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ATTENUATION, RIPPLE AND PHASE STABILITY IN VLA WAVEGUIDE TRANSMISSION SYSTEM

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In waveguide transmission line, there are many sources causing ripples in attenuation characteristics. Some of these affect the IF transmission characteristics and local oscillator system stability. These ripples are especially bothersome because they are affected by small waveguide length changes and hence are unstable.

In this report, all the ripple sources that can be considered are reviewed and stability against waveguide length change is calculated. The analysis will be done in the case of many ripple sources and the specification of couplers and some other components will be reviewed.

1. GENERAL RELATIONSHIP BETWEEN SPURIOUS SIGNAL LEVEL AND PHASE SHIFT

Under the following definitions:

- A ; TE_{01} mode amplitude (shown in vector)
- A_0; TE_{01} mode main signal amplitude (will be set to 1.0 in this report)
- C ; TE_{01} mode spurious signal amplitude
- ϕ ; phase shift caused by spurious signal
- ψ ; phase difference between main and spurious signals

The amplitude and phase shift of TE_{01} mode after addition of a spurious signal can be expressed as follows:

$$
\begin{aligned}
A &= |A|e^{j\phi} \\
&= A_0 + Ce^{j\psi} \qquad (1\text{-}1) \\
&= 1 + Ce^{j\psi}
\end{aligned}
$$

When ψ changes with frequency, ripple is caused in $|A|$ and its amplitude is given as:

$$
\begin{aligned}
R_{max}^{P-P} &= 20 \log (1+C) - 20 \log (1-C) \\
&\doteqdot 17.3C \quad (dB) \quad (C \leq 0.1)
\end{aligned}
\qquad (1\text{-}2)
$$

Also the phase shift caused by a spurious signal can be expressed as follows:

$$\Delta\phi = \sin^{-1} (C\sin\psi) \qquad (1\text{-}3)$$

and maximum peak-peak variation becomes;

$$\Delta\phi_{max} = 2 \sin^{-1} (C). \qquad (1\text{-}4)$$

Figure 1-1 shows the relationship between spurious signal level, R_{max}^{P-P} and $\Delta\phi_{max}^{P-P}$.

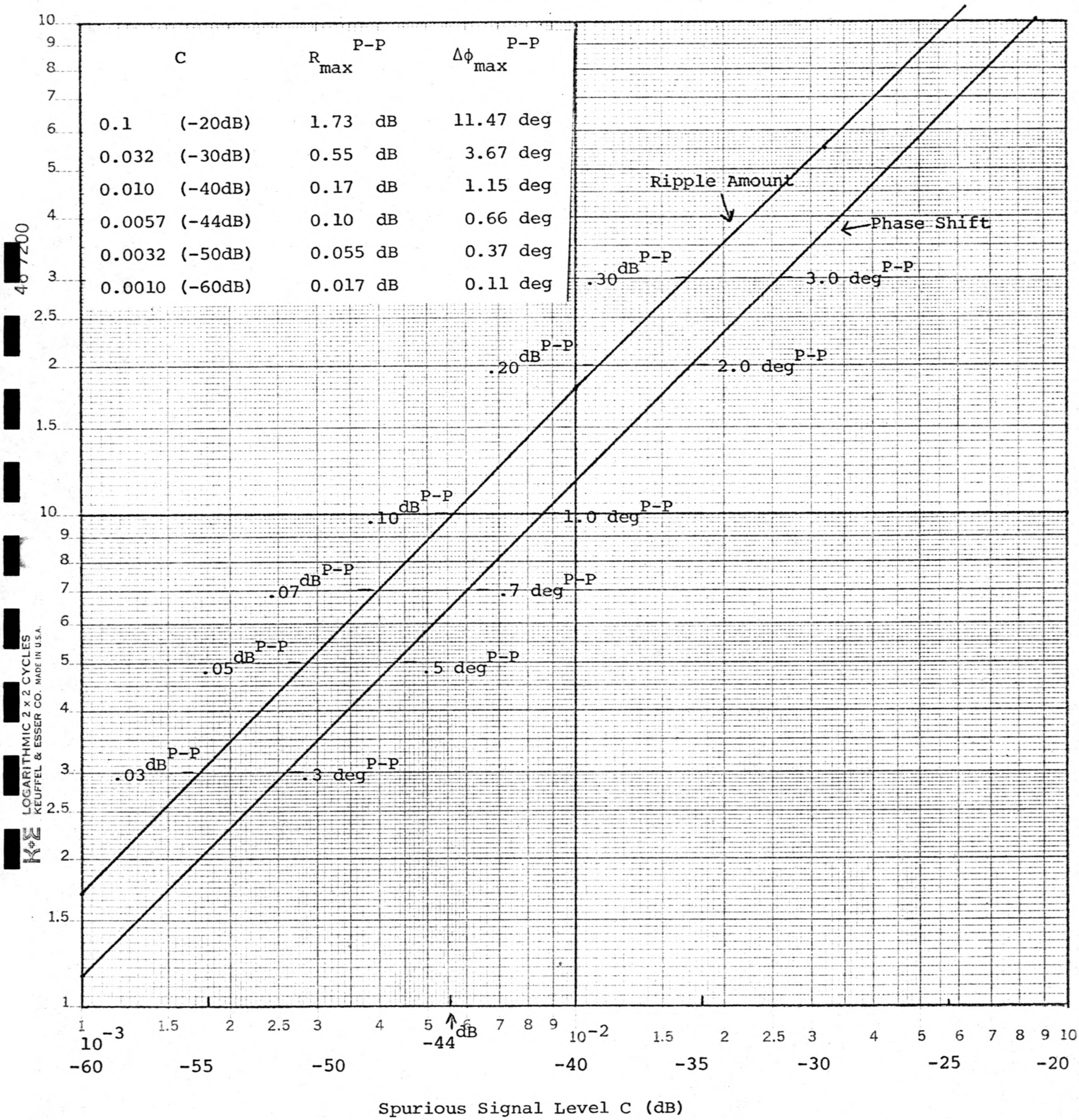
2. SPURIOUS SIGNAL SOURCES IN TE_{01} MODE CIRCULAR WAVEGUIDE SYSTEM

The TE_{01} mode spurious signals generated in waveguide can be classified into two kinds as follows:

A. Spurious signal caused by mode conversion-reconversion between TE_{01} mode and unwanted modes.

B. Spurious signal caused by TE_{01} mode reflections.

In both cases, one source of mode conversion or reflection does not cause ripple, but combinations or interactions of



C		R_{max}^{P-P}	$\Delta\phi_{max}^{P-P}$
0.1	(-20dB)	1.73 dB	11.47 deg
0.032	(-30dB)	0.55 dB	3.67 deg
0.010	(-40dB)	0.17 dB	1.15 deg
0.0057	(-44dB)	0.10 dB	0.66 deg
0.0032	(-50dB)	0.055 dB	0.37 deg
0.0010	(-60dB)	0.017 dB	0.11 deg

FIGURE 1-1: THE RELATIONSHIP BETWEEN SPURIOUS SIGNAL LEVEL AND PHASE STABILITY

more than two sources do cause ripple. Generally the ripple period depends on the distance between those sources, and the larger the distance becomes, the smaller the period becomes. For the VLA Waveguide System, it is important to suppress the ripples of less than 50 MHz period to a level of $\leq$ 1 dB peak-to-peak and ripples of less than 10 MHz period to $\leq$ 0.2 dB peak-to-peak.

2.1 Spurious Signal Caused by Mode Conversion Effect

Actual cases of the spurious signal caused by mode conversion effect can be listed as follows:

	SOURCE	SPURIOUS MODES
A.	Between taper waveguides	TE_{on} $(n \geq 2)$
B.	Between flexible waveguides	$TE_{11} TE_{12} \cdots TE_{21} TE_{22} \cdots TM_{11}$
C.	Between imperfect connections	$TE_{11} TE_{12} \cdots TE_{21} TE_{22} \cdots TM_{11}$
D.	In uniform or random dimensional imperfections	Depends upon deformation
E.	Between TE_{on} mode generation source and helical coupler	TE_{on} $(n \geq 2)$ (especially TE_{02})
F.	Between Michelson interferometer couplers (beam splitter couplers)	TE_{on} $(n \geq 2)$

Out of those, C and D are small enough to neglect when helix waveguides with random length (as NRAO Spec.) are used. Much attention must be paid to A, E, and F, because TE_{on} $(n \geq 2)$ cannot be absorbed without special mode filter and the attenuation of TE_{on} mode in the main waveguide is too small to attenuate this effect.

2.2 Spurious Signal Caused by TE_{01} Mode Reflection Effect

As TE_{01} mode reflection effects in the VLA waveguide system, the following can be listed:

1. Between couplers (in main line)

A. Reflection from front port and back port

B. Reflection from front port and reverse coupling

2. Between a coupler (front port) and signal distributor

3. Between a coupler (coupled port) and circular-rectangular adapter

3. MODE CONVERSION EFFECT

3.1 Ripple Amount

When there are two mode conversion sources, as shown in Figure 3-1, the output TE_{01} amplitude is as follows:

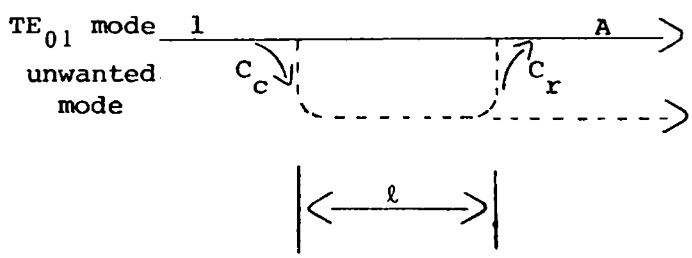


FIGURE 3-1

$$A = |A|e^{j\phi}$$

$$= \sqrt{1-C_c^2} \cdot \sqrt{1-C_r^2}\, e^{-\alpha_{01}\ell - j\beta_{01}\ell} - C_c C_r\, e^{-\alpha x\ell - j\beta x\ell} \qquad (3\text{-}1)$$

$$\doteq e^{-\alpha_{01}\ell - j\beta_{01}\ell} \left(1 - C_c C_r\, e^{-\Delta\alpha\ell} \cdot e^{-j\Delta\beta\ell}\right)$$

C_c, C_r; mode conversion coefficients

$\alpha_{01} + j\beta_{01}$; propagation constantant of TE_{01} mode

$\alpha x + j\beta x$; propagation constantant of spurious mode (unwanted)

$\Delta\alpha$ $(=\alpha x-\alpha_{01})$; the difference in attenuation constant

$\Delta\beta$ $(=\beta x-\beta_{01})$; the difference in phase constant

ℓ; the distance between two mode generation sources

In this case $C_c C_r \cdot e^{-\Delta\alpha\ell}$ corresponds to C in Formula 3-1 and to suppress R_{max}^{P-P} less than 0.1 dB

$$20 \log|C_c \cdot C_r \cdot e^{-\Delta\alpha\ell}| \leq -44 \text{ dB} \qquad (3\text{-}2)$$

must be fulfilled.

