NATIONAL RADIO ASTRONOMY OBSERVATORY
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VLA COMPUTER MEMORANDUM #112

GEOMETRY ROUTINES - GEOM10, GEOMLR, GEOMA, GEOMDL
Preliminary Specification

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I. INTRODUCTION

Preliminary discussions of the VLA geometry calculations are given in VLA Computer Memorandum #103. That memorandum discusses the computation of the quantities necessary to drive the various array devices from elementary quantities and derivatives, whose computation is discussed in VLA Computer Memorandum #105. The nomenclature of the tasks is given in VLA Computer Memorandum #106, and the allocation of storage to the relevant variables is given in VLA Computer Memorandum #108.

This memorandum covers essentially the same ground as VLA Computer Memorandum #103, and, essentially, replaces it. It is hoped that this document is the current explanation of the array geometry and thus, for most purposes, supercedes Memo #103. For historical reasons, though, I shall here summarize the changes from the considerations of Memo #103. First, and most important, it has been decided to do many of the calculations in floating point. Therefore this memo must concern itself with the conversions between fixed and floating numbers. Second, the actual storage allocations for the quantities in question have been made, so that we can discuss the calculations in rather more specific terms than were used in Memo #103. Third, to make the coordinate system used here consistent with the conventions used in the discussions of the asynchronous system, and to make the coordinates right handed, the sign of the baseline parameter BY has been reversed. Fourth, an error has been corrected, in that, for an az-el telescope, the axis intersection defect term is not a constant of the source (as it is for equatorial mountings) but varies with elevation, as pointed out by C. Wade in VLA Test Memo #104. Finally, the length of a major cycle of the array has been changed from 9.6 seconds to 10 seconds exactly.

The real substance of the geometry handling routines lies in the task GEOM10, which is activated every 10 seconds. The remaining three tasks, GEOMLR, GEOMA, and GEOMDL, may be regarded as device handlers for running the lobe rotators, antennas, and delay lines respectively. They are activated at intervals of 1.25 seconds, 104-1/6 miliseconds, and 52-1/12 miliseconds respectively.

II. GEOM10 SPECIFICATION

GEOM10 acts only to change values in control blocks. It changes, in order, the Array Control Block, the Subarray Control Block, the IF Group Control Block (this is concerned only with synthesizer control for line observations, and will not be specified at this time), the Antenna Control Block and the IF Control Block.

1. GEOM10-ARACB

The time-of-day, last 10 second interupt, is left (by the clock routine) in words 46 and 47. This is incremented by 192 (to look ahead to next 10 seconds), floated, and multiplied by $2\pi/(8640*192)$, to make a time T in IAT radians, which is stored in words 46-48. This time is multiplied by 1.002737811908 to convert it to a mean sidereal interval, and by 1 + d(EEQ)/dt (words 60 and 61) to convert it to an apparent sidereal interval. Note that here we assume that the derivative of the equation of the equinox also contains the derivative of UT1 with respect to IAT. The result is added to the apparent LST of midnight IAT (words 62-64) to get the current apparent sidereal time, which is stored in words 49-51. The cosine and sine of this sidereal time are stored in words 40-45.

2. GEOM10--SCB

The trig functions of the source position are updated by the formulae below. In these formulae, the numbers in parenthesis give the words of the SCB involved.

$$\cos(dec)\ (92\text{-}94) = \cos(dec_o)\ (71\text{-}73) - \sin(dec_o)\ (74,75)$$
$$*d(dec)/dt\ (85,86)*T(ARACB\ 46,47)$$

$$\sin(dec)\ (95\text{-}97) = \sin(dec_o)\ (74\text{-}76) + \cos(dec_o)\ (71,72)$$
$$*d(dec)/dt\ (85,86)*T(ARACB\ 46,47)$$

$$\cos(RA)\ (98\text{-}100) = \cos(RA_o)(77\text{-}79) - \sin(RA_o)\ (80,81)$$
$$*d(RA)/dt\ (83,84)*T(ARACB\ 46,47)$$

$$\sin(RA)\ (101\text{-}103) = \sin(RA_o)\ (80\text{-}82) + \cos(RA_o)\ (77,78)$$
$$*d(RA)/dt\ (83,84)*T(ARACB\ 46,47)$$

cos(HA) (104-106) = cos(ST) (ARACB 52-54) * cos(RA)
+ sin(ST) (ARACB 55-57) * sin(RA)

sin(HA) (107-109) = sin(ST) (ARACB 55-57) * cos(RA)
- cos(ST) (ARACB 52-54) * sin(RA)

All of the above calculations are done in floating point.

