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The Response of the VLA to Interfering Signals

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I. INTRODUCTION

This memorandum examines the maximum tolerable strengths of interfering signals as a function of their distance in frequency from the center of the observing band to which the VLA receiving system is tuned. The response profiles thus obtained allow an estimate to be made of the impact of adjacent-band signals on the performance of the VLA. They also enable one to see what additional filters may be required to eliminate the expected interference or to provide frequency responses that are matched to the assigned radio astronomy bands. The four observing frequency bands of the VLA and the corresponding radio astronomy bands are given in Table I.*

VLA RECEIVING BAND	RADIO ASTRONOMY BAND	NOMINAL WAVELENGTH	BAND DESIGNATION
1.35-1.73 GHz	1.400-1.427 GHz	21 cm	L
4.5 - 5.0 GHz	4.99 - 5.0 GHz	6 cm	C
14.4 - 15.4 GHz	15.35 - 15.4 GHz	2 cm	Ku
22 - 24 GHz	23.6 - 24.0 GHz	1.25 cm	K

TABLE I.

VLA Receiving Bands and the Corresponding Radio Astronomy Assigned Bands

It is often suggested that the differences in the nature of celestial and man-made signals should enable a distinction between them to be made at some stage in the data processing. However, in radio astronomy one is usually dealing

*The VLA receiving bands given refer to the frequency ranges over which the system can be tuned. At any instant the maximum received bandwidth is confined to two 50 MHz bands for each polarization, and additional filtering is provided to further limit the response as desired.

with signals near the limit of detectability and integration times of minutes or hours are required. Any identifying modulation on the man-made signals is thereby generally lost. The remaining characteristic of the interference is its spectrum, and the use of a spectral line system certainly facilitates the detection of interference. Interference recognition may well become a good reason for using a spectral line system for continuum observations, a mode of operation which has also been suggested to help overcome the limitations on sky coverage of broadband single-channel systems. The present discussion, however, is concerned mainly with the single-channel continuum capability of the VLA, and here the use of filters to restrict the frequency response appears to be the only practicable method of dealing with interference. Two previous VLA electronics memoranda (Napier 1973, Thompson 1973) have also been concerned with interference, but in the present one an attempt is made to examine the instrumental response in more detail, with particular attention to filter requirements.

II. ASSUMPTIONS USED

In the calculations which follow it has been assumed that the interfering signals are of constant strength and come from a transmitter that is fixed in position with respect to the array. To determine the signal power received at any antenna the equivalent isotropic collecting area $\lambda^2/4\pi$ is used and a factor of 1/2 is included to take account of the random orientation of the polarization angles of the signal and antennas. In practice, of course, the strengths of the interfering signals received will fluctuate as the antennas track, and thus their effects on gains and noise figures cannot be calibrated out. The calculations are necessarily very approximate and the results should be interpreted as referring to mean levels except where high power pulses, which cause overloading, are concerned.

III. THRESHOLDS AND RESPONSE CURVES

In calculating the tolerable interference levels three effects of the signals will be considered. First we are concerned mainly with small signals that fall within the range of maximum sensitivity of the instrument. These add a component to the output of the multipliers and thus distort the final map. Because the output from the interfering source does not contain the fringe frequency variations produced by the motion of the source under observation across the sky, the response to the interfering signals is significantly reduced in the data processing. The extent of this reduction is derived in Appendix I, where expressions for the rms level of the interference relative to a point source and to the noise are derived. In the case of broadband interference the response is further decreased by the delay times in the paths to the multiplier inputs which are equalized for radiation from the direction of the wanted source but not the interference. For any antenna pair the delay difference is of the order of d/c. where d is the antenna spacing, the maximum possible value being 2d/c. For the D configuration the mean value of d/c is approximately 1.3 μs and for the A configuration 56 μs. Hence strong decorrelation will not occur for signal bandwidths less than about 800 kHz for the D configuration or 18 kHz for the A configuration. Thus with the D configuration decorrelation of interference is probably not very helpful, and it has been omitted from the present calculations.

As explained in Appendix I the threshold of significant interference has been taken to be that level at which the rms contribution to the map is one tenth that of the rms noise. The corresponding signal levels are designated S_1 and are given in Table II together with some instrumental parameters used. Note that for the above interference effect slightly higher levels of interference can be tolerated with the longer configurations.

The values of S_1 in Table II correspond to the case where the interference falls within the frequency range of maximum sensitivity. If the signal moves off in frequency the tolerable threshold will increase in a manner defined by the frequency response of the system. A simplified block diagram of the VLA receiving system is shown in Figure 1. The frequency response is controlled mainly by the two filters F_1 and F_2, which have -3 dB bandwidths of 60 MHz, and by the IF Receiver. F_1 and F_2 have identical 6-pole responses and were chosen to have a 1 dB bandwidth of 50 MHz, the maximum usable bandwidth of an IF channel. They are centered at 1325, 1425, 1575 and 1675 MHz in the four IF channels. The IF Receiver has a maximum response extending from 1 to 50 MHz and contains filters of bandwidth 24, 12, 4, 1.5 and 0.5 MHz which can be switched in as desired. These filters have center frequencies of 25 MHz and as a result the mixer input of the IF Receiver has an image response 50 MHz below its main response. Figure 2a shows the combined response of filters F_1 and F_2, the response of the IF Receiver with full bandwidth, and the overall system response. Figure 2b shows the same curves for the 12 MHz bandwidth of the IF Receiver. Note that in both examples the IF Receiver response is centered on that of F_1 and F_2 although the fine tuning capability allows it to be centered as desired within the 50 MHz band.

The four ordinate scales at the right-hand edge of Figures 2a and 2b indicate the threshold interference levels in each of the four observing wavelength bands. These curves are drawn so that the zero-dB level of the response curves corresponds to the values of S_1 for the D configuration in Table II. Thus at any point on the overall response curve the value on the appropriate ordinate scale shows the level at which the rms interference is one tenth the rms noise. This represents the worst case since for larger configurations higher values of S_1 apply.

If we consider an interfering signal moving away from the center of the receiving band, a threshold level is reached at which a second effect must be considered. The Frequency Converter in Figure 1 contains an automatic level control (alc) loop, and when the interfering signal becomes comparable in power to the noise in the IF band, the level of the noise (which contains the wanted signal) is reduced at the input of the waveguide transmission system. If the interfering signal is cut out by using one of the narrow bandwidths of the IF Receiver the level of noise will be restored by the second alc loop at the IF Receiver and Sampler. However, the signal-to-noise ratio in the waveguide transmission system may be only 16 dB for the more distant antennas (Predmore 1974) so a drop in signal level at the waveguide input could introduce significant errors. As a threshold level for errors due to alc effects we use here a signal level, S_2, equal to the noise in a 50 MHz band:

$$S_2 = (5\mathrm{x}10^7) \times 8\pi k\ T_s/\lambda^2 \qquad (\mathrm{Wm}^{-2})$$

where T_s is the system noise temperature and k is Boltzmann's constant. Values of S_2 for the four wavelength bands are given on Table II.

In Figure 1 F_1 is the only filter preceding the first alc loop and this filter therefore determines the frequency response of the S_2 threshold. Figure 3 shows the response of F_1 and the signal strength ordinate scales for the four wavelength ranges have been included as before.

Again as one considers an interfering signal moving away from the band center a threshold level is reached at which a third effect becomes important.

The parametric amplifier suffers gain compression when the signal level within it becomes too high. The overall system gain is restored by the action of the alc loops but for 1 dB compression there is also a 1 dB increase in the second stage contribution to the noise figure. This increase amounts to about 0.25°K and would cause an amplitude error of about 0.5% in the visibility data. One dB of gain compression occurs with an input level of -43 dBm, and this defines a third signal-strength threshold*:

$$S_3 = (5\text{x}10^{-8}) \text{ x } 8\pi/\lambda^2 \qquad (\text{Wm}^{-2})$$

Values of S_3 are given in Table 2. The frequency response of this effect is determined by the antenna feeds, the waveguide runs and the parametric amplifiers themselves. It varies very little over the frequency scales in Figures 2 & 3. Damage to the parametric amplifiers does not occur until input levels of +20 dBm are reached, and the corresponding signal levels are therefore 63 dB greater than the values of S_3 given in Table II.

The overall interference threshold curve for any wavelength can be drawn by combining the responses corresponding to S_1, S_2 and S_3. An example of this is shown in Figure 4 for the 21 cm band using the 12 MHz bandwidth of the IF Receiver. The shaded area represents the range of frequencies and signal strengths for which the tolerable levels are exceeded.