3.2 Ripple Period

Ripple period of this kind of ripple is determined by $\Delta\beta$ and ℓ.

Since

$$\frac{d(\Delta\beta x)}{df} = B_x = \frac{2\pi}{C}\left\{\frac{1}{\sqrt{1-(f_{cx}/f)^2}} - \frac{1}{\sqrt{1-(f_{c01}/f)^2}}\right\} \qquad (3\text{-}3)$$

f_{c01}; cut off frequency of TE_{01} mode

f_{cx}; cut off frequency of unwanted mode

C; velocity of light $(= \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.98 \times 10^8$ m/sec)

ripple period f_{px} can be shown to be:

$$f_{px} = \frac{2\pi}{B_x \cdot \ell} \qquad (3\text{-}4)$$

The values[1] of $|f_{px} \cdot \ell|$ $(= \frac{2\pi}{Bx})$ for critical spurious modes are shown in Figure 3-2 and the actual period of TE_{02} mode and $e^{-\Delta\alpha\ell}$ for different lengths are shown in Figure 3-3.

3.3 Contribution to Phase Stability of Pilot Signals

When the length of waveguide line changes with the temperature change, mode conversions that are mentioned above affect stability.

Two effects must be considered:

1) The phase of 5 MHz and 600 MHz pilot signals transmitted to the antennas through the waveguide are affected by both delay changes and mode-conversion

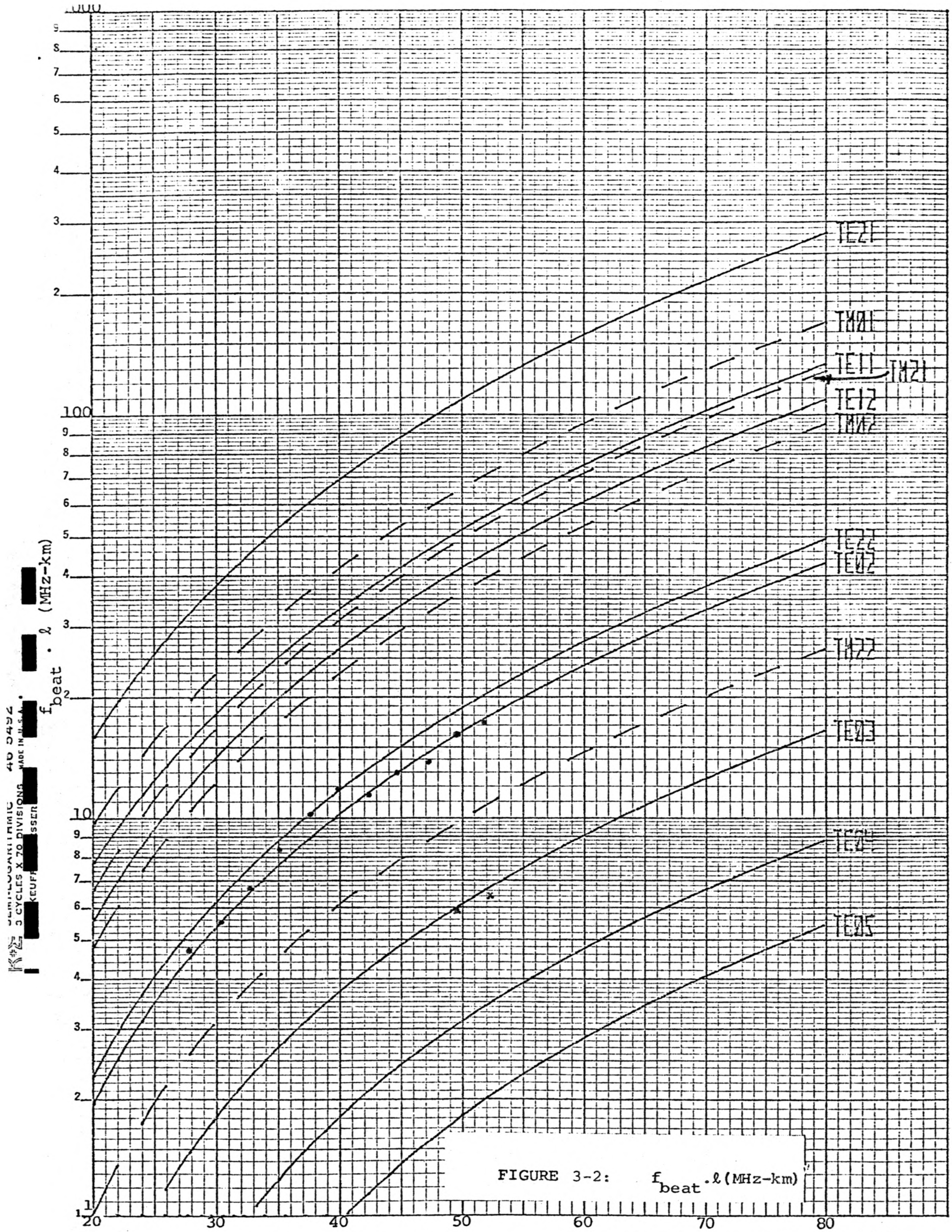


FIGURE 3-2: $f_{beat} \cdot \ell$ (MHz-km)

· experimental values of TE_{02} mode

x experimental values of TE_{03} mode

f_o (GHz)

{originally from reference (1)}

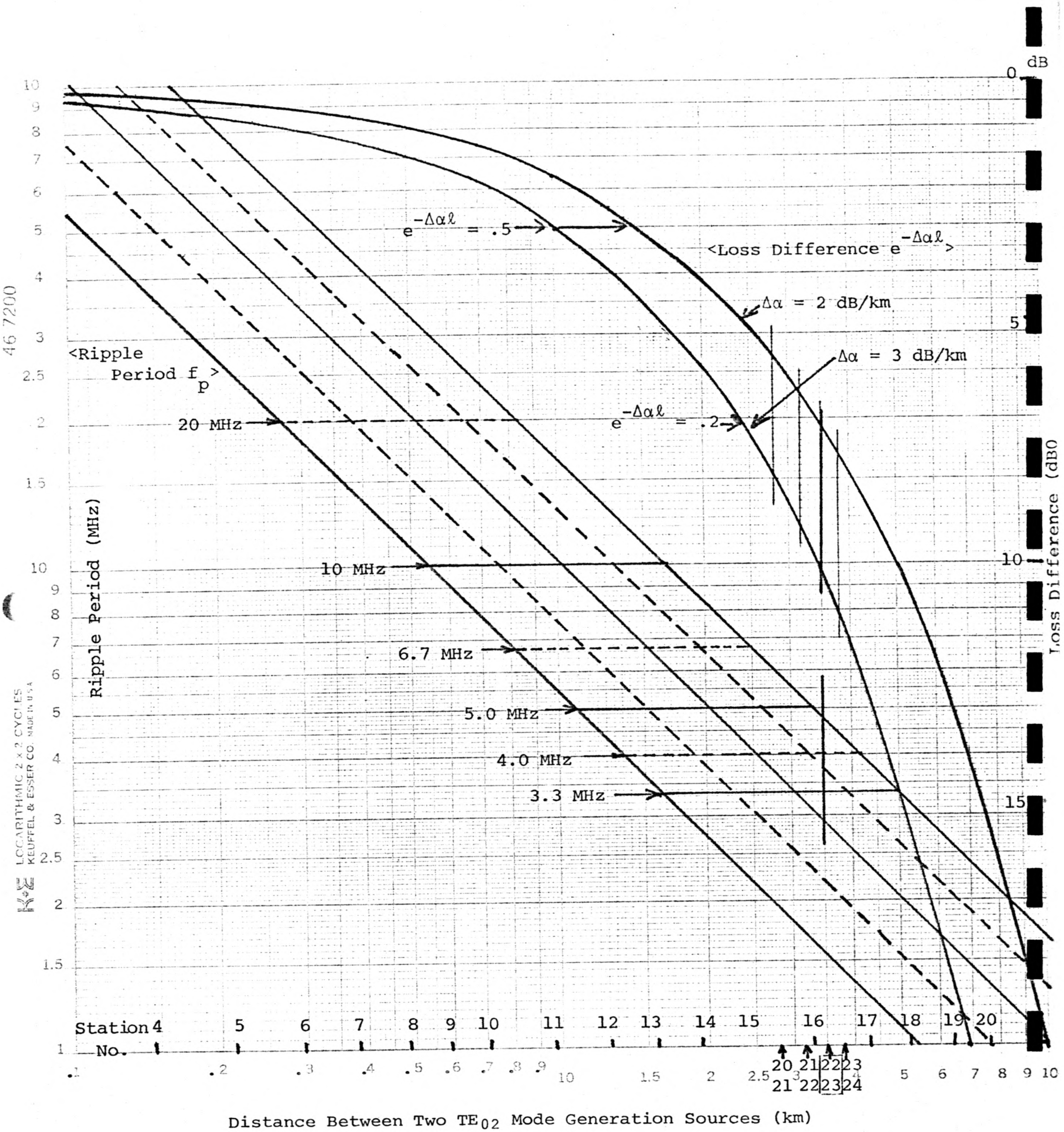


FIGURE 3-3: RIPPLE PERIOD CAUSED BY TE_{02} MODE CONVERSION EFFECT

phase changes. However, since the waveguide path is reciprocal, both of the changes are corrected by the L.O. round-trip measuring system. It is worthwhile, however, to calculate how much additional one-way instability is caused by mode conversion.

