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The Bandwidth Effect ('Delay Beam') for A Synthesis Array and Related Requirements for the IF Filter Characteristics

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This memorandum examines the degradation of the beam of a synthesis array which occurs in the outer parts of the field and results from the finite bandwidth of the receiving system. The importance of the subject with regard to possible optimisation of the band shape of the IF filters of the VLA was pointed out by R. D. Ekers in a meeting of the VLA Advisory Committee on November 7, 1973.

1. The Delay Beam of a Two Element Interferometer

Consider a single antenna-pair in a synthesis array in which the antennas track a point in the sky which is referred to as the center of the field of view. The compensating delays are continuously adjusted to equalize the time delays of signals from the field center to the multiplier inputs. Following the usual conventions, positions in the synthesized field are measured in the (x,y) coordinate system, the origin of which is the field center. The component of the antenna spacing normal to the direction of the (x,y) origin is measured in the usual (u,v) coordinates. For signals from a point (x_1,y_1) the time delays to the multiplier inputs are not equal but differ by an interval τ given by

$$\tau = (ux_1 + vy_1)\ /f_o \qquad (1)$$

where u & v are measured in wavelengths, x_1 & y_1 in radians and f_o is the center frequency of the receiving system. To determine the response to a point source at $(x_1,\ y_1)$ it is convenient to specify the frequency response characteristics of the two receiving channels as a function of f', the frequency measured relative to the center frequency f_o . Then if A_1 & A_2 are the complex voltage transfer functions the response of the interferometer is the real part of

$$\int_0^{\infty} A_1\,(f-f_o)\; A_2^*\,(f-f_o)\; e^{-i2\pi f\tau}\,df$$

$$= e^{-i2\pi f_o\tau} \int_{-\infty}^{\infty} A_1\,(f')\; A_2^*\,(f')\quad e^{-i2\pi f'\tau}\,df' \qquad (2)$$

The factor $e^{-i2\pi f_o\tau}$ represents the fringes, and their amplitude is seen to be proportional to the Fourier transform of $A_1(f')\ A_2^*\ (f')$. For brevity $A_1\ (f')\ A_2^*\ (f')$ will be replaced by $G(f')$, the cross power spectrum, the Fourier transform of which is $g\ (\tau)$. If, for example, $G\ (f')$ is represented by a rectangular function of width Δf and unit area,

$$G\ (f') = \frac{1}{\Delta f} \sqcap \left(\frac{f'}{\Delta f}\right)$$

where

$$\sqcap\ (\xi) = 1.0\ ,\ |\xi| \leqslant 1/2$$
$$= 0\ ,\ |\xi| > 1/2$$

then

$$g\ (\tau) = \sin\ (\pi\Delta f\tau)/\pi\Delta ft \qquad (3)$$

Since τ is a function of the position (x_1,y_1) of the source the strength of the fringe pattern will vary as though the radiation were being received by antennas with a beam shape corresponding to $g\ (\tau)$ and this effect is therefore often referred to as the delay beam.

The delay beam is a well known characteristic and has been used to limit the reception pattern of long wavelength interferometers (see, for example, Goldstein 1959, Moseley et. al. 1970). The subsiduary maxima of the sinc function act as sidelobes of the delay beam, and if these are deemed undesirable it is clearly necessary to shape the receiving passband to eliminate the sharp frequency cutoffs. The question naturally arises of whether there is an optimum passband shape for the VLA, particularly when the synthesized field is much smaller than the primary beam of the antennas and the bandwidth effect is used to reduce the response to confusing sources outside the field.

2. The Bandwidth Effect for A Synthesis Array

For a given receiving passband and center frequency the width of the delay beam for a pair of antennas is a constant number of fringe widths, but varies in angle inversely as the spacing. With a synthesis array in which measurements are made over an essentially continuous range of antenna spacings the concept of a delay beam as outlined above does not apply. To examine the bandwidth effect for such an array we now calculate the response to a point source as a function of its position (x_1,y_1).