IV. OBSERVED AND EXPECTED INTERFERENCE LEVELS, AND RELATED FILTER REQUIREMENTS

The extent to which the operation of the VLA will be limited by interference can be predicted by comparing the frequency response curves derived above with the measured or expected interference levels. The levels of man-made signals at the Plains of San Augustin have been monitored over the frequency ranges 1.2 to 2.2 and 4.1 to 5.1 GHz since December 1972. So far as is known there is no significant interference in the two higher-frequency bands of the VLA. The measurements thus far indicate the possibility of serious interference only in the 1.35-1.73 GHz band. Here the only signals that approach the gain compression threshold of $2.9\text{x}10^{-5}$ Wm^{-2} have been found in the range 1.24 to 1.33 GHz. and have strengths of 10^{-7} to 10^{-5}Wm^{-2}†. A band stop filter may be obtained to insert in the L-band input waveguide to reduce these signals to tolerable levels. Signals of strengths up to 10^{-9} Wm^{-2} are more numerous, and

*This calculation neglects the small up converter gain in the 21 cm band and the mixer losses in the 2 and 1.25 cm bands, none of which amount to more than 4 dB.

†See Appendix II

Figure 4 indicates that at this level the system is vulnerable to interference over a band 105 MHz wide because of alc effects. This width is independent of the responses of the IF Receiver which effectively controls the bandwidth for signals up to $10^{-13}Wm^{-2}$ only. In the 21 cm band some further filtering will clearly be required. In the three higher frequency bands, however, the system should have no significant interference problems with the signal levels observed to date. The strong signals near 1.3 GHz will not cause gain compression in the higher bands because they are below the cut-off for the input waveguides.

As a first step toward improving the response of the system the center frequencies of the filters in the IF Receiver can be increased to 40 MHz. The image response is them 80 MHz below the main response and is much more strongly suppressed by filters F_1 and F_2. This change of center frequency would apply to the filters with 12, 4 and 1.5 MHz bandwidths. The 24 MHz bandwidth filter can have its center frequency increased only to 38 MHz since the response must remain within the overall 1-50 MHz passband. Also the 0.5 MHz bandwidth filter should remain at 25 MHz center frequency since this results in the minimum percentage bandwidth (2%) available in the design used. A further step would be to change F_1 and F_2 from 6 pole filters with 60 MHz (-3 dB) bandwidth to 8 pole filters with 50 MHz bandwidth. Figure 5 shows the expected threshold curve for the same conditions as Figure 4 but with the increased center frequency in the IF Receiver and the suggested changes for F_1 and F_2. The new response curve is a significant improvement, especially for the interference at the level 10^{-13} to $10^{-10}Wm^{-2}$. At $10^{-9}Wm^{-2}$ the width is decreased from 105 to 85 MHz.

Further improvement of the interference response curve calls for the ability to insert narrower filters in series with F_1. In discussing this it is important to consider how the system will operate in the long term if only the bands assigned to radio astronomy remain clear of man-made signals. If such signals extend up to the very edges of the radio astronomy bands, so that the sensitivity of the VLA receiving system must have fallen by 50 dB or more at the band edges, the maximum useful bandwidth of the instrument cannot be much more than half the width of the assigned band. The present selectivity would be adequate for the 23.6-24 GHz band with its 400 MHz width, but for the three lower frequency bands further filtering would be required. The filter bandwidths would probably be 13.5 MHz for the 1.40-1.427 GHz band, 5 MHz for the 4.99-5.0 GHz band and 25 MHz for the 15.35 to 15.40 GHz band. These filters, inserted in series with F_1, would restrict the frequency response of interference entering directly into the map or causing disturbances to the alc system.

As an example of the effect of such a narrower filter Figure 6 shows the response curve with a filter of bandwidth 13.5 MHz which would be appropriate for the 21 cm assigned band. The filter response is taken from a curve for an 8-pole tubular bandpass filter with 10% bandwidth in the June 1973 catalog of Telonic Inc. The IF receiver bandwidth is 12 MHz, but the first filter has a greater effect on the overall selectivity. As a result of this the alc effect does not enter into the interference threshold curve. At $10^{-7}Wm^{-2}$ the width of the curve is 31 MHz and the tolerable level at the edges of a 27 MHz-wide band is $10^{-9}Wm^{-2}$.

V. IMPLEMENTATION OF REQUIRED FILTERS

The effect of the phase response of a filter like the one in Figure 6 has not yet been investigated. If the corresponding filter at each antenna had an identical response there would, of course, be no loss in sensitivity, but with the normal tolerances the deviations of the individual phase responses from the mean must be significant. Filters with steep amplitude characteristics have highly non-linear phase-frequency characteristics which makes matching of these characteristics difficult, and small differences in the center frequencies become important. It appears that to maximize the overall sensitivity of an interferometer there must be an optimum filter design. This would be a compromise between achieving a response close to the rectangular ideal, which maximizes the usable bandwidth, and obtaining matched phase responses which maintain the theoretical sensitivity. This point deserves further investigation. In an earlier report* which is now incorporated in a study of Working Group 2-D of the CCIR (Doc. 2/152-E) a maximum usable bandwidth of 75% of the assigned band was suggested. This referred mainly to cases where the phase response is not important, and in the above discussion of the VLA a width of 50% of the assigned band was deemed more appropriate.

A purely practical problem concerned with inserting filters with bandwidths of 5 to 25 MHz in the position of F_1 in Figure 1 is that the center frequencies result in percentage bandwidths of 0.3 to 1.9. A bandwidth of 1% is generally considered the minimum for common types of filters, and also the use of a lower center frequency will result in reducing temperature-induced changes in the center frequencies of the filters.

Figure 7 shows a possible system in which the input frequencies are reduced by 1200 MHz for channels A and B and 1800 MHz for channels C and D. Filters F_7 and F_8 are to prevent IF signals from leaking along the local oscillator line. A unit for one channel could be built into a single module at a cost of about $1550 as the following estimate shows:

2 mixers, Relcom MIJ	$ 202
1 amplifier, 3 Avantek GPD modules plus mounting	115
2 switches, Transco 144C70110	460
6 filters	420
1 power divider, OSM20493	107
1 control card	100
Module and assembly	150
TOTAL	$1554

*'The Sensitivity of Radio Astronomy Receivers to Interference from Adjacent-Band Signals', reporty by A. R. Thompson, July 1973.

The filter arrangement just described could be added without modification to the prototype electronics. Although it may appear simpler to convert directly from the 4.5-5.0 GHz band to the frequencies of filters F_3, F_4 and F_5 in Figure 7, a problem with an image response would then arise, because these frequencies are less than half the bandwidth of the signals at the mixer input.

A remaining problem concerns spurious responses which can provide channels for the reception of interfering signals. The most serious case of this effect results from a response at the input of the mixer that converts the 4.5-5.0 GHz signals. The spurious response occurs for signals at frequencies of twice the local oscillator frequency minus the intermediate frequency. IF channel A is free from this effect and it is most serious for channels C and D. The level of the spurious response is 10 to 40 dB below the wanted response and varies with the type of mixer used. Possible ways of reducing or eliminating the problem include using a specially chosen mixer, changing the IF frequencies or using a high-side instead of a low-side local oscillator at the mixer. The best approach has not yet been determined. Spurious responses which are below the wanted system response by more than 20 dB would, of course, generally be tolerable if it were not for effects of interference.

VI. WORKING WITHIN THE ASSIGNED RADIO ASTRONOMY BANDS

Up to the present time radio astronomers have not generally found it necessary to design instruments that are sensitive only to frequencies within the assigned bands. This will hopefully continue to be the case for some time, but we must design the VLA so that it can still operate if only the assigned bands are clear. Under such conditions would the VLA have adequate sensitivity?

In Table II figures are given for the flux density of a point source for which the peak amplitude is equal to the rms noise. S_4 is calculated for a bandwidth of 100 MHz which assumes that four 50 MHz-wide IF bands (two for each polarization) will be used for each antenna. S_5 is calculated for a bandwidth equal to half the width of the corresponding radio astronomy band, or 100 MHz for the 1.25 cm band.

The following formula was used:

$$\frac{\text{Peak signal amplitude}}{\text{rms noise}} = \frac{0.81\ A\ F\ \sqrt{B\tau\ n\ (n-1)}}{2\ k\ T_s}$$

where A is the collecting area of an antenna (equal to $491\,m^2$ x aperture efficiency), F is the flux density of the source, B is the bandwidth, τ is the integrating time (12 hours assumed), and n is the number of antennas. The factor 0.81 takes account of the digital sampling process (Cooper, 1970), and the formula applies to an array that uses both sine and cosine multipliers and receives both polarizations simultaneously.