2) The round-trip L.O. correction system will have an offset in the 1 KHz to 100 KHz range between outgoing and return pilot signals. Any phase change which occurs between pilot signals differing by this offset frequency will <u>not</u> be corrected by the round-trip system. Hence it is important to predict the magnitude of this phase change.

Both of the above effects can be analyzed as follows:

$$/A_0 = e^{-\alpha 01 \ell} \cdot e^{-j\beta 01 \ell} \left(1 - C_c \cdot C_r \cdot e^{-\Delta\alpha \cdot \ell}\, e^{-j\Delta\beta\ell}\right) \quad (3\text{-}5)$$

$$/A_0' = e^{-\alpha 01' \ell} \cdot e^{-j\beta 01' \ell} \left(1 - C_c \cdot C_r \cdot e^{-\Delta\alpha' \cdot \ell}\, e^{-j\Delta\beta'\ell}\right) \quad (3\text{-}6)$$

$/A_0$, $/A_0'$; amplitude of TE_{01} mode shown in vector at frequency f_{p1} and f_{p2}

α_{01}, α_{01}'; attenuation constant of TE_{01} mode at f_{p1} and f_{p2}

β_{01}, β_{01}'; phase constant of TE_{01} mode at f_{p1} and f_{p2}

$\Delta\alpha$, $\Delta\alpha'$; the difference in attenuation constant between TE_{01} mode and spurious mode at f_{p1} and f_{p2}

$\Delta\beta$, $\Delta\beta'$; the difference in phase constant between TE_{01} mode and spurious mode at f_{p1} and f_{p2}

Phase shift caused by mode conversions Φ_0 and Φ_0' can be shown as follows:

$$\Phi_0 = \Phi_\ell + \Phi_c$$

$$= -\beta_{01}\cdot\ell + \sin^{-1}(C_c\cdot C_r\cdot e^{-\Delta\alpha\ell}\cdot\sin\Delta\beta\ell) \qquad (3\text{-}7)$$

$$\doteqdot -\beta_{01}\cdot\ell + C_c\cdot C_r\cdot e^{-\Delta\alpha\ell}\sin\Delta\beta\ell$$

$$\Phi_0' = \Phi'_\ell + \Phi'_c$$

$$\doteqdot -\beta_{01}'\cdot\ell + C_c'C_r'\cdot e^{-\Delta\alpha'\ell}\sin\Delta\beta'\ell \qquad (3\text{-}8)$$

(in case of $C_cC_re^{-\Delta\alpha\ell}\leq 0.01$)

where

Φ_0, Φ_0'; phase shift of TE_{01} mode at f_{p1} and f_{p2}

Φ_ℓ, Φ_ℓ'; phase shift caused by first order delay

Φ_c, Φ_c'; phase shift caused by mode conversion

3.4 The Instability of Pilot Signals

The phase shift difference $\Delta\Phi_p$ between two pilot frequencies f_{p1} and f_{p2} can be calculated as follows:

$$\Delta\Phi_p = \Phi_0 - \Phi_0' \qquad (3\text{-}9)$$

$$= -(\beta_{01} - \beta_{01}')\cdot\ell + C_c\cdot C_r\cdot e^{-\Delta\alpha\cdot\ell}\sin\Delta\beta\ell - C_c'\cdot C_r' e^{-\Delta\alpha'\ell}\sin\Delta\beta'\ell$$

When f_{p1} and f_{p2} are close enough to assume $C_c = C_c'$, $C_r = C_r'$, $\Delta\alpha = \Delta\alpha'$, $\Delta\Phi_p$ becomes:

$$\Delta\Phi_p = \Delta\Phi_\ell + \Delta\Phi_c$$

$$= -(\beta_{01} - \beta_{01}')\ell + C_c\cdot C_r\cdot e^{-\Delta\alpha\ell}(\sin\Delta\beta\ell - \sin\Delta\beta'\ell) \qquad (3\text{-}10)$$

$$= -(\beta_{01} - \beta_{01}')\ell + C_c\cdot C_r\cdot e^{-\Delta\alpha\ell}\cdot 2\cos\frac{\Delta\beta\ell+\Delta\beta'\ell}{2}\sin\frac{\Delta\beta\ell-\Delta\beta'\ell}{2}$$

where

$\Delta\Phi\ell$; phase difference derived from the first order delay between f_{p1} and f_{p2}

$\Delta\Phi_c$; phase difference derived from mode conversion effect between f_{p1} and f_{p2}

Using Formula (3-3) and (3-4), $\Delta\Phi_c$ can be expressed as follows:

$$\Delta\Phi_c = C_c \cdot C_r \; e^{-\Delta\alpha\ell}2 \cdot \cos \frac{(\Delta\beta+\Delta\beta')\ell}{2} \; \sin \frac{\Delta f_p}{f_{px}} \cdot \pi \qquad (3\text{-}11)$$

where

Δf_p; frequency difference between two pilot signals

f_{px}; ripple period around pilot signals $(=\frac{2\pi}{B_x\ell})$

Therefore, the mode conversion that causes the ripple whose period is $f_{px} = \frac{\Delta f_p}{m}$ (m = 1, 2,$\cdots$) does not affect the stability of L.O. measuring system. As our f_{px} and $C_c \cdot C_r e^{-\Delta\alpha\ell}$ can be got from loss measurement, $\Delta\Phi_c$ can be presumed from this formula.

$\Delta\Phi_c$ can also be shown in graph as a function of distance ℓ. One example is shown in Figure 3-4. It shows, even the same amount of the distance change happens, the change of $\Delta\Phi_c$ is different from distance to distance and depends upon the initial distance. The smaller mechanical ripple period ℓ_1

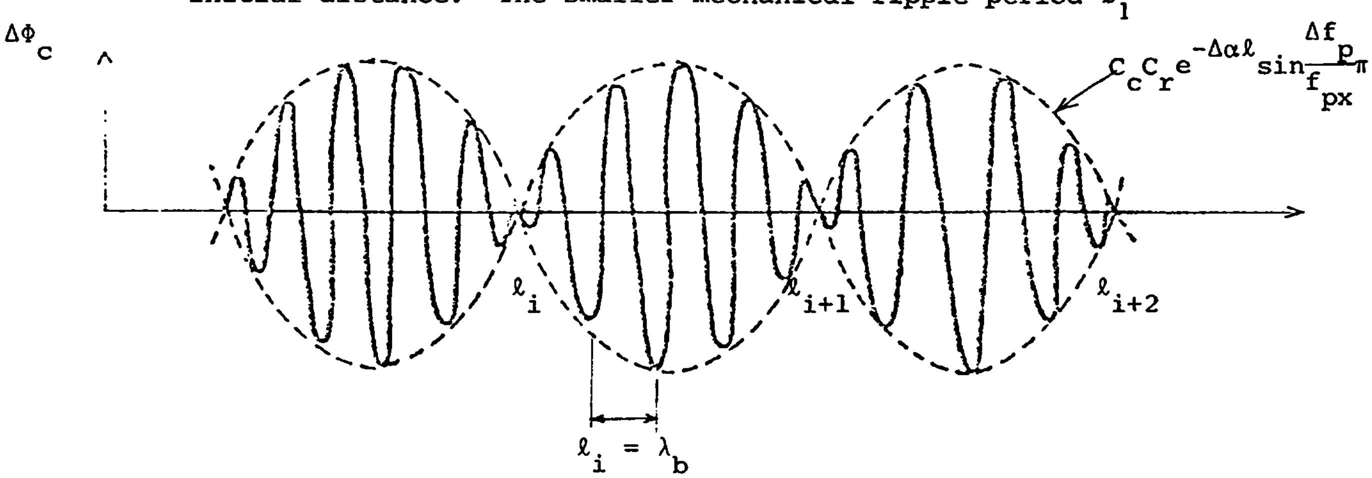


FIGURE 3-4

can be expressed as follows:

$$\ell_1 = \frac{4\pi}{\Delta\beta+\Delta\beta'} \tag{3-12}$$

$$\doteqdot \frac{2\pi}{\Delta\beta} = \lambda_b$$ (beat wavelength between TE_{01} mode and spurious mode, shown in Figure 3-6)

The distances that offer $\Phi_c = 0$ can be expressed as follows:

$$\ell_i = \frac{2\pi}{\Delta\beta-\Delta\beta'}\ i \quad (i = 1, 2, \cdots)$$

$$= \frac{2\pi}{B_x \Delta f_p}\ i \tag{3-13}$$

As shown in Figure 3-2, $\frac{2\pi}{B_x}$ depends upon frequency and spurious mode and it is 7.8 (MHz·Km) for TE_{02} mode at 35 GHz in 60 mm waveguide.

When pilot signals of both 5 MHz and 600 MHz are concerned, $|\ell_{i+1} - \ell_i|$ becomes more than 7.5 km and much bigger than ℓ_1.