In mapping a point source with an array the constant fringe visibility is modified by S(u,v), the spectral sensitivity of the array, T(u,v), the applied beam-shaping taper, and $g(\tau)$ which represents the bandwidth effect. The time delay τ is given by eqn. (1) and the point source response is therefore the Fourier transform of

$$S(u,v)\ T(u,v)\ g\ [(ux_1+vy_1)/f_o] \qquad (4)$$

It is assumed that G(f') is the same for all antenna pairs. The Fourier transform of S(u,v)T(u,v) is simply the synthesized beam free from bandwidth effects which will be referred to as the undegraded beam, B(x,y). To examine g, let $x_1 = r_1 \sin\theta$ & $y_1 = r_1 \cos\theta$, θ being the position angle of the source as shown in Figure (1).

The function $g\left[\frac{r_1}{f_o}(u \sin\theta + v\cos\theta)\right]$ is shown in Figure (2) and is seen to consist of a series of corrugations running in a direction $\theta + \pi/2$. For a rectangular passband these corrugations have a sinc-function cross section. The Fourier transform of $g\left[\frac{r_1}{f_o}(u \sin\theta + v\cos\theta)\right]$ is shown in Figure (3). In the $\theta+\pi/2$ direction it is a delta function and in the θ direction it is $\frac{f_o}{r_1} G\left[\frac{f_o}{r_1}(x \sin\theta + y\cos\theta)\right]$ (This last result can be obtained by working in terms of the distance of a point from a line through the origin at angle $\theta+\pi/2$ which is $(u \sin\theta + v\cos\theta)$ in the (u,v)-plane and $(x \sin\theta + y\cos\theta)$ in the (x,y)-plane.

The Fourier transform of (3), the point source response, is

$$B(x,y) ** \frac{f_o}{r_1} G\left[\frac{f_o}{r_1}(x \sin\theta + y\cos\theta)\right] \delta\left[x \sin(\theta + \pi/2) + y\cos(\theta + \pi/2)\right] \qquad (5)$$

where the double asterisk represents two dimensional convolution. This shows that in the θ direction, i.e. the radial direction through the origin, the beam is broadened by convolution with the bandpass function G, but in the circumferential direction it is not broadened.* If B(x,y) is symmetrical in the θ direction and if G(f') is symmetrical the peak response is not displaced by the convolution, but high spatial frequencies are attenuated and the peak response is decreased.

To examine the decrease in the peak amplitude consider a source on the x-axis at (x_1,o) and let B(x) be the one dimensional profile of the undistorted beam. The decrease factor is

$$D = \frac{f_o}{x_1}\int_{-\infty}^{\infty} B(x)\, G\left(\frac{f_o x}{x_1}\right) dx \qquad (6)$$

where B and G are normalized so that B(o) = 1.0 and $\int_{-\infty}^{\infty} G(\xi)\, d\xi = 1.0$. As an illustrative example we again use the rectangular function for G and eqn. (6) becomes

$$D = \frac{2f_o}{x_1 \Delta f}\int_{o}^{\frac{x_1\Delta f}{2f_o}} B(x)\, dx \qquad (7)$$

*An alternative derivation of this result (Fomalont & Wright 1973) considers the broad-band response to be the sum of a series of narrow band responses, the angular scales of which vary in proportion to the frequency.

Since D depends upon B(x) it also depends upon the applied beamshaping taper. Two extreme examples of tapering will therefore be considered. In the first a strong Gaussian taper is used, and B(x) is then very closely Gaussian:

$$B(x) = e^{-x^2/2\sigma^2}$$

The half power beamwidth, b, is 2.355σ, so

$$D_1 = \frac{2f_o}{x_1 \Delta f} \int_o^{\frac{x_1 \Delta f}{2f_o}} e^{-(2.355x)^2/2b^2} \, dx$$

$$= \frac{\sqrt{2\pi}}{2.355} \left(\frac{bf_o}{x_1 \Delta f}\right) \text{erf} \left(\frac{2.355}{2\sqrt{2}} \frac{x_1 \Delta f}{bf_o}\right) \qquad (8)$$

Figure 4 shows a curve of D_1 as a function of $\frac{x_1 \Delta f}{bf_o}$. Note that x_1/b is the distance of the response from the field center measured in units of the undistorted beamwidth and $\Delta f/f_o$ is the fractional bandwidth of the receiving system.