The scientific requirements on the sensitivity of the VLA are discussed in "A Proposal for a Very Large Array Radio Telescope", Vol. I, pp.2-8 to 2-9, where it is stated that the sensitivity should approach 10^{-4} flux units (10^{-30} Wm^{-2} Hz^{-1}) as closely as proves feasible and should not be less than about 10^{-3} flux units (10^{-29} Wm^{-2} Hz^{-1}). These figures referred to a wavelength of about 10 cm and should be compared with the values of S_4 and S_5 for the two longer wavelength bands. For these bands Table II shows that a source of 10^{-30} Wm^{-2} Hz^{-1} results in a signal to noise ratio of about 10 with the 100 MHz bandwidth and 2.5 to 3 with the restricted bandwidth. Thus in either case the performance satisfies the stated requirement. The greatest effect of restricting the bandwidth to half that of the assigned band occurs at 6 cm where the loss in sensitivity is a factor of 4.4 which gives a signal to noise ratio of 25 for

10^{-3} flux units. The loss in the data collecting rate is not necessarily as great, however, since a narrower bandwidth allows larger fields to be mapped at the large configurations.

The most serious consequence of being forced to observe only within the assigned bands is in the spectral line area. Lines of OH, formaldehyde, excited hydrogen and water fall within the frequency range of the VLA but not within the assigned radio astronomy bands. Furthermore the 1400 MHz lower limit of the 21 cm band corresponds to a distance of 80 megaparsecs for the red-shifted H line, and thus observations of only the nearby galaxies can be made within the assigned band. In such cases where observations must continue to be made in bands assigned to other services, the chances of success will be enhanced if the system response extends no further than necessary beyond the range of interest. Since the tunability of the local oscillators at the antennas is limited to the 20 and 30 MHz increments of the 2-4 GHz synthesizer module, it is likely that a number of sets of filters with center frequencies matched to specific lines or red-shifts will be required. The cost of a set of 54 tubular bandpass filters of 6 or 8 pole design, which would be required for each new line frequency, would be about \$3000. Such filters, and the necessary coaxial switches for selecting them, would be added only as required. Provision must be made, however, so that they can be installed easily and at an intermediate frequency for which the percentage bandwidths are not too low.

VIII. CONCLUSIONS

1. Filters will be provided to limit the response of the system to the assinged radio astronomy bands. The basic performance of the VLA when limited to these bands is satisfactory. Operation outside the assigned bands allows enhanced sensitivity and a wider choice of spectral line observations, and the system will be able to take advantage of this as the spectrum coverage by other users permits. Two of the proposed filter modules (Figure 7) will be required at each antenna, one for each polarization. The other two IF channels can be used for observations in parts of the spectrum where there is little or no other usage, as is presently the case in the three highest frequency bands.

2. Cost estimates for the electronic system should include two filter modules (approximately $1500 each) per antenna.

3. Study of the optimum filter characteristics for an interferometer type system, as discussed in Section V, should be pursued.

4. Measurements of the effect of the radar signals near 1.3 GHz (see Appendix II) must be made during the interferometer testing to decide whether a filter to attenuate them is required in the input waveguide.

5. The modifications described in Section IV concerning the center frequencies of the filters in the IF Receiver module and the specifications of filters F_1 and F_2 should be implemented.

6. Measurements should be made to detect all unwanted (spurious) responses before deciding on any modifications required.

7. It would be useful to have a computer program to generate response curves of the type shown in Figures 2-6 for each arrangement of observing frequencies, bandwidths, etc. chosen by an observer. Residual effects of unwanted responses should be included.

WAVELENGTH CONFIGURATION	21 cm D	21 cm A	6 cm D	6 cm A	2 cm D	2 cm A	1.25 cm D	1.25 cm A
System Temp., T_s	50K		50K		350K		500K	
Max. Antenna Spacing in wavelengths, q_{max}	4.9×10^3	1.7×10^5	1.7×10^4	6.1×10^5	5.1×10^4	1.8×10^6	8.2×10^4	2.9×10^6
S_1 (Wm^{-2})	1.5×10^{-16}	8.6×10^{-16}	3.3×10^{-15}	2.9×10^{-14}	3.7×10^{-13}	2.2×10^{-12}	1.7×10^{-12}	1.0×10^{-11}
S_2 (Wm^{-2})	2.0×10^{-11}		2.4×10^{-10}		1.5×10^{-8}		5.5×10^{-8}	
S_3 (Wm^{-2})	2.9×10^{-5}		3.5×10^{-4}		3.1×10^{-3}		8.1×10^{-3}	
Aperture Efficiency	0.52		0.71		0.62		0.46	
S_4 (Wm^{-2} Hz^{-1})	1.2×10^{-31}		8.9×10^{-32}		7.1×10^{-31}		1.4×10^{-30}	
S_5 (Wm^{-2} Hz^{-1})	3.3×10^{-31}		4.0×10^{-31}		1.4×10^{-30}		1.4×10^{-30}	

TABLE II. INTERFERENCE THRESHOLDS, SENSITIVITY LIMITS, AND RELATED PARAMETERS

APPENDIX I

The Response of a Correlator Array to Interfering Signals

I. THEORETICAL DISCUSSION

A multiplying interferometer tends to reject signals from directions other than that in which it is programmed to observe because of two effects. First, the unwanted output does not come at the expected fringe frequency and second, the time delays in the paths through the different antennas are equalized for the observing direction only, and broadband unwanted signals are to some extent decorrelated. This analysis is concerned only with the fringe frequency effect.

In the VLA, phase changes are introduced in the local oscillators to slow the fringe frequency to zero for a source at the center of the field under observation. Signals from a source which is stationary with respect to the antennas, as the interfering source is assumed to be, produce outputs at the multipliers at frequencies equal to the natural fringe frequencies for the direction of observation. These unwanted outputs are reduced by the subsequent averaging process. In discussing this effect it is convenient to work in the (u',v')-plane where $u' = u$ and $v' = v \operatorname{cosec} \delta$, u and v being the usual spacial frequency coordinates and δ the observing declination. All 12-hour tracks in the (u',v')-plane are semi circles centered at points on the v' axis and generated by radius vectors that rotate at the angular velocity of the earth. Figure A-1 shows a track for an east-west baseline. It will be assumed that the multiplier outputs are interpolated by cell summing or cell averaging, (Thompson and Bracewell, 1973), to obtain visibility values on a rectangular grid of points with spacings $\Delta u'$ and $\Delta v'$ as shown in the figure.

The averaging time is not constant along any circular track because the path lengths in the cells vary, but the mean value is equal to the mean path length through a cell divided by $\omega_0 q'$ where ω_0 is the angular rotation velocity of the earth and q' is the radius of the track in wavelengths. The mean path length through a cell in the direction ϕ in Figure A-2 is equal to the cell area divided by the projected width $W = \Delta u' \sin \phi + \Delta v' \cos \phi$. If $\Delta u' = \Delta v'$ the mean over all angles becomes:

$$\frac{2\Delta u'}{\pi} \int_0^{\pi/2} \frac{d\phi}{\cos\phi + \sin\phi} = 0.79\ \Delta u' \ . \tag{A-1}$$

The mean averaging time is thus $0.79\ \Delta u'/\omega_0 q'$. The natural fringe frequency at any point along a track is equal to $\omega_0 u' \cos\delta$, and the averaging can be regarded as convolution with a rectangular function followed by sampling. If the interfering signal produces an output of unit peak amplitude before averaging, the amplitude after averaging will be:

$$\sin\,(0.79\pi\ \Delta u'\ u'\ \cos\delta/q')/(0.79\pi\ \Delta u'\ u'\ \cos\delta/q') \tag{A-2}$$

In terms of the angle θ of the radius vector in Figure A-1

$$u' = q' \sin\theta$$

and (2) becomes

$$\sin\,(0.79\pi\ \Delta u'\ \cos\delta\ \sin\theta)/0.79\pi\ \Delta u'\ \cos\delta\ \sin\theta \tag{A-3}$$

To determine the level of the interference in the final map we need to calculate the rms of its contribution to the visibility values in the (u',v')-plane. Over any small range of θ the rms of the averaged values is $1/\sqrt{2}$ times expression (A-3), and the mean squared value over a 12-hour track is:

$$\frac{1}{\pi} \int_0^{\pi/2} \frac{\sin^2\,(0.79\pi\ \Delta u'\ \cos\delta\ \sin\theta)\ d\theta}{(0.79\pi\ \Delta u'\ \cos\delta\ \sin\theta)^2} \tag{A-4}$$