Therefore, when one spurious mode generation exists at the station, the channel that has the frequency close to the frequency fulfilling the following formula should be used.

$$\frac{2\pi}{B_x} = \frac{\Delta f_p \ell_s}{i} \quad (i = 1, 2, \cdots) \tag{3-14}$$

where

ℓ_s; the distance between the station and another spurious mode generation source

In such a case, the distance change from ℓ_s to $\ell_s + \Delta\ell$ causes the following phase difference change $\Delta\Delta\Phi_c$:

$$\Delta\Delta\Phi_c = 2\cdot C_c\cdot C_r\cdot e^{-\Delta\alpha\ell}\cos\frac{(\Delta\beta+\Delta\beta')(\ell_s+\Delta\ell)}{2}\cdot\sin\frac{(\Delta\beta-\Delta\beta')(\ell_s+\Delta\ell)}{2}$$

$$\doteqdot (-1)^i C_c\cdot C_r\cdot e^{-\Delta\alpha\ell}\cos\left[\frac{(\Delta\beta+\Delta\beta')(\ell_s+\Delta\ell)}{2}\right]\cdot(\Delta\beta-\Delta\beta')\Delta\ell \tag{3-15}$$

$\Delta\Delta\Phi_c$ for MHz pilot signals on carrier around 35 GHz becomes as follows:

$$\begin{aligned} \Delta\Delta\Phi_c &\leq C_c \cdot C_r\, e^{-\Delta\alpha\ell} \cdot B_x \Delta f_p \cdot \Delta\ell \\ &\doteqdot 4.68 \times 10^{-4} \cdot C_c \cdot C_r e^{-\Delta\alpha\ell} \Delta\ell \,(\text{deg}) \end{aligned} \tag{3-16}$$

When $C_c \cdot C_r$ is specified below 0.0057 (-44 dB) this is negligible compared with the contribution by the first order delay shown below:

$$\begin{aligned} \Delta\Delta\Phi_\ell &= -(\beta_{01} - \beta_{01}{}') \Delta\ell \\ &\doteqdot .012\ \Delta\ell \text{ deg } (\Delta\ell;\ \text{in mm}) \end{aligned} \tag{3-17}$$

But if channel allocation to the station was very bad, $\Delta\Delta\Phi_c$ could increase as follows:

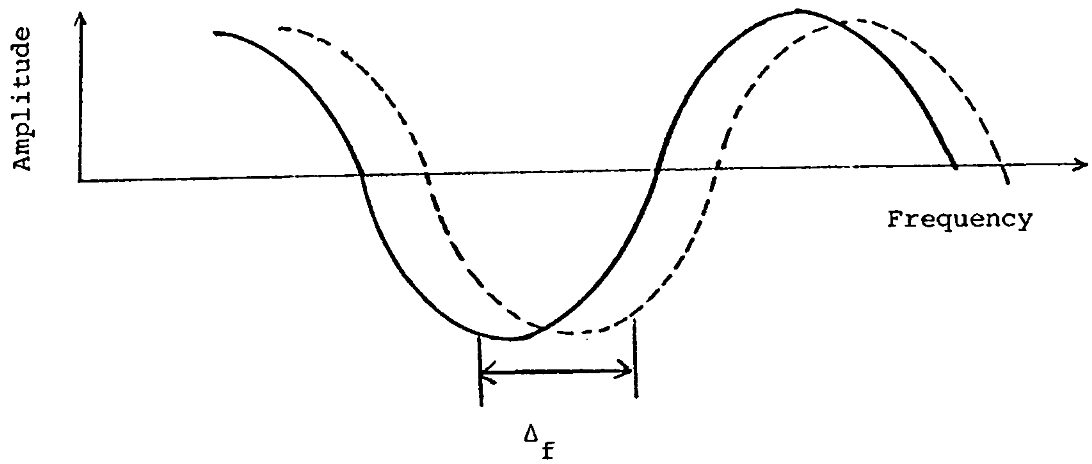
$$\begin{aligned} \Delta\Delta\Phi_c &\doteqdot C_c \cdot C_r\, e^{-\Delta\alpha\ell} 2\Delta\beta \cdot \Delta\ell \\ &= 4 \cdot C_c \cdot C_r e^{-\Delta\alpha\ell} \cdot \frac{\Delta\ell}{\lambda_b} \pi \\ &\doteqdot 3.4 \times C_c \cdot C_r \cdot e^{-\Delta\alpha\ell} \cdot \Delta\ell \ (\text{deg}) \end{aligned} \tag{3-18}$$

This could be comparable with $\Delta\Delta\Phi_\ell$ in actual case. It means when the distance is carefully set, the effect of first delay could be reduced by mode conversion.

3.5 Instability of Amplitude Response

If mode conversion sources exist in line, the distance change between these sources does not only cause affect to the stability of the L.O. system but also to the amplitude response. The amplitude response to frequency involving mode conversion effect has ripples of which properties are shown in 3-1 and 3-2. It can be shown on Figure 3-5. When the distance becomes longer by $\Delta \ell$ the frequency response shifts toward lower frequency as shown in Figure 3-5. And the relationship between Δf and $\Delta \ell$ can be shown as follows:

$$\Delta f = \frac{\Delta \beta}{B_x} \cdot \frac{\Delta \ell}{\ell} \qquad (3\text{-}19)$$



And the change $\Delta \ell_p$ that gives $\Delta f = f_{px}$ can be expressed as follows:

$$\Delta \ell_p = \frac{2\pi}{\Delta \beta} (= \lambda_b) \text{ (beat wavelength between } TE_{01} \text{ and spurious mode, shown in Figure 3-6)} \qquad (3\text{-}20)$$

Effect of 1-100 KHz offset frequency will be discussed in Appendix A.

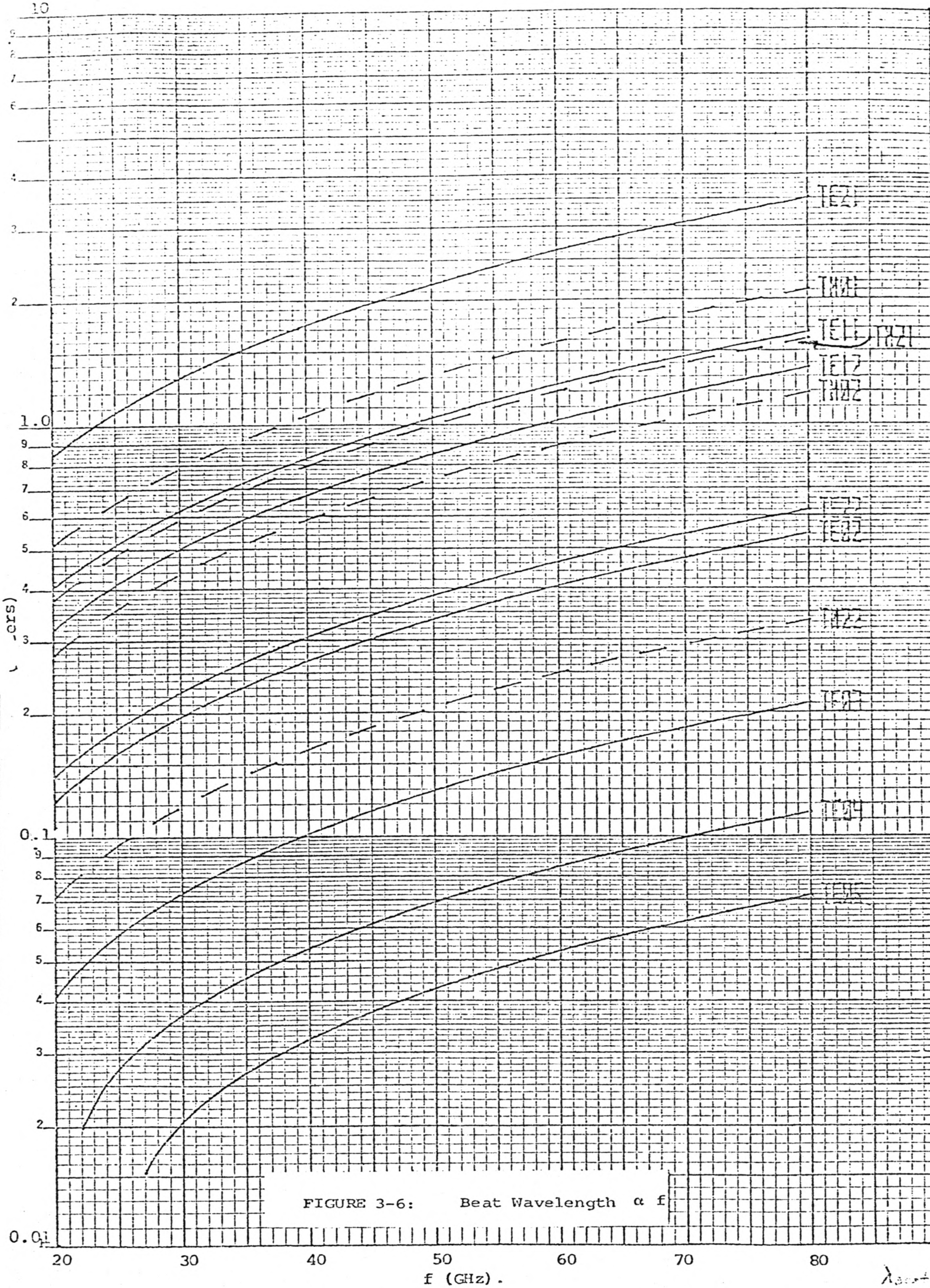


FIGURE 3-6: Beat Wavelength α f

4. TE_{02} MODE COUPLING EFFECT IN HELICAL COUPLER

A helical coupler couples not only to TE_{01} mode but also to TE_{on} mode. This effect can be considered similarly to mode conversion effect as explained in Section 3.