At the other extreme consider a case with no applied tapering and suppose that the spectral sensitivity is uniform within a circle of radius of q centered on the (u,v) origin. Then

$$B(x) = \frac{J_1(2\pi q x)}{\pi q x}$$

or in terms of the half power beamwidth

$$B(x) = \frac{J_1(4.42x/b)}{2.21x/b}$$

($2J_1(\xi)/\xi \to 1$ as $\xi \to 0$, and $J_1(2.2124)/1.1062 = 0.5$). So for this case

$$D_2 = \frac{2f_o}{x_1 \Delta f} \int_o^{\frac{x_1 \Delta f}{2f_o}} J_1(4.42x/b)/(2.21x/b)\, dx \qquad (9)$$

Numerical values of D_2 have been computed by S. D. Burgan and are also plotted in Figure 4.

The curves for D_1 and D_2 are remarkably similar, and are not greatly dependent upon the form of the filter response. This last point can be illustrated by using for G(f') a Gaussian of half-power width Δf

$$G(f') = \frac{2.355}{\sqrt{2\pi}\Delta f}\, e^{-\left[\frac{2.355\, f'}{\sqrt{2}\, \Delta f}\right]^2}.$$

Then the decrease in the peak response in the heavily tapered, Gaussian beam case is

$$D3 = \frac{2.355\, f_o}{\sqrt{2\pi}\, \Delta f\, x_1} \int_{-\infty}^{\infty} e^{-\left[\frac{2.355\, x\, f_o}{\sqrt{2}\, \Delta f\, x_1}\right]^2} e^{-\left[\frac{2.355\, x}{\sqrt{2}\, b}\right]^2} dx$$

$$= \sqrt{\frac{1}{1 + \left(\frac{x_1^{\,2} \Delta f}{b^2\, f_o^{\,2}}\right)}} \qquad (10)$$

The curve of D_3 is also plotted in Figure 4.

All three curves in Figure 4 drop to half amplitude for values of $x_1 \Delta f/b\, f_o$ between 1.7 and 2.2. Thus the width of the region of the map out to where the point source response is decreased by 0.5 is approximately 4b $f_o/\Delta f$. It would be misleading however to regard the curves in Figure 4 as defining a beam in the way in which that term is applied to an antenna, since the volume under the response for any source is not affected and thus the total flux density contributed to the map remains the same. The function D is better regarded as a degradation in resolution.

In observations with the VLA in the high resolution configurations it is expected that the synthesized field will often cover only a small central section of the area within the antenna beams because of computing considerations. Sources within the antenna beams but outside the synthesized field will then produce images which are shifted into the map by the replication inherent in Fourier transformation of data in a rectangular array. If the receiving bandwidth is chosen so that the half amplitude width of the degradation function is comparable to the field width, sources external to the field which are not wide compared to the undegraded beam will produce images elongated in the direction of the (x,y) origin. After being shifted into the map the elongation of these images will no longer appear radial, but the possibly spurious nature of any elongated responses can be tested by displacing them integral numbers of field widths in the x and y directions. The bandwidth effect can thus be a useful means of reducing and identifying spurious responses from narrow sources. This facility can possibly be enhanced by comparing maps made with different bandwidths (a genuinely elongated source should look the same) which could be made simultaneously using different IF channels or by switching between two different filters in each IF amplifier.

3. Requirements for the Filter Characteristics

The IF filters which determine the frequency responses are discussed in VLA E.M. #115. Unlike the two-element interferometer case, the use of a rectangular passband with the full array does not result in zeros and sidelobes in the response function, and as Figure 4 shows the filter shape makes very little difference in this respect* The rectangular passband indeed appears desirable since a spurious image replicated into the map will not be smeared over quite so large an area as would occur with a filter with more extended skirts. Since also the rectangular bandpass is best for interference rejection it appears to be the optimum choice. The most satisfactory filters that have been found to date are the B51 series of K&L Microwave Inc. which use an 0.05 dB Chebyshev design. This is about as close to a rectangular passband as can readily be obtained and it is planned to use 4 or 6-section filters of this type.