The form of the sinc-squared function in the integral is shown in Figure A-3. The integral is intractable without simplification, which fortunately is possible since generally $\Delta u'$ will not be less than about 50 wavelengths. Then for $\theta = 0.1$ radians the denominator of the sinc-squared function is

$$(0.79\pi\ \Delta u'\ \cos\delta\ \sin\theta)^2 \gtrsim 154\ \cos^2\delta$$

So in the cases of interest the sinc-squared function in (A-4) will generally have fallen to 10^{-2} or less in the range over which $\sin\theta \simeq \theta$. The integral can then be approximated by

$$\frac{1}{\pi}\int_0^{\infty}\frac{\sin^2(0.79\pi\ \Delta u'\ \cos\ \delta\theta)\ d\theta}{(0.79\pi\ \Delta u'\ \cos\ \delta\theta)^2}=\frac{1}{1.6\pi\ \Delta u'\ \cos\ \delta} \qquad \text{(A-5)}$$

Thus, if the interfering signal produces a constant fringe-frequency output of unit peak amplitude at the multiplier, its rms contribution to the visibility values will be

$$(1.6\pi\ \Delta u'\ \cos\ \delta)^{-1/2} \qquad \text{(A-6)}$$

Note that the above result is independent of q', the radius of the track in the (u',v')-plane, and is not affected by shifts of the center of the circular arc on the v' axis. It therefore holds for all baseline lengths and orientations between antenna pairs. This is because for long baselines the high fringe frequencies are offset by the short cell-crossing times. Note also that, as Figure A-3 indicates, the contribution of the interference is concentrated along the v' axis, and hence it will result in adding to the map of the sky structures which contain high frequencies in the north-south direction only, which thus should appear elongated in the east-west direction. In some cases it may be useful to reject data close to the v' axis to reduce the interference contribution, although this will add unwanted east-west sidelobes to the beam.*

Now compare the amplitude in the (x',y')-plane of the response to a point source at the origin with the rms level of the interfering signal. Suppose that the source also produces an output of unit peak amplitude at each multiplier and that there are N sampled cells in the (u',v')-plane. Then after Fourier transformation the rms interference level will be less than the

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*The use of such a procedure to remove interference from the map raises some interesting points for further thought. For example, the use of a Gaussian interpolating function would reduce the residual interference level at points well away from the v' axis. However, the limit to which the interference can be reduced by averaging may depend upon the constancy of the interfering signal.

peak amplitude of the point source by a factor

$$(1.6\pi \; N \; \Delta u' \; \cos \delta)^{-1/2} \tag{A-7}$$

To compare the rms of the interference contribution with the rms noise the following formula for the output of a multiplying interferometer when observing a point source can be used:

$$\frac{\text{peak fringe amplitude}}{\text{rms noise}} = \frac{AF \sqrt{B\tau}}{\sqrt{2} \; k \; T_s} \tag{A-8}$$

Here A = collecting area of a single antenna, F = flux density of the source, B = system bandwidth, τ = averaging time, k = Boltzmann's constant, and T_s = system noise temperature. Now in (A-8) the point source can be replaced by an interfering signal of strength S which produces the same power in the receiving system,

$$S\lambda^2/8\pi = 1/2 \; AFB, \tag{A-9}$$

τ can be replaced by the mean cell crossing time, and using (A-6) one then obtains

$$\frac{\text{rms of interfering signal}}{\text{rms noise}} = \frac{S\lambda^2}{8\pi^{3/2} \; k \; T_s \sqrt{B\omega_0 \; q' \; \cos \delta}} \tag{A-10}$$

Note that this applies to the data values from a single multiplier after averaging to obtain the values at points on a rectangular grid. In the VLA each antenna pair feeds a sine and a cosine multiplier. The noise outputs of the two multipliers are uncorrelated. The interference outputs are of the form

$$\int_{T_1}^{T_2} \sin 2\pi ft dt \quad \text{and} \quad \int_{T_1}^{T_2} \cos 2\pi ft dt$$

where T_1, T_2 and f are unrelated and vary from cell to cell along a track, so the sine and cosine components behave much as though they were uncorrelated. The result in (A-10) can therefore be applied to the combined outputs of each sine and cosine multiplier pair. Finally, by Parseval's theorem the ratio of the rms contributions of the interference and noise in the final map are also given by (A-10).

The above result can be used to determine a maximum tolerable value of S such that the interference does not produce features in the map with amplitudes significantly greater than the noise. The noise is spread essentially uniformly over the map, and if the interference is also uniformly distributed the required value for S can be obtained by equating (A-10) to unity. In the worst case where all of the frequency components that represent the interference add in phase to form one east-west ridge across the map, the peak level would be greater than rms level by a factor equal to the square root of the dimension of the transformation array. Values used for this dimension lie in the range of 128 to 2048, and since the formation of one ridge sums an unlikely extreme, the maximum tolerable level of S will be taken to represent the case where the rms interference contribution is 0.1 of the rms noise. This maximum level designated S, is given by

$$S_1 = \frac{0.8\pi^{3/2}\, k\, T_s \sqrt{\sqrt{B\omega_0}\, q' \cos\delta}}{\lambda^2} \tag{A-11}$$

This result should be directly applicable to analysis in the (u,v)-plane since changing the scale factor of one dimension of a map should not affect the relative levels of the noise and other features. Equation (A-11) was used to determine the values of S_1 in Table II, using the mean value of 1/2 for cos δ and $q_{max}/2$ for q' where q_{max} is the distance in wavelengths between the extreme southeast and southwest antennas for any configuration.

II. COMPARISON WITH AN OBSERVATIONAL TEST

The predictions of equation (A-11) may be compared with results of a test on the Green Bank interferometer by Hogg and Dolan (1973) using an unmodulated signal generator and a small antenna to generate an interfering signal at 8110 MHz. Relevant parameters were as follows:

$\lambda = 3.7$ cm

$B = 30$ MHz

$T_s = 120$ K

$q' = 4.9 \times 10^4$ (mean antenna spacing 5920 ft)

Hogg and Dolan obtained maps in which the interfering signal introduced ridges at regular intervals distributed in a fairly uniform manner over the whole map. They concluded that these features would be at a level comparable with the noise at a signal strength of 5×10^{-13} Wm^{-2}. An rms interference level equal to the rms noise corresponds to ten times S_1 in equation (A-11), and using the above parameters this turns out to be 3.9×10^{-13} Wm^{-2}. This almost exact agreement with the observed value must be to some extent fortuitous, considering all of the approximations involved. Hogg and Dolan considered that a signal level of 10^{-13} Wm^{-2} would be acceptable, and the criterion used here, requiring the rms interference level to be one tenth the rms noise, is therefore quite conservative.

The ridges on Hogg and Dolan's map do not run in an east-west direction as predicted above but at an angle which appears to be related to the azimuth of the interferometer baseline. However, only three spacings were used, and the incompleteness of the coverage in the (u,v)-plane might be expected to be the dominant influence in shaping features in the map.