Also in the Subarray Control Block are the az el coordinates of the source (in this memo, the elevation h will be used instead of the zenith distance used in memo's #103 and #108).

sin(h) (110,111) = 0.5603318 * sin(dec) (95,96)
+ 0.8282682 * cos(dec) (92,93)*cos(HA) (104,105)

cos(h) (112,113) = SQRT(1.0-sin(h)**2)

cos(A) = (sin(dec) (95,96) - 015603318 * sin(h))
/ (0.8282682*cos(h))

sin(A) = cos(dec) (92,93) * sin(HA) (107,108) / cos(h)

cos(η) = 0.5603318 * cos(h) - 0.8282682 * sin(h)*cos(A)

sin(η) = sin(A) * 0.8282682

The layout of the area for these trig functions was not optimally set out in memo #108. Instead, we shall lay it out (within the two areas 53-68 and 110-125 as follows. First, sin(h) and cos(h) floating short (2 words each) for use of phase calculations. Then, cos(h), sin(h), cos(A), sin(A), cos(η), and sin(η) single precision fixed scaled B+0 for use of the pointing correction routines et al. Finally, the double precision angles themselves in words 65-68 and 122-125 as shown. The words 63, 64 and 120, 121 are unused.

Therefore, the trig functions calculated above are used to calculate the actual angles (via ATAN2), h and A, in floating point, and are then fixed (scaled B+0) and stored in locations 114,115,116, 117,118 and 119 for sin(h), cos(h), cos(A), sin(A), cos (η), and sin (η) respectively. The angles themselves (h and A) are converted to double precision fixed point scaled S+1 and stored in locations 122,123 and 124,125 respectively. It is expected that the truncation errors in the calculation due to the single precision floating point will be less than 2" for all cases. The output scaling is S+1 rather than the conventional S+0 because the azimuth coordinate sent to the antenna may exceed 360°.

3. GEOM10 -- IFGCB (Specification omitted - spectral line only).

4. GEOM10--ACB

We start by calculating, in extended precision floating point, the intermediate quantities, the pure geometric delay and, separately, its equatorial component.

$$D_{eq} = BX\ (22\text{-}24) * \cos(HA)\ (SCB\ 107\text{-}109) - BY(25\text{-}27) * \sin(HA)\ (SCB\ 110\text{-}112)$$

$$D_g = D_{eq} * \cos(dec)\ (SCB\ 92\text{-}94) + BZ\ (28\text{-}30) * \sin(dec)\ (SCB\ 95\text{-}97) + BA\ (31,32) * \cos(h)\ (SCB\ 112,113)$$

This interferometer geometric delay is converted to fixed point nanoseconds (scaled B+19) and stored in word 49 (not 3's of nanoseconds as erroneously stated in Memo #108). The equatorial delay, D_{eq} is numerically equal to the second derivative of delay, and is stored in words 53 and 54. The additional delay path introduced by the atmosphere is given by

$$D_a(38,39) = N\ (ARACB\ 67,68)\ H\ (36,37)/\sin h\ (SCB\ 110,111) + S/R_\theta\ (ARACB\ 69,70)*G/\sin h\ (SCB\ 110,111)\ **2$$

The total delay (including cables, etc.) is given by

$$D_t\ (55\text{-}57) = D_g + BC\ (33\text{-}35) + D_a$$

This delay is also divided by ten (to put it in terms of IF sample times), fixed, and stored in locations 58 and 59 for the use of the delay line setting routines.

Note that in Memo #108 this quantity was erroneously assigned a single location. Two are necessary. To make room for this extra word, the previous 10 second delay is stored in words 20 and 21 and the current delay in words 58 and 59. The delay change since the interupt is stored in word 60, and the addresses of the antenna pointing constants are incremented by two. To make room for this second word in the IF Control Block, the addresses of all the quantities after word 6 are incremented by one.

The partial derivatives of the phase are calculated by the formulae below. The calculation for u is carried out in extended precision, because the accuracy is necessary to run the lobe rotator, but the calculation for v may be done in single precision.

$$u = BX(22\text{-}24)*\sin(HA)\ (SCB\ 107\text{-}109) + BY\ (25\text{-}27) * \cos(HA)\ (SCB\ 104\text{-}106)$$

$$v = -D_{eq} * \sin(dec)\ (SCB\ 95,96) + BZ\ (28,29)*\cos(dec)\ (SCB\ 92,93)$$

Both u and v are fixed, scaled B+19, and stored in words 47 and 48 respectively, from whence they shall be picked up and inserted in the output record. Note that this scaling permits u and v to go to 100 miles in steps of 16 feet. Is this quantization interval too coarse? If so, we should change these to floating point, as I hate to sacrifice the 100 mile coverage.

The floating point version of u, words 50-52, is used for setting the lobe rotator and for predicting the phase within 10 second intervals. Rather than being u proper, it is u cos δ (SCB 92-94) times 1 plus $d\alpha/dt$ (SCB 83,84) + v $d\delta/dt$ (SCB 85,86).