4.1 Ripple Amplitude

Output to rectangular waveguide is shown as follows:

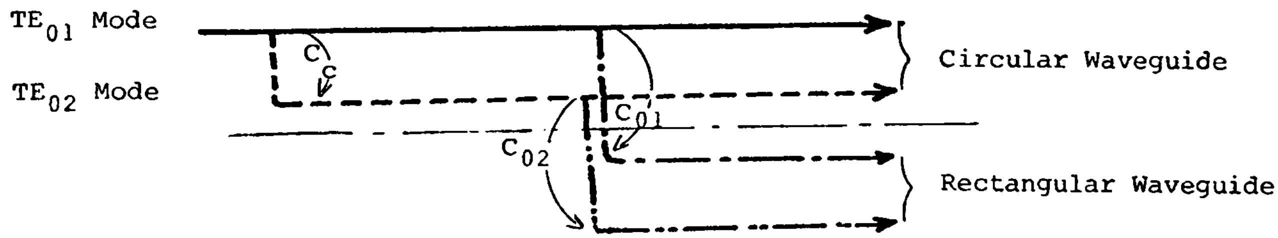


FIGURE 4-1

$$A = e^{-\alpha_{01}\cdot\ell}\cdot e^{-j\beta_{01}\cdot\ell}\cdot C_{01}\{1 - C_c\cdot e^{-\Delta\alpha_{02}\ell}\,(\frac{C_{02}}{C_{01}})\,e^{j\Delta\beta_{02}}\} \quad (4\text{-}1)$$

C_c; TE_{02} mode generation at taper waveguide

C_{01}, C_{02}; TE_{01}, TE_{02} mode coupling in helical coupler

α_{01}; TE_{01} mode loss (shown in amplitude)

$\Delta\alpha_{02}$; difference in attenuation constant between TE_{01} mode and TE_{02} mode

$\Delta\beta_{02}$; difference in phase constant between TE_{01} mode and TE_{02} mode

ℓ; distance between taper waveguide or other TE_{02} mode generation source and helical coupler

In this case $C_c\cdot e^{-\Delta\alpha\ell}\cdot(\frac{C_{02}}{C_{01}})$ corresponds to C in Formula 1-1 and to suppress R_{max}^{P-P} less than 0.1 dB

$$20\log|C_c\cdot e^{-\Delta\alpha\cdot\ell}\cdot(\frac{C_{02}}{C_{01}})| \leq -44 \text{ dB} \quad (4\text{-}2)$$

is required. In the conventional design of helical coupler

$$20 \log \left| \frac{C_{02}}{C_{01}} \right| \doteqdot -10 \text{ dB}$$

at the edges of 1 GHz frequency band.

In the case of using such helical couplers

$$\left| 20 \log C_c + 20 \log (e^{-\Delta\alpha \cdot \ell}) \right| \geq 34 \text{ dB}$$

is required.

4.2 Ripple Period

The same argument to that presented in Section 3.2 is valid in this case.

4.3 Contribution to Pilot Signal Phase Stability

The similar arguments to that presented in Section 3.3, 3.4 and 3.5 are valid in this case.

5. TE_{on} MODE CONVERSION EFFECT CAUSED BY MANY TE_{on} MODE CONVERSION SOURCES

When there are many spurious mode conversion sources along waveguide lines as shown in Figure 5-1 the attenuation characteristics of TE_{01} mode can be expressed as follows:[2]

$$A = e^{-\alpha_{01} L - j\beta_{01} L} \cdot e^{-\gamma} \qquad (5\text{-}1)$$

where

$$\gamma = (1-a_1-a_2\cdots a_n) + \sum_{i,j>i}^{n} C_i C_j \; e^{-\Delta\alpha\cdot\ell ij}\cdot e^{-j\Delta\beta\cdot\ell ij} \quad (5\text{-}2)$$

$a_1, a_2\cdots a_n$; insertion loss of waveguide components such as couplers or tapers

L = total length

ℓij; the distance between i th component and j th component

The second term in Formula 5-2 causes ripple in attenuation, and variation of attenuation, σ^2, is given by:

$$\sigma^2 = \frac{1}{2} \sum_{i,j>i}^{n} \{C_i\cdot C_j \; e^{-\Delta\alpha\ell ij}\}^2 \quad (\text{neper}^2) \quad (5\text{-}3)$$

And maximum variation of this attenuation R_{max}^{P-P} can be shown to be:

$$R_{max}^{P-P} = 2 \sum_{i,j>i}^{n} |C_i\cdot C_j|\cdot e^{-\Delta\alpha\ell ij} \quad (\text{neper}) \quad (5\text{-}4)$$

When mode conversion amounts of all sources are equal as follows:

$$C_1 = C_2 = \cdots = C_n = C_0 \quad (5\text{-}5)$$

those values are simplified as follows:

$$\sigma^2 = \frac{1}{2} \sum_{i,j>i}^{n} (e^{-\Delta\alpha\ell ij})^2\cdot C_0^{\,4} \quad (\text{neper}^2) \quad (5\text{-}6)$$

$$R_{max}^{P-P} = 2 \sum_{i,j>i}^{n} e^{-\Delta\alpha\ell ij}\cdot C_0^{\,2} \quad (\text{neper}) \quad (5\text{-}7)$$

MODE GENERATION SOURCE NUMBER i	1	2	3	4	5
	AW5	AW6	AW7	AW8	AW9
	ℓ_{12}	ℓ_{23}	ℓ_{34}	ℓ_{45}	
DISTANCE FROM THE CENTER (Km)	7.66	10.47	13.64	17.16	21.0
MODE CONVERSION COEFFICIENT BETWEEN TE_{01} AND TE_{02} MODES	C_1	C_2	C_3	C_4	
CHANNEL ASSIGNED	M_4	M_3	M_2	M_1	M_0
COUPLING VALUE	C_{05}	C_{06}	C_{07}	C_{08}	0

<Some possibility on design and channel allocation>

1. Channel M_0 and M_1 should be adjacent to each other when it is hard to get sufficient TE_{02} mode suppression in wide band.
2. To decrease coupled power loss, M_0 and M_1, should not be adjacent to each other.

FIGURE 5-1: DIAGRAM FOR TE_{02} MODE CONVERSION EFFECT

When the specification for the total transmission performance is available in the term of σ^2 or R_{max}^{P-P}, the requirement for C or maximum allowable n can be determined.

As an actual case in the VLA waveguide system, couplers that generate some amount of TE_{02} mode at stations AW5 to AW8 on the southwest arm can be considered. In this case σ^2 and R_{max}^{P-P} can be written as follows with some assumption of $\Delta\alpha$.

$$\sigma^2 = 0.42 \cdot C_0^4 \text{ (in case of } \Delta\alpha = 2 \text{ dB/km)} \qquad \text{(neper)}$$

$$R_{max}^{P-P} = 17.3 \times 2.04 \times C_0^2 \text{ (in case of } \Delta\alpha = 2 \text{ dB/km)} \quad \text{(dB)}$$

When the specifications are $4\sigma \leq 0.1$ dB or $R_{max}^{P-P} \leq 0.1$ dB, the maximum allowable C_0 is:

$$C_0 \leq 0.067 \text{ (-23.5 dB)} \quad \text{for} \quad 4\sigma \leq 0.1 \text{ dB}$$

$$C_0 \leq 0.053 \text{ (-25.5 dB)} \quad \text{for} \quad R_{max}^{P-P} \leq 0.1 \text{ dB}$$

Maximum allowable C_0 for the same requirement in different cases is shown in Figure 5-3.

The above argument is applicable for the evaluation of Michelson interferometer couplers* that look promising as tight couplers but generate TE_{on} $(n \geq 2)$ modes.

Judging from Figure 5-3 and some other preliminary test results of this coupler, it looks possible to have at least three of these on one arm. But it may be necessary to suppress TE_{02} mode to have more than four of these couplers.

*also referred to as beamsplitter couplers

n	Combination Of TE_{02} Mode Generation Sources	The Distance Between Two Sources	Loss Difference In Main Line $(e^{-\Delta\alpha\ell})$		Remarks
			$\Delta\alpha$=2 dB/km	$\Delta\alpha$=3 dB/km	
1.	AW5-AW6	2.81 km	0.53	0.38	
2.	AW6-AW7	3.17 km	0.49	0.33	
3.	AW7-AW8	3.52 km	0.45	0.30	
4.	AW5-AW7	5.99 km	0.25	0.13	
5.	AW6-AW8	6.68 km	0.21	0.10	
6.	AW5-AW8	9.49 km	0.11	0.038	
Case I.	Couplers* are installed at AW5~8	$\sum_{n=1}^{6} (e^{-\Delta\alpha\ell_n})^2$	0.842	0.372	
Case II.	Couplers* are installed at AW6~8	$\sum_{n=2,3,5,6} (e^{-\Delta\alpha\ell_n})^2$	0.499	0.210	
Case III.	Couplers* are installed at AW7~8	$\sum_{n=3} (e^{-\Delta\alpha\ell_n})^2$	0.202	0.09	

*couplers generate TE_{02} mode in the main line

Maximum Allowable Mode Conversion

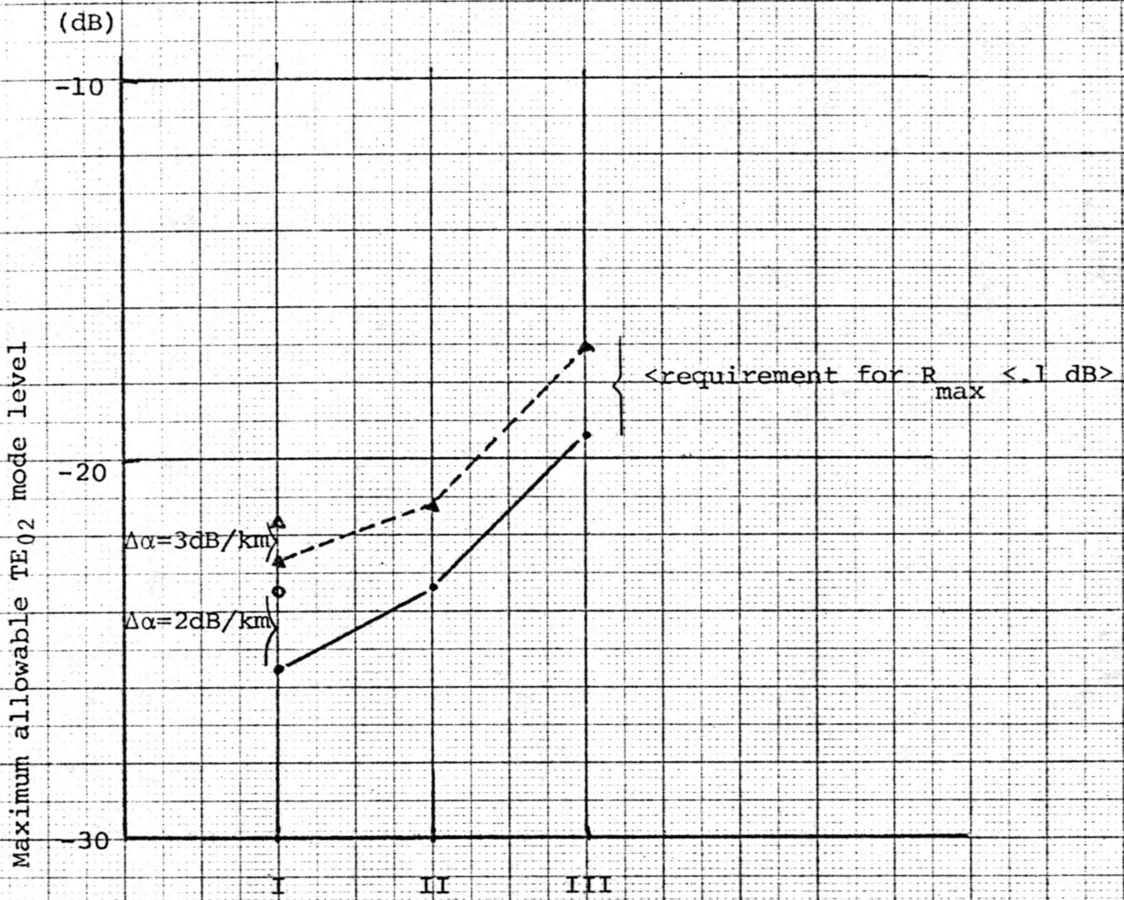
A. Requirement: $4\sigma<0.1$ dB

	$\Delta\alpha$=2 dB/km	$\Delta\alpha$=3 dB/km
Case I.	0.047(-26.5 dB)	0.058(-24.7 dB)
Case II.	0.054(-25.8 dB)	0.067(-23.5 dB)
Case III.	0.068(-23.4 dB)	0.083(-21.6 dB)

