = 23^\circ\!.4 \sin \alpha$$

V. Refraction

The current refractivity is approximately given by

$$N = n-1 = 7.76 \times 10^{-5} (P/T_s) + .373 \, T_s^{-2} \exp \begin{cases} (T_D+22.8)/13.10 & T_D < 10^\circ \text{ C} \\ (T_D+33.5)/17.34 & T_D > 10^\circ \text{ C} \end{cases}$$

Where P is the surface pressure in milibars, T_s is the surface temperature in Kelvins, and T_D is the dew point temperature in degrees centigrade.

The atmospheric phase path is approximately given by

$$\frac{S}{R_\oplus} = \frac{1}{R_\oplus} \int_o^\infty N\, dh \simeq 3.65 \times 10^{-8}\ P + 1.17 \times 10^{-4}\ T_s^{-2}$$

assuming an isothermal atmosphere and a watervapor scale height of 2.0 km.

Each antenna height is given by

$$H = B_x \cos \phi' + 0.99329\ B_z \sin \phi' + \frac{1}{2R_\oplus} [B_x^2 \sin^2 \phi' + B_y^2 + B_z^2 \cos^2 \phi']$$

Where ϕ' is the geocentic latitude of array center, 33°886. The antenna pointing term D5 is given (in radians) by

$$D5 = N + \frac{S}{R_\oplus}$$

VI. Disclaimer

Many of the above coefficients are based on a single, unchecked calculation. The formulae will be carefully checked in the coming months, and this memo will be issued in a second edition before VLA operation. I fully expect that several errors in signs arguments or coefficients will be discovered.

VII. References

For Section I.A. Explanatory Supplement (ES), p. 30, 31.

For Section I.B. Newcomb, APAE, VI, 1898 and Porter and Sadler, M.N. 110, 1950.

For Section I.C. ES p.44, 45.

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NATIONAL RADIO ASTRONOMY OBSERVATORY

Charlottesville, Virginia

October 18, 1973

VLA COMPUTER MEMORANDUM #105 ADDENDUM

B. G. Clark

VLA TELESCOPE POINTING ANALYSIS

The VLA Computer Memorandum's Table I is 1) Seriously in error, and 2) Not a very good way of organizing things.

Instead, let us take

$$V_1 = \frac{dx}{dt} = \Sigma\, C_i \cos\,(\omega_i\, T_x + arg_i)$$

$$V_2 = \frac{dy}{dt} = \Sigma\, C_i \sin\,(\omega_i\, T_x + arg_i)$$

$$V_3 = \frac{dz}{dt} = 0\overset{''}{.}0008 \cos\,(483202\overset{\circ}{.}0 * T_x + 191^\circ)$$

Where ω_i, arg_i, and C_i are taken from the attached Table I.

TABLE I

ω degrees/Julias century	Arg degrees	C 10^{-4} arc sec	Origen
36000.76953	189.69638	204901	☉
71999.819	188.173	3424	2 ☉
1.719	11.221	3425	E
-35997.	13	4	☉
107998.9	186.6	64	3☉
58519.	73	3	♀
81038.	136	3	2 ♀ - ⊕
45038.	346	2	4⊕ - 2 ♀
68965.	232	4	2⊕ - ♃
-29928.	286	2	⊕ - 2 ♃
35981.	138	3	⊕ - L
36021.	61	3	⊕ + L
3036.3	145.1	86	♃
1224.	177	19	♄
430.	153	2	♅
220	355	2	♆
6071	13	4	2 ♃
481267.9	180.4	87	☾
958466.7	117	5	2 ☾