APPENDIX II

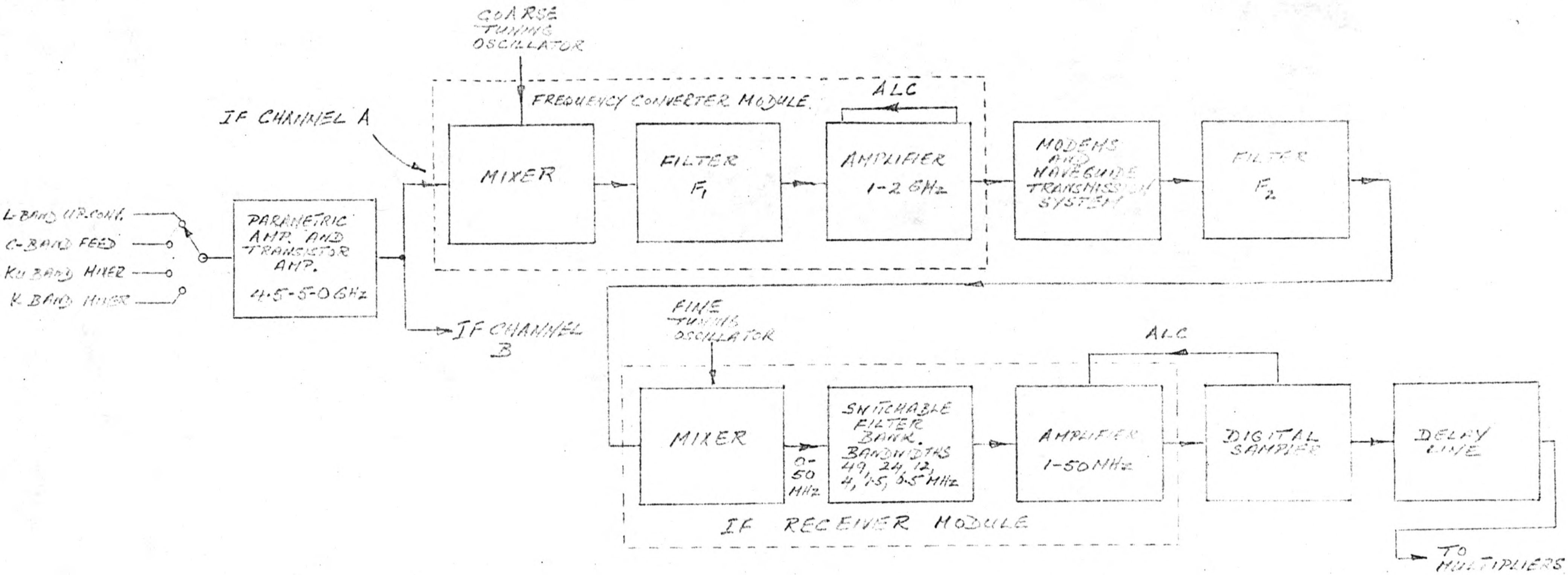
The Strong L-Band Radar Signals

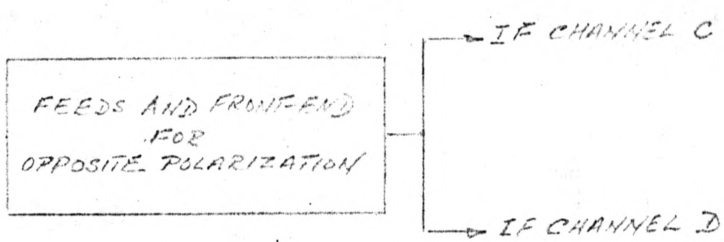
Results of the interference survey at the VLA site are discribed in VLA Electronics Memorandum #108 and subsequent addenda and reports by J. L. Dolan and G. A. Bonebrake. The strong signals near 1.3 GHz have been identified as coming from the FAA radar at West Mesa near Albuquerque. Parameters of this radar are believed to be as follows:

1. Frequency Range 1250-1350 MHz; usually has transmitters at 1.31 and 1.33 GHz, one operating and one on standby.
2. Peak Power - 2.5 MW
3. Antenna Gain - 10^3 to 10^4 (deduced from signal strength variation as antenna rotates)
4. Pulse Length - 5 μs
5. Pulse Repetition Frequency - 345-375 pps
6. Rotation Period of Antenna - 12s

The strong signals (10^{-7} to 10^{-5} Wm^{-2}) are received only for about 0.1s every 12s when the antenna beam is pointing toward the site. The radar is located about 100 miles from the site and the observed peak signal strengths correspond to a path attenuation of about 50 dB below free space transmission.

Preliminary measurements on the front end indicate that the effects of strong pulsed signals of low duty cycle may not be particularly serious. When a level is reached at which overloading occurs the effects depend more on the duty cycle than on the peak signal strength. Without tests on the prototype system operating in the interferometer mode it is difficult to be certain whether an input waveguide filter will be required.





IF CHANNEL	F_1, F_2 CENTER FREQ.
A	1325 MHz.
B	1425 "
C	1575 "
D	1675 "

Figure 1. Simplified block diagram of VLA receiving system showing locations of filters and a.l.c. loops.

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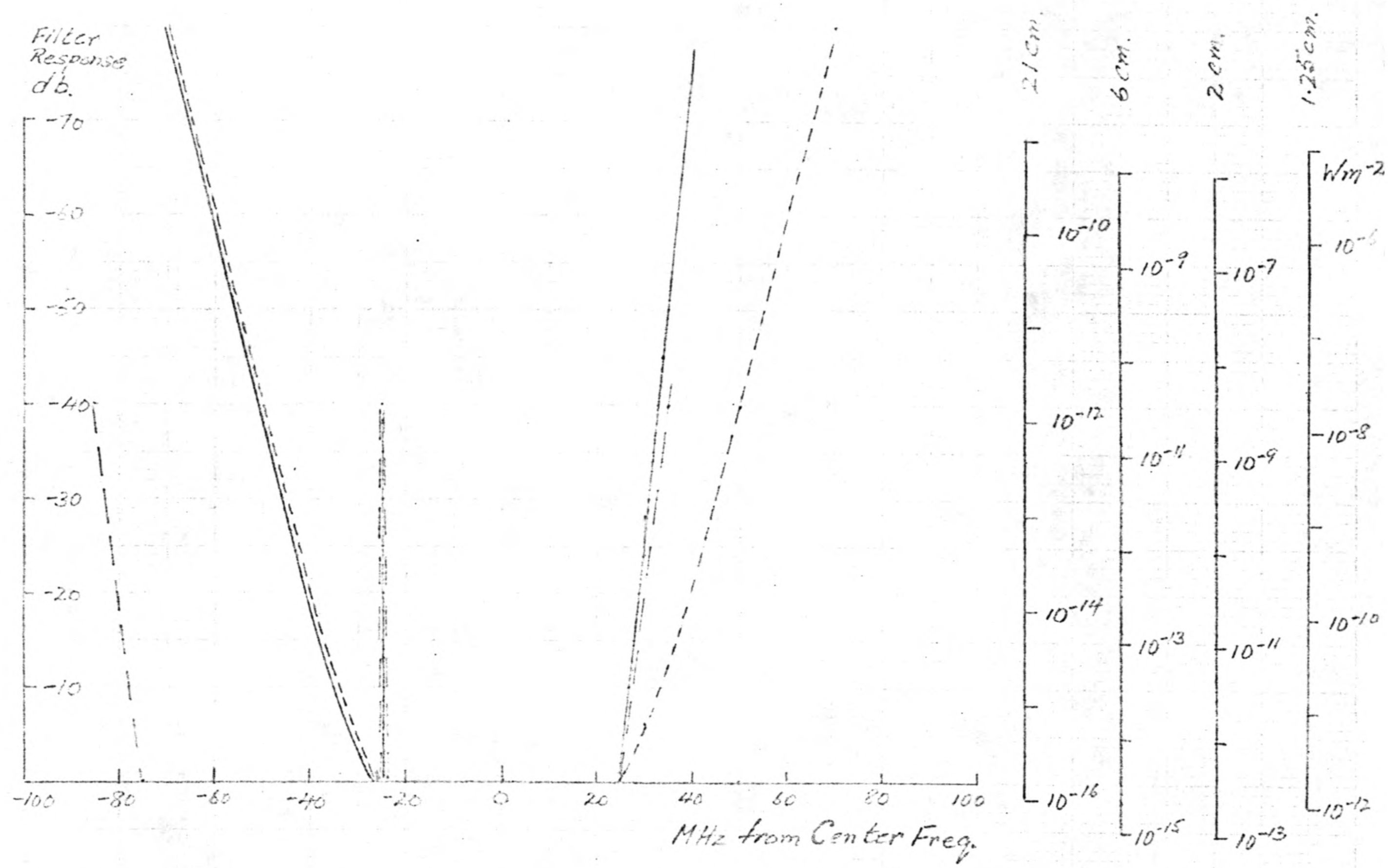

Figure 2a. Frequency response curves for filters F_1 & F_2 combined (- - -), for the IF Receiver with full bandwidth (— — —), and the overall response curve (unbroken line). The corresponding threshold levels, S_1, are indicated by the ordinate scales at the right.