B. Requirement: $R_{max}^{P-P} \leq 0.1$ dB

Case I.	0.053(-25.5 dB)	0.073(-22.7 dB)
Case II.	0.068(-23.4 dB)	0.087(-21.2 dB)
Case III.	0.107(-19.4 dB)	0.139(-17.1 dB)

FIGURE 5-2: EVALUATION OF TE_{02} MODE CONVERSION EFFECT



Case I: Couplers that have some amount of TE_{02} mode generation are installed at AW5~AW8 on the southwest arm.

Case II: Couplers that have some amount of TE_{02} mode generation are installed at AW6~AW8 on the southwest arm.

Case III: Couplers that have some amount of TE_{02} mode generation are installed at AW7~AW8 on the southwest arm.

FIGURE 5-3: REQUIREMENT FOR TE_{02} MODE SUPPRESSION OF COUPLER INSTALLED AW5~AW8

6. LOSS CHANGE AND PILOT SIGNAL PHASE STABILITY UNDER ANTENNA ROTATION

6.1 Loss Change

When there are two mode generation sources at both sides of the rotary joint (like the configuration around elevation rotary joint in the case shown in Figure 6-1), the rotation of the joints causes loss changes as shown below even if the rotary joint is perfect:

$$A = e^{-\alpha 01 \ell}\{1 - C_{f1} \cdot C_{f2} \cdot e^{-\alpha a} \cdot e^{-j\Delta\beta\ell} \cdot e^{j(n\phi+\phi_i)}\} \qquad (6\text{-}1)$$

where

n; the circumferential order of spurious mode (unwanted)

C_{f1}, C_{f2}; mode conversion in flexible waveguide

20 log $e^{-\alpha a}$; spurious mode attenuation between two flexible waveguides (dB) (is caused by 40" rigid helix waveguide in the actual case)

α_{01}; attenuation constant of TE_{01} mode

$\Delta\beta$; the difference in phase constant between spurious mode and TE_{01} mode

others; shown in Figure 6-1

Usually flexible waveguide generates TE_{11}, TE_{12}, TM_{11} modes because of its bending and TE_{21}, TE_{22}, because of its elliptical deformation. These modes are not pure modes in solid waveguide but hybrid modes. Thus $\Delta\beta$ is different from that in solid waveguide.

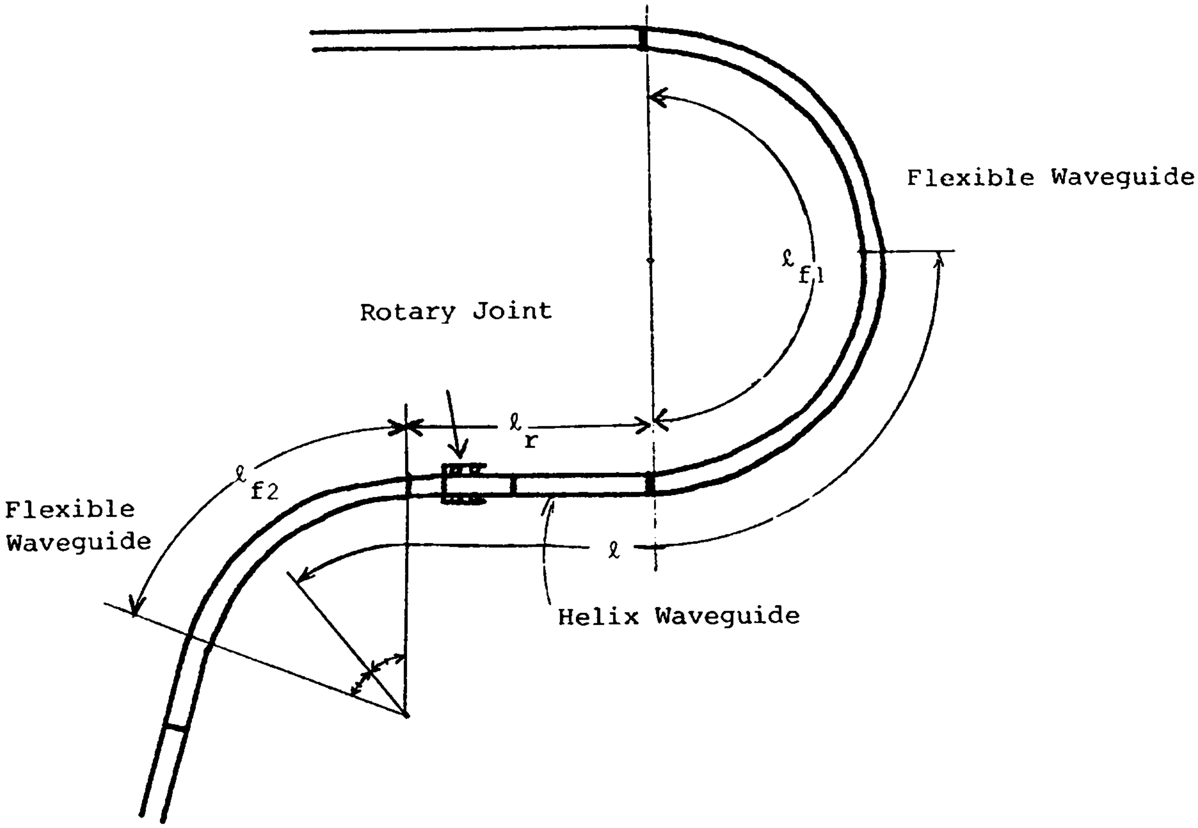
In such cases loss variation R_{max}^{P-P} becomes as follows:

$$R_{max}^{P-P} = 20 \log (1 + C_{f1} \cdot C_{f2} \cdot e^{-\alpha a}) - 20 \log (1 - \frac{1}{2} C_{f1} \cdot C_{f2} \cdot e^{-\alpha a})$$

$$\fallingdotseq 13.0\ C_{f1} \cdot C_{f2} \cdot e^{-\alpha a}\ \text{(dB)} \quad \text{(for } n=1) \qquad (6\text{-}2)$$

$$R_{max}^{P-P} = 20 \log (1 + C_{f1} \cdot C_{f2} \cdot e^{-\alpha a}) - 20 \log (1 - C_{f1} \cdot C_{f2} \cdot e^{-\alpha a})$$

$$\fallingdotseq 17.3\ C_{f1} \cdot C_{f2} \cdot e^{-\alpha a}\ \text{(dB)} \quad \text{(for } n=2) \qquad (6\text{-}3)$$



$$\ell = (\ell_{f1} + \ell_{f2})/2 + \ell r$$

ϕ_0; angle deviation from initial angle

(-30^0 ~ 80^0 in our case)

(-30^0 ~ 90^0 is used for analysis)

ϕ_i; initial angle ($=180^0$ in above case)

ℓ_{f1}, ℓ_{f2}; the length of bend part of flexible waveguides

ℓ; equivalent length between two bended waveguides

FIGURE 6-1: ANTENNA WAVEGUIDE CONFIGURATION AROUND ELEVATION ROTARY JOINT

When the bend or elliptical deformation is uniform along the bent sections, mode conversion amount C_{f1}, C_{f2} depends upon frequency and length of flexible waveguides and is given as follows:[3]

$$C_{f1} = C_0 \sin(\Delta\beta\ell_{f1}/2) \quad (6\text{-}3)$$

$$C_{f2} = C_0 \sin(\Delta\beta\ell_{f2}/2) \quad (6\text{-}4)$$

where

C_0; maximum conversion

This means that at certain frequencies C_{f1} and C_{f2} become zero and loss variation does not occur even under rotation of antenna. The frequency period that has no loss change is defined as $f_p/2$ for later.

6.2 Pilot Signal Phase Stability

Mode conversions at both sides of the rotary joint cause not only loss change but also phase difference change of the two pilot signals. The latter can be analyzed as follows:

$$\Phi_r = \text{Sin}^{-1}\{C_c \cdot C_r \cdot \text{Sin}\Delta\beta\ell \cdot \text{Sin}(n\phi_0 + \phi_i)\}$$
$$\doteqdot C_c \cdot C_r \cdot \text{Sin}\Delta\beta\ell \cdot \text{Sin}(n\phi_0 + \phi_i) \quad (6\text{-}5)$$

$$\Phi_r' \doteqdot C_c' \cdot C_r' \cdot \text{Sin}\Delta\beta'\ell \cdot \text{Sin}(n\phi_0 + \phi_i) \quad (6\text{-}6)$$

$$\Delta\Phi_r = \Phi_r - \Phi_r'$$
$$= \text{Sin}(n\phi_0 + \phi_i)\ \{C_c \cdot C_r \cdot \text{Sin}\Delta\beta\ell - C_c' \cdot C_r' \text{Sin}\Delta\beta'\ell\} \quad (6\text{-}7)$$

where

Φ_r, Φ_r'; TE_{01} mode phase shift caused by mode conversions in both sides of rotary joint at frequency f_{p1} and f_{p2} on which two pilot signals are sent.

