NATIONAL RADIO ASTRONOMY OBSERVATORY
SOCORRO, NEW MEXICO
VERY LARGE ARRAY PROGRAM

VLA COMPUTER MEMORANDUM NO. 154

# A NOTE ON POLARIZATION CALIBRATION

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December 1979

Motivated by D'Addario's comment in VLA Scientific Memorandum No. 130 that "the existing software makes some approximations not appropriate for large circular polarization", I would like to point out that, having determined the instrumental polarization by the usual means, a correction valid for strongly polarized sources of arbitrary polarization is easily made. The correction, which includes all ellipticity terms (a similar development would follow for linear feeds), is achieved by solving a 4 x 4 system of linear equations. The correction, of course, is valid only to the extent that the feed responses are indeed elliptical and the beams coaxial.

The responses of the four correlators are (see figure):

$$\tilde{V}_{R_iR_j} = g_{R_i}\,\bar{g}_{R_j}\,[V_{RR} + \varepsilon_{R_i}\,\bar{\varepsilon}_{R_j}\,V_{LL} + \bar{\varepsilon}_{R_j}\,e^{-2iq}\,V_{RL} + \varepsilon_{R_i}\,e^{2iq}\,V_{LR}]$$

$$\tilde{V}_{L_iL_j} = g_{L_i}\,\bar{g}_{L_j}\,[\varepsilon_{L_i}\,\bar{\varepsilon}_{L_j}\,V_{RR} + V_{LL} + \varepsilon_{L_i}\,e^{-2iq}\,V_{RL} + \bar{\varepsilon}_{L_j}\,e^{2iq}\,V_{LR}]$$

$$\tilde{V}_{R_iL_j} = g_{R_i}\,\bar{g}_{L_j}\,[\bar{\varepsilon}_{L_j}\,V_{RR} + \varepsilon_{R_i}\,V_{LL} + e^{-2iq}\,V_{RL} + \varepsilon_{R_i}\,\bar{\varepsilon}_{L_j}\,e^{2iq}\,V_{LR}]$$

$$\tilde{V}_{L_iR_j} = g_{L_i}\,\bar{g}_{R_j}\,[\varepsilon_{L_i}\,V_{RR} + \bar{\varepsilon}_{R_j}\,V_{LL} + \varepsilon_{L_i}\,\bar{\varepsilon}_{R_j}\,e^{-2iq}\,V_{RL} + e^{2iq}\,V_{LR}].$$

$i$ and $j$, as subscripts, are telescope numbers; the $\tilde{V}$'s, the observed fringe visibilities; the $V$'s, the true (unknown) visibilities (for compactness of notation, the $i$, $j$ subscripts have been dropped); the $\varepsilon$'s, the ellipticity terms; $q$, the parallactic angle; and the $g$'s, the system gains (determined, say, by ANTSOL).

The solution of these equations can be written in the matrix form:

$$\begin{pmatrix} V_{RR} \\ V_{LL} \\ e^{-2iq}\, V_{RL} \\ e^{2iq}\, V_{LR} \end{pmatrix} = \begin{pmatrix} 1 & \varepsilon_{R_i}\,\bar{\varepsilon}_{R_j} & \bar{\varepsilon}_{R_j} & \varepsilon_{R_i} \\ \varepsilon_{L_i}\,\bar{\varepsilon}_{L_j} & 1 & \varepsilon_{L_i} & \bar{\varepsilon}_{L_j} \\ \bar{\varepsilon}_{L_j} & \varepsilon_{R_i} & 1 & \varepsilon_{R_i}\,\bar{\varepsilon}_{L_j} \\ \varepsilon_{L_i} & \bar{\varepsilon}_{R_j} & \varepsilon_{L_i}\,\bar{\varepsilon}_{R_j} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \tilde{V}_{RR}/(g_{R_i}\,\bar{g}_{R_j}) \\ \tilde{V}_{LL}/(g_{L_i}\,\bar{g}_{L_j}) \\ \tilde{V}_{RL}/(g_{R_i}\,\bar{g}_{L_j}) \\ \tilde{V}_{LR}/(g_{L_i}\,\bar{g}_{R_j}) \end{pmatrix}$$