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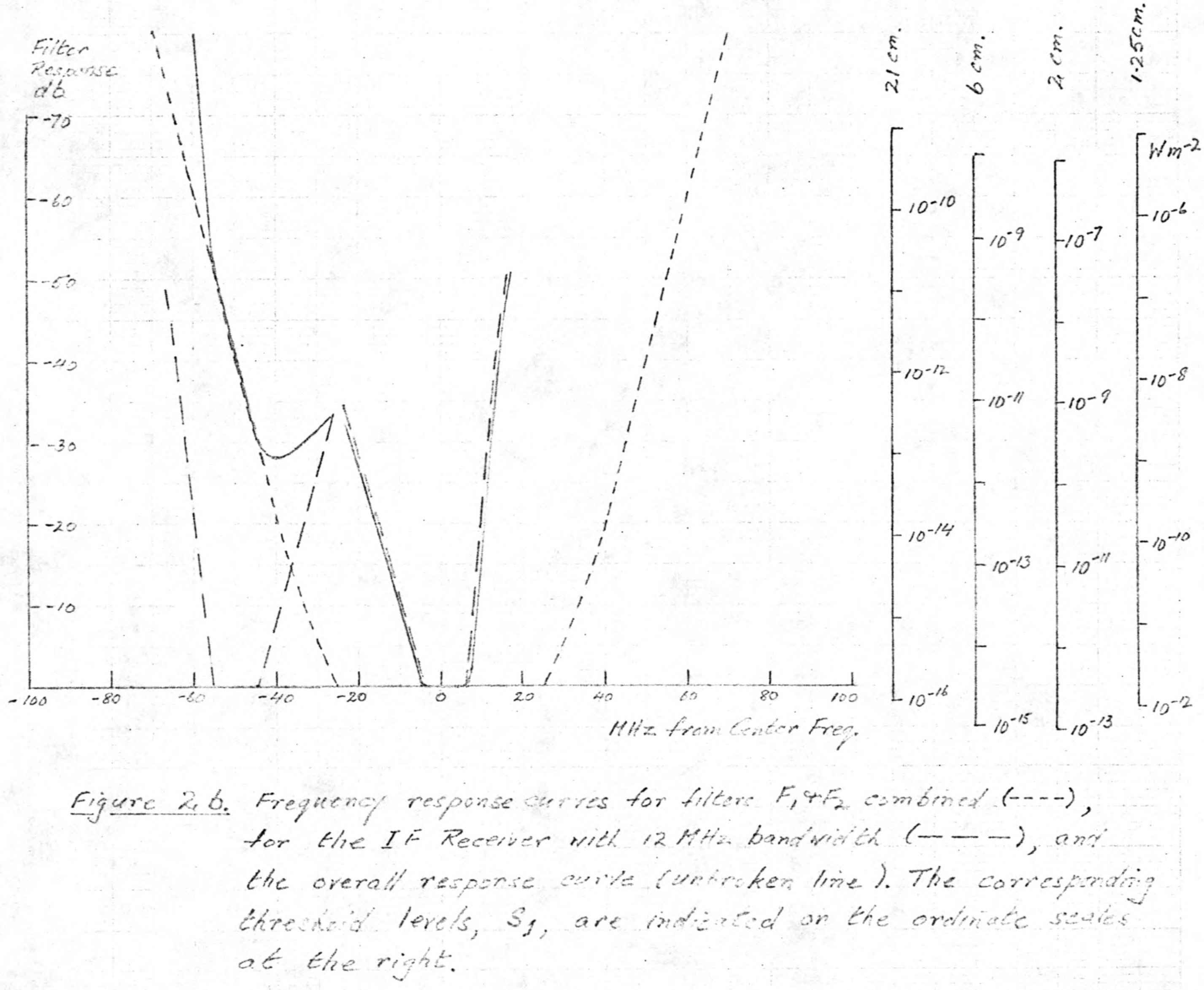

Figure 2.b. Frequency response curves for filters F_1 & F_2 combined (----), for the IF Receiver with 12 MHz bandwidth (— — —), and the overall response curve (unbroken line). The corresponding threshold levels, S_1, are indicated on the ordinate scales at the right.

A.R.T. 12/18/74.

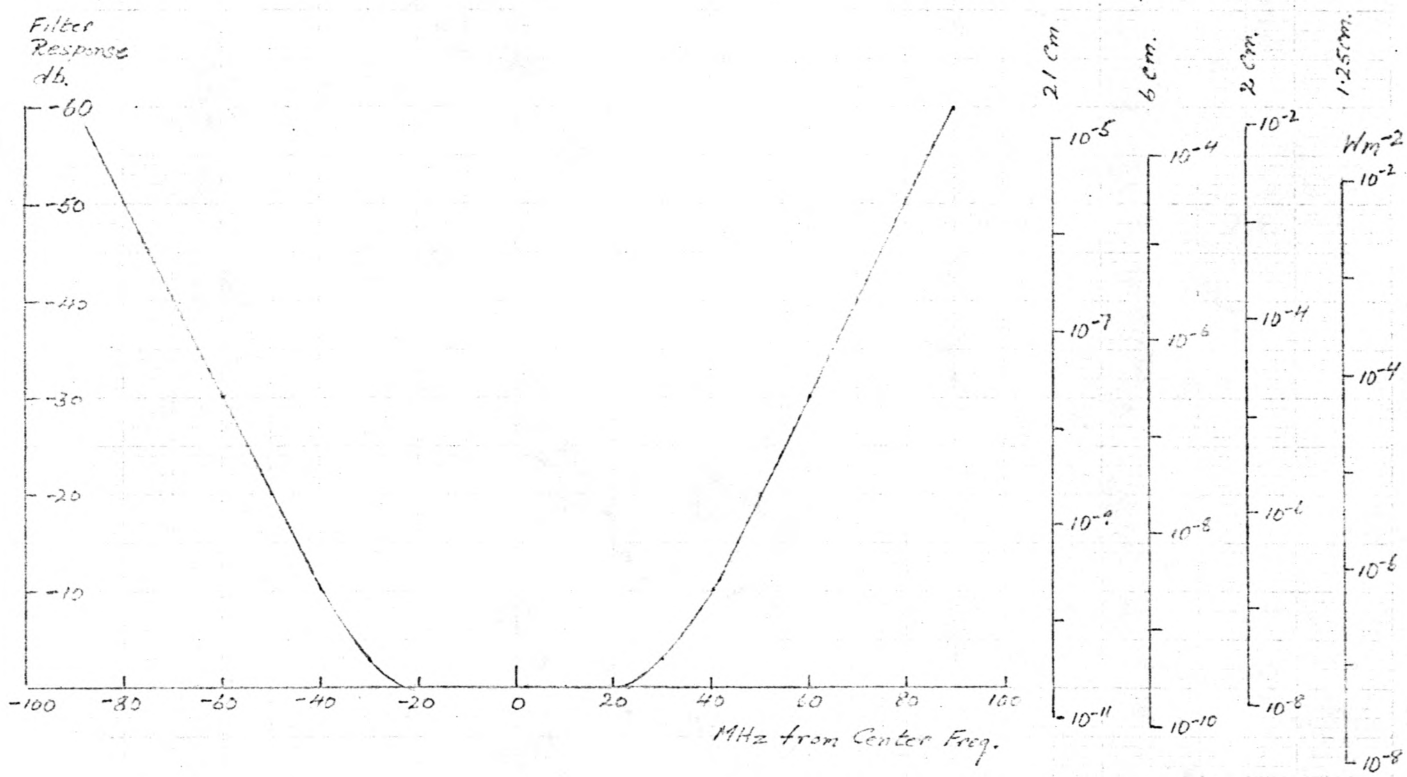

Figure 3. Response curve of filter F_1 and corresponding threshold levels S_2 resulting from a.l.c. effects.