In the case of 5 MHz pilot signals, $C_c = C_c'$ and $C_r = C_r'$ can be assumed and $\Delta\Phi_r$ becomes as follows:

$$\Delta\Phi_r = \text{Sin}(n\phi_0+\phi_i)\cdot C_c\cdot C_r\{\text{Sin}\Delta\beta\ell - \text{Sin}\Delta\beta'\ell\}$$

$$\doteqdot \text{Sin}(n\phi_0+\phi_i)\cdot C_c\cdot C_r\cdot(\Delta\beta-\Delta\beta')\cdot\ell \qquad (6\text{-}8)$$

Maximum deviation $\Delta\Delta\Phi_{rmax}^{P-P}$ becomes as follows in the area of $-30^0 \leq \phi_0 \leq 90^0$; $(\phi_i = +180^0)$

$$\Delta\Delta\Phi_{rmax} = 1.5\cdot C_c\cdot C_r\cdot(\Delta\beta-\Delta\beta')\cdot\ell \qquad \text{(for n=1)} \qquad (6\text{-}9)$$

$$\Delta\Delta\Phi_{rmax} = 2\cdot C_c\cdot C_r\cdot(\Delta\beta-\Delta\beta')\cdot\ell \qquad \text{(for n=2)} \qquad (6\text{-}10)$$

From Formulas (6-2) and (6-3),

$C_c\cdot C_r$ can be derived from R_{max}^{P-P} as follows:

$$C_c\cdot C_r = \frac{R_{max}^{P-P}}{13.0} \quad - \quad \text{(for n=1)} \qquad (6\text{-}11)$$

$$C_c\cdot C_r = \frac{R_{max}^{P-P}}{17.3} \qquad \text{(for n=2)} \qquad (6\text{-}12)$$

From the definition of f_p

$(\Delta\beta-\Delta\beta')\cdot\ell$ can be shown to be:

$$(\Delta\beta-\Delta\beta')\ell = \frac{\Delta f_p}{f_p}\cdot 2\pi \quad \text{(rad)} \qquad (6\text{-}13)$$

where

f_p; the frequency period of no loss variation under antenna rotation (150~2000 MHz)

Δf_p; the frequency difference between two pilot signals

From (6-9) and (6-10)

Thus $\Delta\Delta\Phi_{rmax}$ can be written as follows:

$$\Delta\Delta\Phi_{rmax} = 3.1\text{x}10^2\ \frac{R_{max}^{P-P}}{f_p} \qquad \text{(degree)} \qquad (6\text{-}14)$$

The measured values of R_{max}^{P-P} and f_p on No. 2 antenna are 0.1~0.2 dB^{P-P} and 150~2000 MHz. So in the worst case

$$\Delta\Delta\Phi_{rmax\ worst} = 0.41 \quad \text{(degrees)} \tag{6-15}$$

In the case of 600 MHz pilot signals

$$\begin{aligned}\Delta\Delta\Phi_r &\leq 1.5\cdot(C_c\cdot C_r + C_c'\cdot C_r') \\ &= 3.12\cdot(R_{max}^{P-P} + R_{max}^{P-P'}) \quad \text{(deg)}\end{aligned} \tag{6-16}$$

R_{max}^{P-P}, $R_{max}^{P-P'}$; maximum variation caused by antenna rotation at f_p and f_p'

To maintain $\Delta\Delta\Phi_r$ less than 1 degree in the worst case, the following is required:

$$R_{max}^{P-P} + R_{max}^{P-P'} \leq 0.32 \text{ (dB)} \tag{6-17}$$

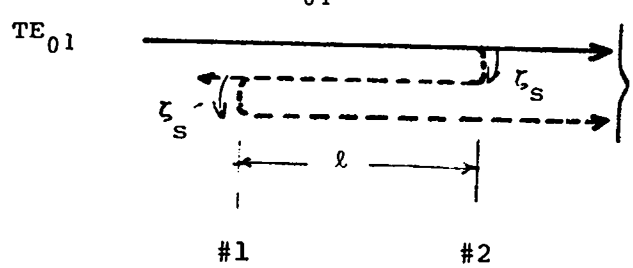
7. TE_{01} MODE REFLECTION EFFECT

7.1 Reflection Sources

Reflection sources in TE_{01} mode circular waveguide have many different properties from rectangular waveguides or TE_{11} mode circular waveguide. In the usual TE_{01} mode waveguides, dimensional imperfections cause forward-going unwanted mode waves, but very few TE_{01} mode reflections. The main sources of TE_{01} mode reflection are circular-rectangular adapters and couplers that are inserted in the main line.

7.2 Ripple Amplitude

When there are two reflection sources as shown in Figure 7-1, the output TE_{01} mode can be given as follows:



#1, #2, reflection sources

FIGURE 7-1

$$A = e^{-\alpha 01 \ell}(\sqrt{1-\zeta_s{}^2} \cdot \sqrt{1-\zeta_s{}'} + \zeta_s \zeta_s{}' \, e^{-2\alpha 01 \ell} \cdot e^{-2j\beta 01 \ell})$$
$$\doteq e^{-\alpha 01 \ell}(1 + \zeta_s \zeta_s{}' \, e^{-2\alpha 01 \ell} \cdot e^{-2j\beta 01 \ell}) \quad (7\text{-}1)$$

where

ζ_s, $\zeta_s{}'$; reflection co-efficient

$\alpha 01$; attenuation constant of TE_{01} mode

$\beta 01$; phase constant of TE_{01} mode

ℓ; the distance between two reflection sources

In such cases, $\zeta_s \cdot \zeta_s{}' \cdot e^{-2\alpha 01 \ell}$ corresponds to spurious signal level C shown in Formula (1-1).

To suppress peak-peak variation of attenuation less than 0.1 dB, the following requirement must be filled:

$$20 \log \left| \zeta_s \cdot \zeta_s{}' \cdot e^{-2\alpha 01 \ell} \right| \leq -44 \text{ dB} \quad (7\text{-}2)$$

7.3 Ripple Period

Ripple period of this kind of ripple is determined by TE_{01} mode phase constant β_{01} and the distance ℓ between reflection sources, and can be expressed as follows:

$$f_p = \frac{C}{2\ell} = \frac{2\pi}{2 \cdot \ell \frac{d\beta 01}{df}} \quad (7\text{-}3)$$

The figures for C at different frequencies are shown in Table 7-1. The higher the operational frequency becomes from cut-off frequency, the closer constant C becomes to 300 (MHz-m).

An antenna waveguide is about 40 m long and when there are reflections at both ends of the antenna waveguide the ripple period is approximately 3.75 MHz.

When this amplitude is too large, it might cause some problem in observation of spectrum line radio sources. This means that specification of TE_{01} mode reflection in rectangular-

	CONSTANT C (MHz m)	
Frequency (GHz)	60 mm Waveguide	20 mm Waveguide
30	304.4	376.0
35	302.8	349.6
40	301.6	335.1
45	300.7	326.2
50	300.2	320.2

TABLE 7-1: MULTIPLE OF RIPPLE PERIOD AND DISTANCE BETWEEN TWO REFLECTION SOURCES

circular adapter, modem input is very important. These should be suppressed to under - 25 dB.

In main line, as the distance between reflection sources becomes much longer except at the center of the "Y", the ripple period becomes much smaller.

7.4 Contribution to Phase Stability of Pilot Signals

When the length of waveguide line changes with the temperature, the ripple that arises from TE_{01} mode reflection affects the phase stability of pilot signals (together with other factors such as the first delay change or mode conversion effects). This contribution can be expressed as follows:

$$\mathbb{A}_0 = e^{-\alpha 01 \cdot \ell} \cdot e^{-j\beta 01 \ell}(1-\zeta_{st}\zeta_{sr} e^{-2\alpha 01 \ell} \cdot e^{-2j\beta 01 \ell}) \tag{7-4}$$

$$\mathbb{A}_0{}' = e^{-\alpha 01' \cdot \ell} \cdot e^{-j\beta' 01 \ell}(1-\zeta_{st}{}'\zeta_{sr}{}' e^{-2\alpha 01' \ell} \cdot e^{-2j\beta 01' \ell}) \tag{7-5}$$

where

$\mathbb{A}_0, \mathbb{A}_0{}'$; amplitude of TE_{01} mode shown in vector at frequency f_{p1}, f_{p2}

$\alpha 01$, $\alpha 01'$; attenuation constant of TE_{01} mode at f_{p1}, f_{p2}

$\beta 01$, $\beta 01'$; phase constant of TE_{01} mode at f_{p1}, f_{p2}

ζ_{st}, $\zeta_{st}{}'$; reflection co-efficient in circular port of transmitting equipment at f_{p1}, f_{p2}

ζ_{sr}, $\zeta_{sr}{}'$; reflection co-efficient in circular port of receiving equipment at f_{p1}, f_{p2}

Phase shifts Φ_0 and Φ_0' between transmitting and receiving ports can be shown as follows:

$$\Phi_0 = \Phi_\ell + \Phi_\zeta$$

$$= -\beta_{01}\cdot\ell + \mathrm{Sin}^{-1}(\zeta_{st}\,\zeta_{sr}\,e^{-2\alpha 01\cdot\ell}\mathrm{Sin}\cdot 2\beta 01\ell) \qquad (7\text{-}6)$$

$$= -\beta 01\cdot\ell + \zeta_{st}\,\zeta_{sr}\,e^{-2\alpha 01\cdot\ell}\mathrm{Sin}\cdot 2\beta 01\ell)$$

$$\Phi_0' = \Phi_\ell' + \Phi_\zeta'$$

$$= -\beta_{01}'\cdot\ell + \zeta_{st}'\cdot\zeta_{sr}'e^{-2\alpha_{01}'\cdot\ell}\mathrm{Sin}2\beta_{01}'\cdot\ell \qquad (7\text{-}7)$$

