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SOLUTIONS FOR ANTENNA GAIN

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Although in the practical case one should solve the equations for antenna gain and phase with the appropriate weights for good signal-to-noise properties, it is nice to have simplified formulae for use for back-of-the envelope calculations and other simple uses. There are explicit inversions of the least-squares matrix available for one simple case, and, since I am always mislaying the piece of paper I have them written on, I would like to insert them into the repository of numbered memos.

The case for which we have a simple explicit solution is that in which we have all correlations present and with equal signal-to-noise ratios rather greater than one. Note that the requirement is for equal signal-to-noise ratio; this development is unsuitable for use with the self-calibration algorithms even if all antennas do have the same sensitivity, because the signal varies from correlator to correlator.

Let c_{ij} be the complex correlation between the i^{th} and j^{th} antennas. Then let

$$a_{ij} = \mathrm{Log}\,(|c_{ij}|)$$
$$(a_{ji} = a_{ij})$$

and $p_{ij} = \mathrm{Argument}\,(c_{ij})$
$(p_{ji} = -p_{ij}$ and

$p_{ii} \equiv 0 \equiv a_{ii})$

We suppose equations of condition, to be solved by least-squares, for A_i and P_i, the antenna amplitude and phase responses respectively.

$$a_{ij} = A_i + A_j$$
$$p_{ij} = P_j - P_i$$

These equations of condition reduce to the equation systems

$$\sum_{j=i}^{N} [1 + (N-2)\, \delta_{ij}]\, A_j = \sum_{j=1}^{N} a_{ij}$$

and

$$\sum_{j=1}^{N} (N\, \delta_{ij} - 1)\, P_j = \sum_{j=1}^{N} p_{ji}$$

where δ_{ij} is the Kronecker delta representation of the unit matrix.

The first of these equations is explicitly solved by

$$A_i = \frac{1}{N-2} \sum_{j=1}^{N} a_{ij} - \frac{1}{(N-1)(N-2)} \sum_{i=1}^{N} \sum_{j=i}^{N} a_{ij}$$

The second matrix is, obviously, singular; if an arbitrary constant is added to any solution, it remains a solution. The general solution of the phase equation is

$$P_i = \frac{1}{N} \sum_{j=1}^{N} p_{ji} + C$$

where C is an arbitrary constant.