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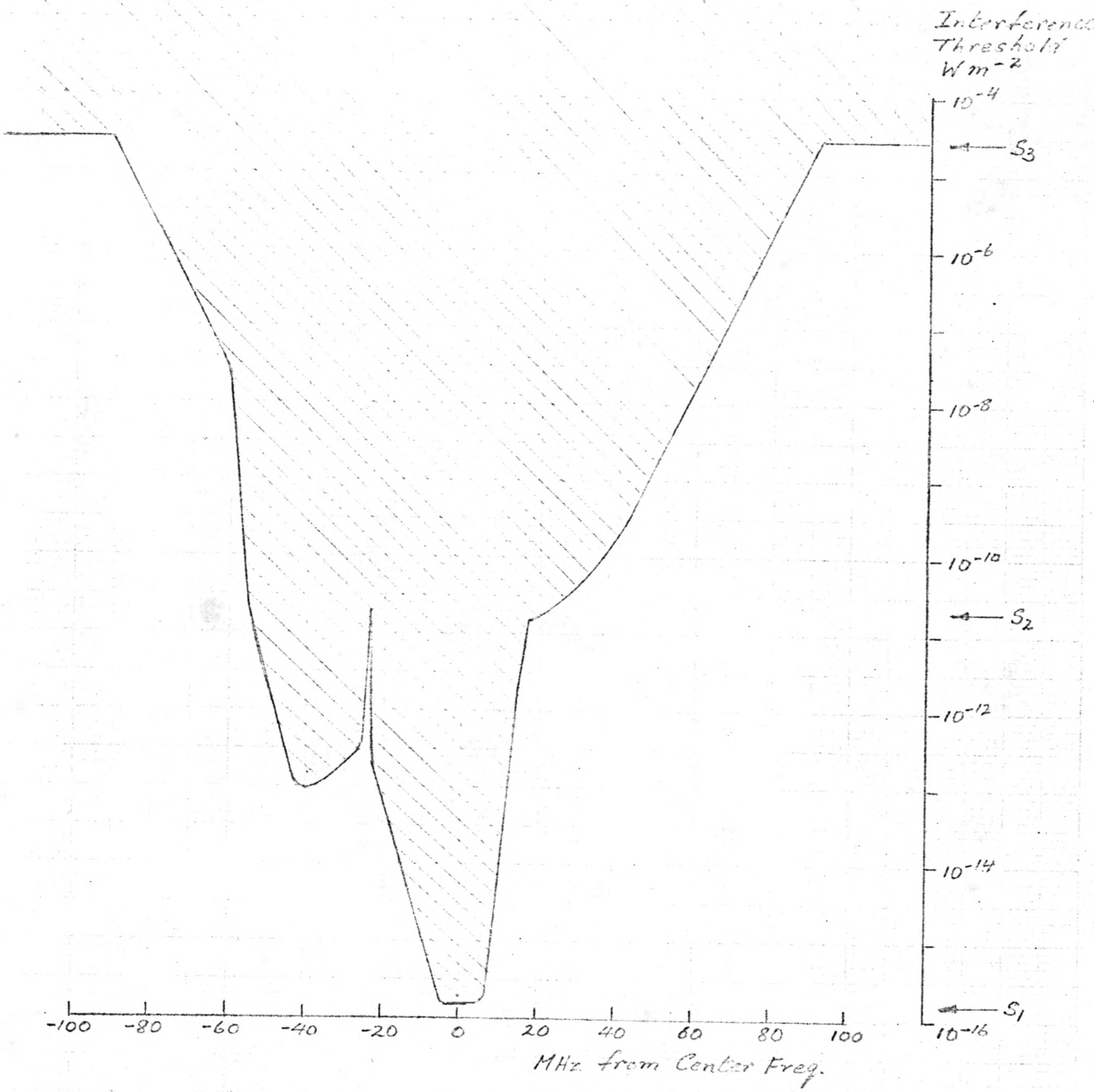


Figure 4 Interference threshold curve for 21 cm. wavelength with IF Receiver set for 12 MHz bandwidth centered on 50 MHz-wide IF band. Shaded area indicates region of signals greater than tolerable threshold.

A.R.T. 12/18/74.

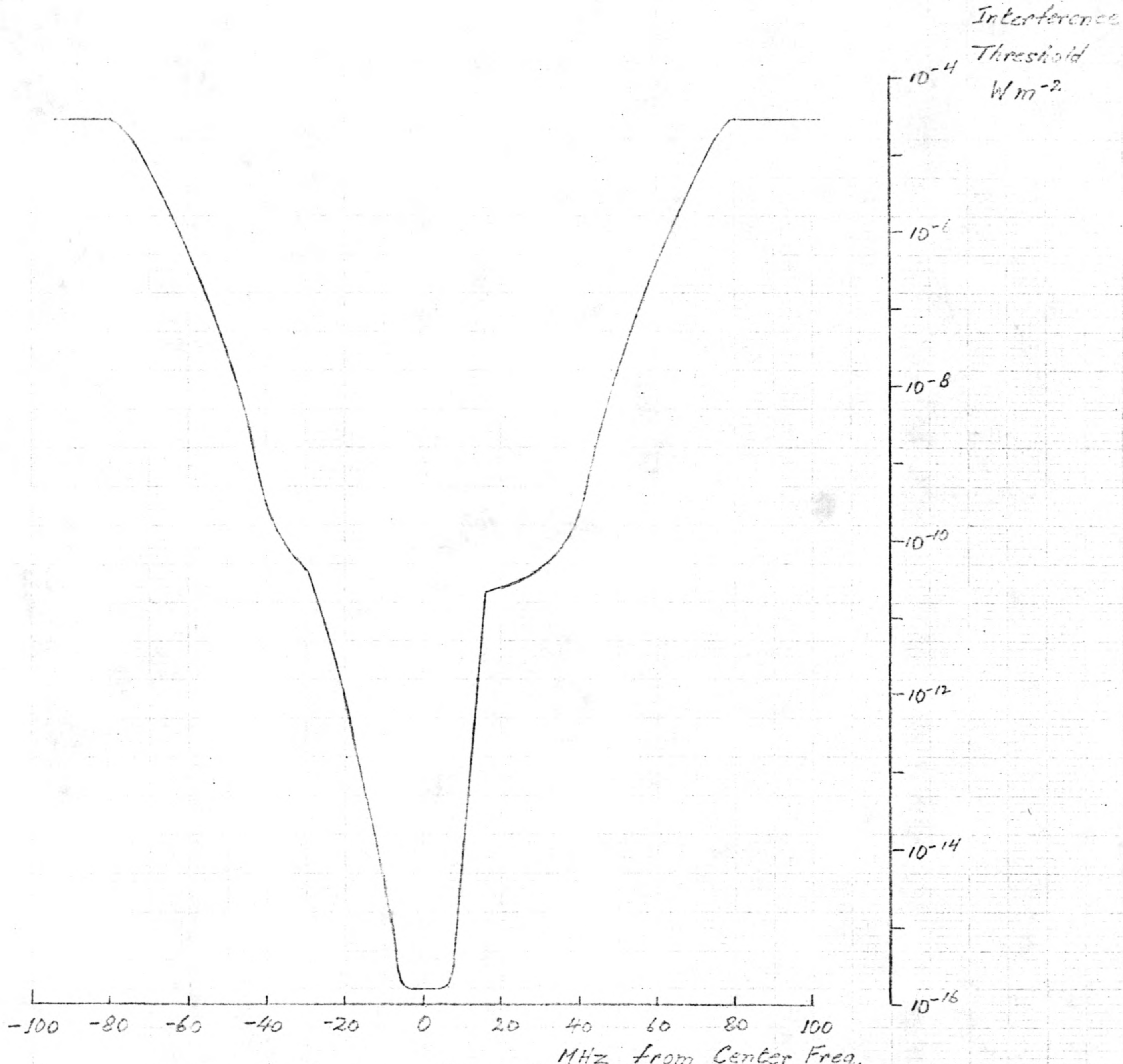


Figure. 5 As Figure 4 but with center frequency of filter in IF Receiver increased from 25 to 40 MHz and F_1 changed to an 8-pole design with -3dB bandwidth of 50 MHz.