The antenna pointing admits of two periodic errors and individual tilt errors in the two directions. In addition, in azimuth there is a colimation error and the axis perpendicularity defect. The calculations for these effects is carried out in fixed point single precision, in order to save the core required to store the constants in floating point. The simple encoder offset is a fixed point double precision number, and contains, as well as the true encoder offset, the decision about which turn of the cable wrap to use and whether to plunge the elevation. In the formulae below, the trig functions come from the SCB and the constants from the ACB.

$$
\begin{aligned}
\Delta A \cos h &= A_1\ (61)\ \cos A\ (114)\ \sin h\ (112) \\
&+ A_2\ (62)\ \sin A\ (115)\ \sin h\ (112) \\
&+ A_3\ (63)\ \cos A\ (114)\ \cos h\ (113) \\
&+ A_4\ (64)\ \sin A\ (114)\ \cos h\ (113) \\
&+ A_5\ (65)\ \sin h\ (112) + A_6\ (64) \\
\Delta h &= D_1\ (69)\ \sin A\ (115) + D_2\ (70)\ \cos A\ (114) \\
&+ D_3\ (71)\ \cos h\ (113) + D_4\ (72)\ \sin h\ (112) \\
A\ (40,41) &= A(\text{SCB } 124,125) + (\Delta A \cos h)/\cos h\ (113) \\
&+ A_7\ (67,68) \\
h\ (42,43) &= h(\text{SCB } 122,123) + \Delta h + (N-1)\ (\text{ARACB } 65) \\
&* \cos h\ (113)/\sin h\ (112) + D_5\ (73,74)
\end{aligned}
$$

5. GEOM10--IFCB

At the time Memo #108 was written, it seemed appropriate to correct for the IF intrinsic delay after the delay has been converted to fixed point. It is now apparent that it is much more convenient to correct for it in floating point in the phase equation. Therefore, the floating point peculiar delay will be appended to the IFCB in locations 24, and 25, as well as included as the fixed point value in word 2.

The last four bits of the IF ID word (word 0) will be a hexidecimal number between 0 and 3 indicating which of the four possible LO frequencies the IF in connected to. These are used to index the four LO frequencies in the SCB, words 41-52. This frequency is used to multiply the sum of the antenna total delay (ACB 55-57) and the IF peculiar delay (24,25), giving the phase to be set. This is converted to fixed point, the integer portion is discarded, the IF peculiar phase, word 1, is added to it, and the result is stored in word 17. The fringe rate is computed by multiplying the floating variable u cos δ (ACB 50-52) by the same frequency, and converting to fixed point. This fringe frequency is stored in words 18 and 19. The quadratic term, word 20, is given by multiplying the second derivative of delay, ACB words 53 and 54, by the same frequency and converting to fixed point.

III. GEOMLR SPECIFICATION

GEOMLR is activated every second and a quarter, to provide lobe rotator phases and rates, from the starting values which GEOM10 has left in the IFCB. These values are, in turn, merely left in the ACB for CMD to transmit to the antenna.

If this is the n'th call to GEOMLR since the even 10 seconds, n-1,2,3,4,5,6,7,8. The mean rate during the interval will be the starting rate (words 9 and 10) plus (n+1/2) times the rate derivative, word 11. The multiplication may be carried out in single precision.

The appropriate starting phase is the 10 second interval starting phase, word 8, plus n times the rate (words 9 and 10, unfortunately a double precision calculation) plus half of the quadratic term (11) times $(n-1/4)^2$. The term is (n-1/4) to make a best fit to the parabola, rather than to fit the phase exactly at the endpoints. Because of the decimal nature of the lobe rotator (which adjusts the phase of a 100 kHz signal in steps of 20 ns) this binary fraction must be converted to a code consisting of three quinary digits followed by a binary fraction for output to the lobe rotator and storage in location 3.

IV. GEOMA SPECIFICATION

GEOMA updates the antenna positions ten times per second, and leaves the current values in the ACB for transmission to the antennas.

In order to facilitate this calculation, it calculates, and leaves in the SCB the amount of motion since the last 10 second interupt. The motion in azimuth and elevation are given respectively by the time elapsed times the sine and cosine of the paralactic angle, words 63,64 and 61,62. The answers are stored, respectively in words 69 and 70.

Then, in the ACB, these quantities are respectively added to the starting values of azimuth and elevation, words 13,14 and 15,16 and the sums stored in locations 8, 9 and 10, 11.

V. GEOMDL SPECIFICATION

GEOMDL is activated every minor interupt (at 52 ms intervals) and changes words in the ACB and IFCB. Since, in addition, it is resident in the computer which is connected to the delay lines, it outputs these values to the delay lines.

The delay change is given by the product of the time times u (ACB 44) times cos δ (SCB 89). It is left in ACB 60.

The sum of this quantity, the starting delay (ACB 20,21) and the IF peculiar delay (IFCB 2) is output to the delay lines and is stored in IFCB 6 and 7.