The center frequencies and bandwidths of individual filters can be expected to vary by a few percent. Variations in the bandwidth will result in fluctuations in that part of the function $g[(ux_1 + vy_1)/f_o]$ shown in Figure 2 in which the amplitude has fallen significantly below unity. This will result in errors of presumably a largely random nature in the map. Such errors are likely to be most troublesome when looking for small differences in the outer regions in two maps of the same area of sky made with different sets of IF channels. This could occur for example when comparing maps made with opposite polarizations or at slightly different frequencies. Attempting to improve the matching of the four filter sets used with any particular antenna would be helpful in this respect, but would limit the interchangability of the IF modules desired for easy servicing. A better solution would be to repeat the observation after having interchanged the IF channels concerned at each antenna, or else to interchange the channels every few seconds whilst making a map. The fastest switching rate that could be used would interchange the channels at the end of each full cycle of the 180° phase-switching sequence at the front ends which take 9.6s. During this time the resolution vector for any antenna pair can rotate up to 7×10^{-4} radians, and to

*One would like the degradation to be small near the field center and then set in rapidly in the outer parts. In this respect curves D_1 and D_2 for the rectangular filter are slightly preferable to D_3.

average out the differences for the two sets of channels the vector for the longest spacing should not cross more than half a cell in the (u,v) plane. The switching would therefore be feasible with array sizes up to about 1500x1500 points.

4. Conclusions

Filters with regular Chebyshev characteristics will be used to determine the IF responses. Tolerances will be specified as tightly as can readily be achieved but no special matching is anticipated.

Terminals will be available on each IF module to allow the use of additional filters with any special characteristics that may be required.

Computer controlled switches to interchange IF channels can be provided if this is likely to be useful. These can most easily be inserted at the 5 GHz outputs of the parametric amplifiers.

The possibility of restoration of the undegraded resolution over all parts of the map has not been discussed. P. J. Napier points out that a version of the Högbom beamcleaning technique modified to take account of the varying beamshape over the field would provide a possible approach. Exact restoration is probably not possible in all cases since if $g(\tau)$ contains zeros some frequency components are entirely lost from the map.

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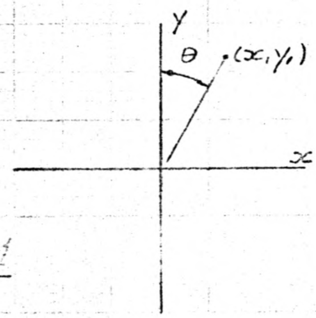