A.R.T. 1/21/74

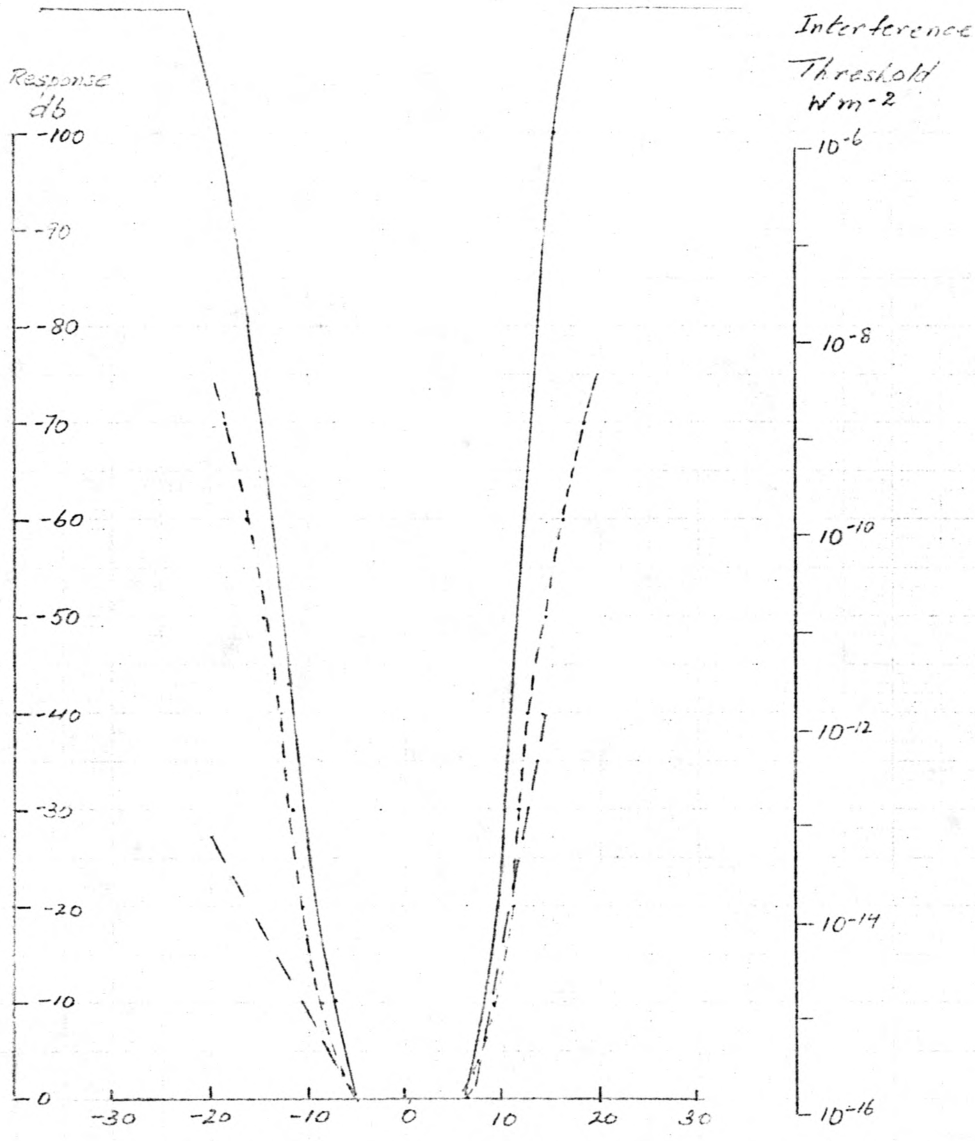
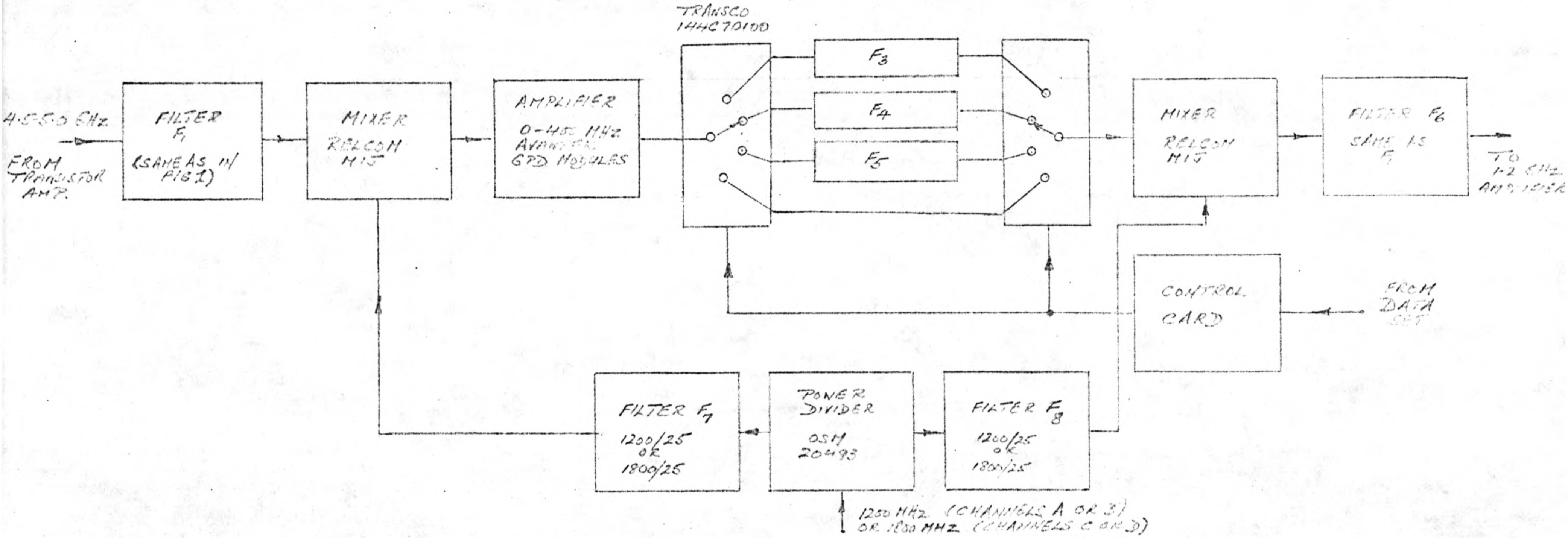


Figure 6. Frequency response curves for 8-pole, 10% bandwidth filter with 13.5 MHz bandwidth (- - -), for IF Receiver with 12 MHz bandwidth (— — —), and overall response curve (unbroken line) with the 8-pole filter replacing F_1.

A.R.T. 12/19/74



IF CHANNEL	F_3	F_4	F_5
A	123.5/13.5	135/5	115/25
B	223.5/13.5	235/5	215/25
C	226.5/13.5	215/5	235/25
D	126.5/13.5	115/5	135/25

Figure 7. Block diagram of a proposed filter module. Filters F_3 F_4 & F_5 are chosen to match the assigned bands 1.420-1.427, 4.99-5.0 & 15.35-15.40 GHz respectively. The module replaces F_1 in Fig. 1.

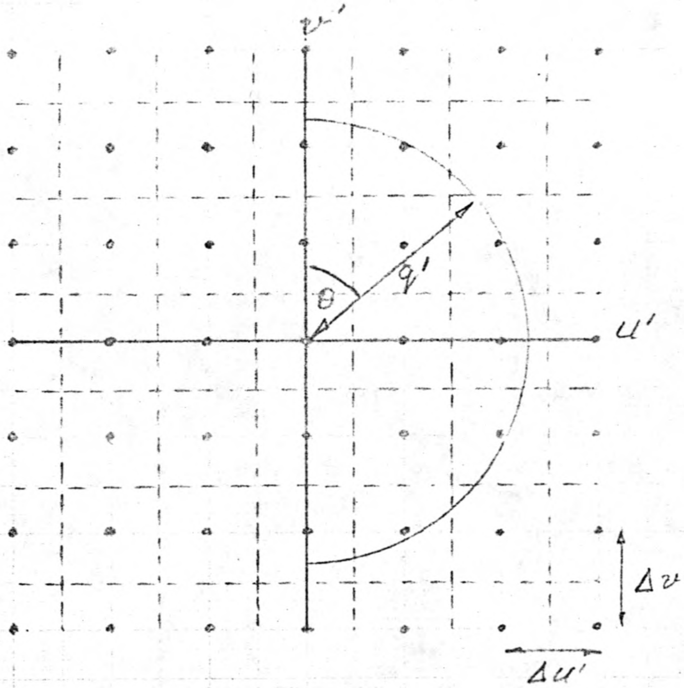
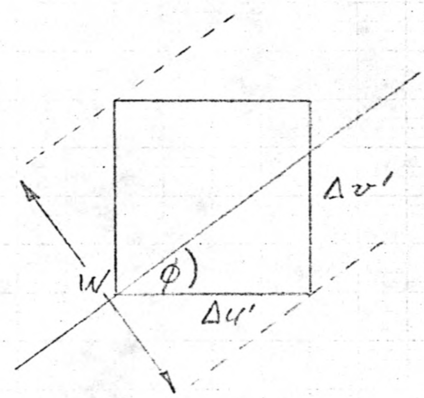


Figure A1 12-hour track in the (u',v')-plane for an east-west antenna pair

Figure A2 Calculation of the mean path length through a cell.



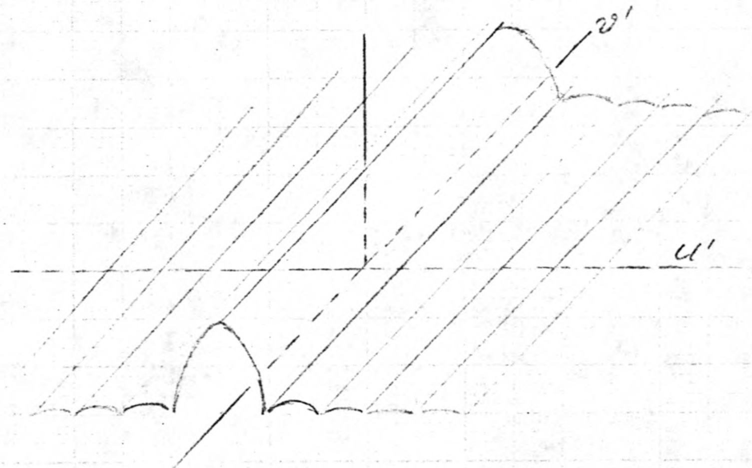


Figure A3 Distribution in the (u',v') plane of the mean squared residual interference after cell summing, as approximated in the present discussion.

A.R.T. 12/20/74.

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