Neglecting the expense in computing the matrix inverse (it would need to be done only once per baseline) and neglecting the divisions by system gains and by complex exponentials in the parallactic angle, the proposed correction would require 3 complex additions and 4 complex multiplications per visibility observation. This compares with roughly 1½-2 additions and one multiplication required by the usual correction for RL and LR observations (see VLA Computer Memorandum No. 136):

$$V_{RL} \simeq [\tilde{V}_{RL}/(g_{R_i}\,\bar{g}_{L_j}) - (\varepsilon_{R_i} + \bar{\varepsilon}_{L_j})\,(\tilde{V}_{RR}/(g_{R_i}\,\bar{g}_{R_j}) + \tilde{V}_{LL}/(g_{L_i}\,\bar{g}_{L_j}))/2]\; e^{2iq}$$

$$V_{LR} \simeq [\tilde{V}_{LR}/(g_{L_i}\,\bar{g}_{R_j}) - (\varepsilon_{L_i} + \bar{\varepsilon}_{R_j})\,(\tilde{V}_{RR}/(g_{R_i}\,\bar{g}_{R_j}) + \tilde{V}_{LL}/(g_{L_i}\,\bar{g}_{L_j}))/2]\; e^{-2iq}.$$

Heretofore, no correction for instrumental polarization has been applied to LL and RR observations by the standard reduction software.

<u>Figure</u>

The sampling of the right circular and left circular components of the E-field incident to the earth's atmosphere can be represented by

$$\begin{pmatrix}\tilde{E}_R\\ \tilde{E}_L\end{pmatrix}=\underbrace{\begin{pmatrix}g'_R & 0\\ 0 & g'_L\end{pmatrix}}_{\substack{\text{system}\\ \text{gain}}}\underbrace{\begin{pmatrix}\eta_R & 0\\ 0 & \eta_L\end{pmatrix}}_{\substack{\text{feed}\\ \text{effi-}\\ \text{ciency}}}\underbrace{\begin{pmatrix}1 & \varepsilon_R\\ \varepsilon_L & 1\end{pmatrix}}_{\substack{\text{eccen-}\\ \text{tricity}\\ \text{of feed}\\ \text{response}}}\underbrace{\begin{pmatrix}e^{-iq} & 0\\ 0 & e^{iq}\end{pmatrix}}_{\substack{\text{feed}\\ \text{rotation}}}\underbrace{\begin{pmatrix}g_A & 0\\ 0 & g_A\end{pmatrix}}_{\substack{\text{atmos-}\\ \text{pheric}\\ \text{refraction}\\ \text{and}\\ \text{absorption}}}\underbrace{\begin{pmatrix}E_R\\ E_L\end{pmatrix}}_{\substack{\text{incident}\\ \text{E-field}}}.$$

Normally, we absorb $g_A$ and the $\eta$'s into "system gain," and, in calibration, determine $\varepsilon$'s and time-dependent g's such that

$$\begin{pmatrix}\tilde{E}_R\\ \tilde{E}_L\end{pmatrix}=\begin{pmatrix}g_R & 0\\ 0 & g_L\end{pmatrix}\begin{pmatrix}1 & \varepsilon_R\\ \varepsilon_L & 1\end{pmatrix}\begin{pmatrix}e^{-iq} & 0\\ 0 & e^{iq}\end{pmatrix}\begin{pmatrix}E_R\\ E_L\end{pmatrix}.$$

The four correlations are $\tilde{V}_{R_iR_j}=\tilde{E}_{R_i}\bar{\tilde{E}}_{R_j}$ , $\tilde{V}_{L_iL_j}=\tilde{E}_{L_i}\bar{\tilde{E}}_{L_j}$ , $\tilde{V}_{R_iL_j}=\tilde{E}_{R_i}\bar{\tilde{E}}_{L_j}$ , and $\tilde{V}_{L_iR_j}=\tilde{E}_{L_i}\bar{\tilde{E}}_{R_j}$ .