Fig 1

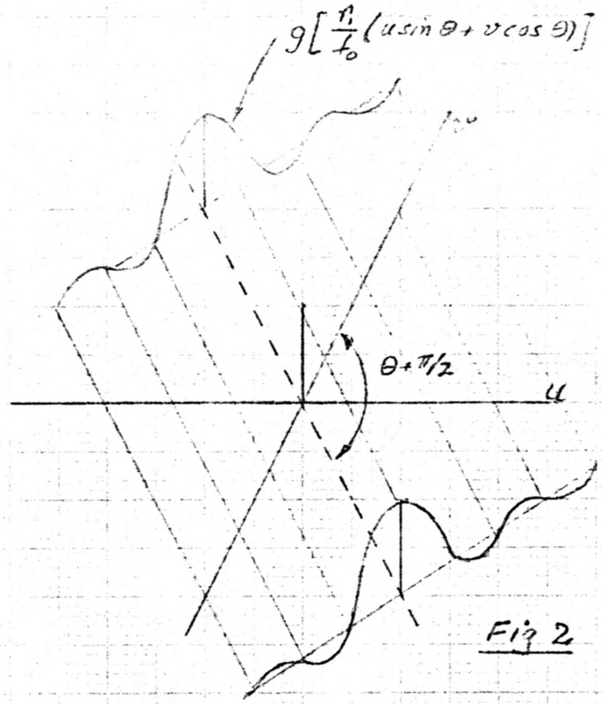

Fig 2

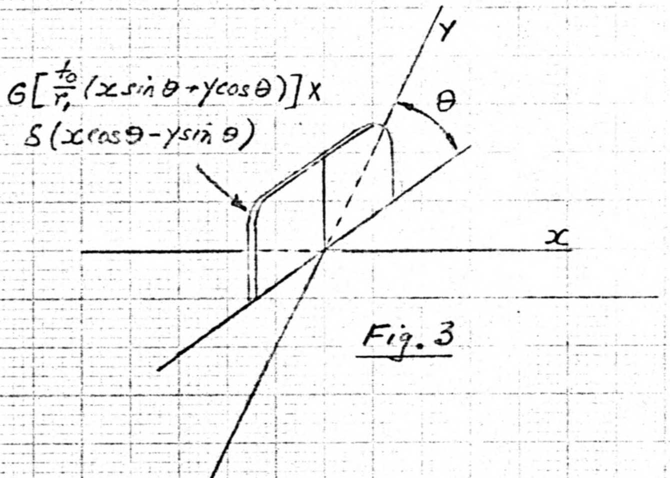

Fig. 3

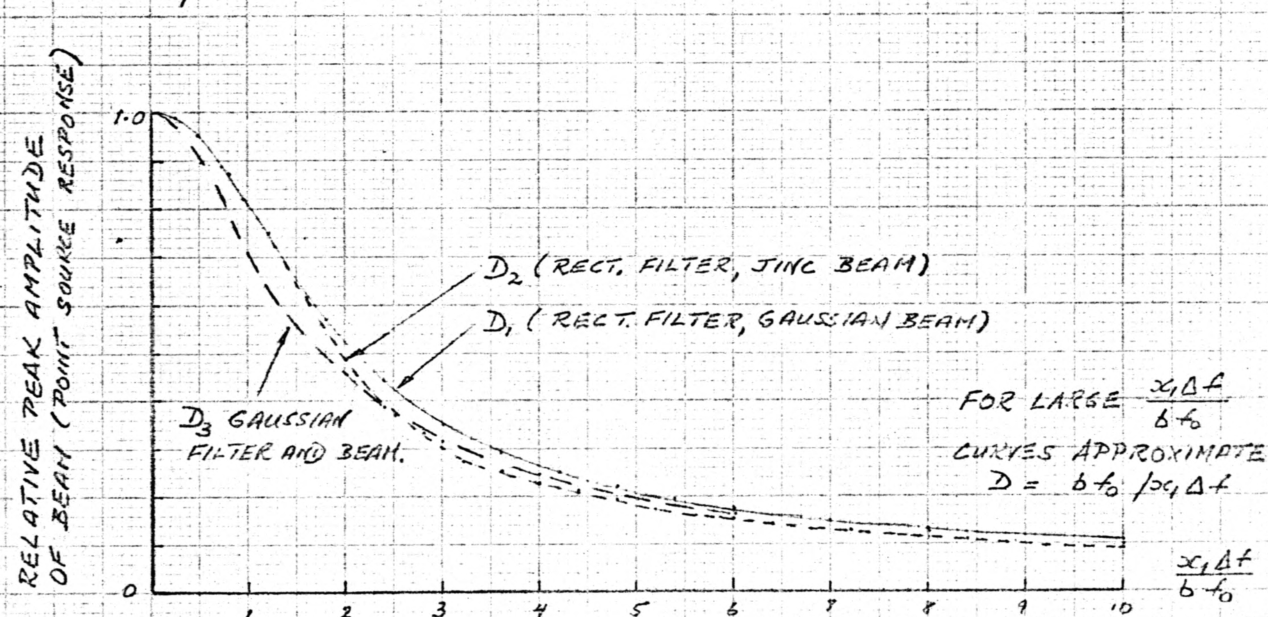

Fig 4 Degradation function for response to a point source.

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