The difference in phase shifts Φ_0, Φ_0' between two pilot frequencies f_{p1} and f_{p2} can be expressed as follows:

$$\Delta\Phi_p = \Phi_0 - \Phi_0'$$

$$= -(\beta 01 - \beta 01')\cdot\ell + \zeta_{st}\zeta_{sr}e^{-2\alpha 01\ell}\mathrm{Sin}2\beta_{01}\ell \qquad (7\text{-}8)$$

$$- \zeta_{st}'\zeta_{sr}'e^{-2\alpha 01\ell'}\mathrm{Sin}2\beta_{01}'\ell$$

When f_{p1} and f_{p2} are close enough to assume $\zeta_{st} = \zeta_{st}'$, $\zeta_{sr} = \zeta_{sr}'$, $\alpha 01 = \alpha 01'$, $\Delta\Phi_p$ becomes as follows:

$$\Delta\Phi_p = \Delta\Phi_\ell + \Delta\Phi_\zeta$$

$$= -(\beta_{01} - \beta_{01}')\cdot\ell + \zeta_{st}\cdot\zeta_{sr}e^{-2\alpha_{01}\ell}(\mathrm{Sin}2\beta_{01}\ell - \mathrm{Sin}2\beta_{01}'\ell)$$

$$= -(\beta_{01} - \beta_{01}')\cdot\ell + \zeta_{st}\cdot\zeta_{sr}e^{-2\alpha_{01}\ell}\cdot 2\cos(\beta_{01}+\beta_{01}')\ell \qquad (7\text{-}9)$$

where $\qquad \sin(\beta_{01}-\beta_{01}')\ell$

$\Delta\Phi_\ell$; phase difference derived from the first order delay between f_{p1} and f_{p2}

$\Delta\Phi_\zeta$; phase difference derived from TE_{01} mode reflection effect

Using Formula 7-3 $\Delta\Phi_\zeta$ can be expressed as follows:

$$\Delta\Phi_\zeta = \zeta_{st} \cdot \zeta_{sr} \cdot e^{-2\alpha_{01}\ell} \cdot 2\cos(\beta_{01}+\beta'_{01})\cdot \ell \sin(\frac{\Delta f_p}{f_p}\cdot\pi) \qquad (7\text{-}10)$$

Δf_p; frequency difference between two pilot signals

f_p; ripple period around pilot signals $(=\frac{C}{x\cdot\ell})$

It shows that the TE_{01} mode reflections that cause the ripple whose period is $f_p = \frac{\Delta f_p}{m}$ (m=1,2,···) do not affect the L.O. system. As $\zeta_{st}\zeta_{sr}e^{-2\alpha_{01}\ell}$ and f_p can be known from loss measurement variation of $\Delta\Phi_\zeta$ can be presumed from this formula.

$\Delta\Phi_\zeta$ can also be shown in graph as a function of distance ℓ. It is shown in Figure 7-2.

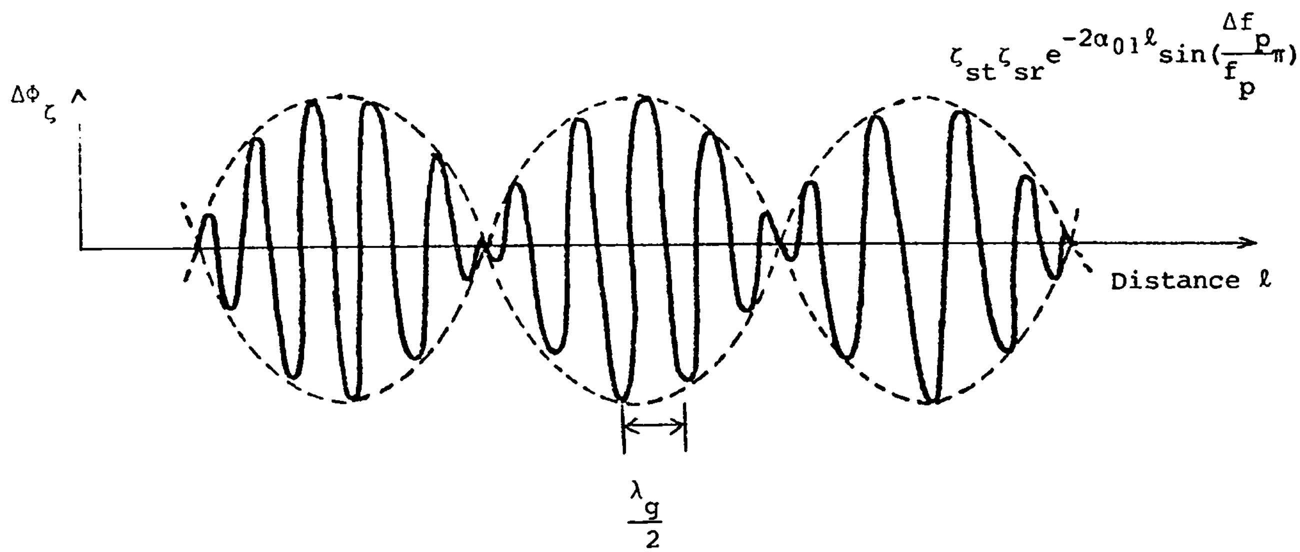


FIGURE 7-2

It shows, even if the same amount of the distance change happens, the change of $\Delta\Phi_\zeta$ varies with the initial distance. In the case of 5 MHz pilot signal, ℓ_2 is almost 15 m and when the distance between two reflection sources becomes integer times of this distance, variation of $\Delta\phi_\zeta$ with distance change $\Delta\ell$ can be minimized and shown as follows:

$$\Delta\Delta\Phi_\zeta = 2 \cdot \zeta_{st}\zeta_{sr}e^{-2\alpha_{01}\ell}\cos(\beta_{01}+\beta_{01}')(\ell_i+\Delta\ell)$$

$$\sin(\beta_{01}-\beta_{01}')(\ell_i+\Delta\ell) \quad (7\text{-}11)$$

$$\doteqdot (-1)^i\cdot 2\cdot\zeta_{st}\zeta_{sr}e^{-2\alpha_{01}\ell}\cdot(\beta_{01}-\beta_{01}')\Delta\ell\cdot\cos(\beta_{01}+\beta_{01}')\ell$$

$\Delta\Delta\Phi_\zeta$ for 5 MHz pilot signals can be got as follows:

$$\Delta\Delta\Phi_\zeta \leq 2.4 \times 10^{-3}\cdot\zeta_{st}\cdot\zeta_{sr}\ e^{-\alpha_{01}\ell}\Delta\ell \quad (7\text{-}12)$$

But when reflection sources exist with unfavorable distance $\Delta\Delta\Phi_\zeta$ could increase as follows:

$$\Delta\Delta\Phi_\zeta \leq 4 \cdot \zeta_{st}\zeta_{sr}\ e^{-2\alpha_{01}\ell} \quad (7\text{-}13)$$

The distance from a coupler to a circular-rectangular adapter in the vertex room is almost 40 m. This is not so bad a length but it seems nice to get pad about 13 m away from the coupler to minimize the unstability in wide frequency range. The pad will also be helpful to reduce the ripple caused between coupler and adapter. The position almost corresponds to the top of the ground.

7.4 Instability of Amplitude Response

Such reflections do not only affect the stability of L.O. system but also affect the stability of amplitude response when the distance between reflection sources changes. The ripple period of which property are explained in 7-1 and 7-2 can be shown in Figure 7-3. When the distance becomes longer by $\Delta\ell$ the frequency response shifts toward lower frequency as shown in Figure 7-3. The relationship between Δf and $\Delta\ell$ can be expressed as follows:

$$\Delta f = \frac{\beta}{\frac{d\beta}{df}} \cdot \frac{\Delta \ell}{\ell} \doteqdot \frac{c}{2\pi} \cdot \frac{\Delta \ell}{\ell} \qquad (7\text{-}14)$$

And, the change $\Delta\ell_p$ that gives $\Delta f = f_p$ can be expressed as follows:

$$\Delta \ell_p = \frac{2\pi}{2\beta'} = \frac{\lambda_g}{2'} \qquad (7\text{-}15)$$

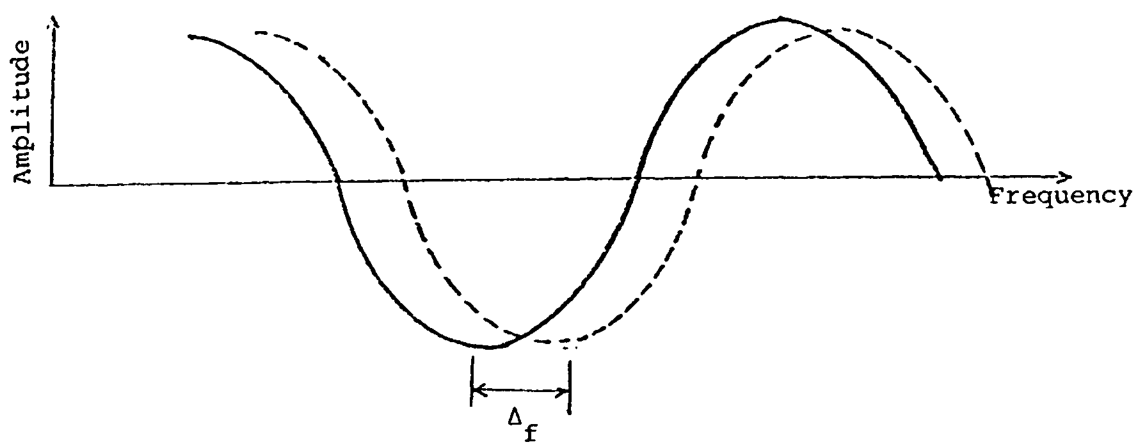


FIGURE 7-3

8. EFFECT OF DIRECTIVITY OF COUPLERS

When some power goes to the reverse direction from main power and it hits reflection sources, it comes back again in the forward direction and becomes a spurious signal to main power. This mechanism is shown in Figure 8-1. In this case, similar effect to

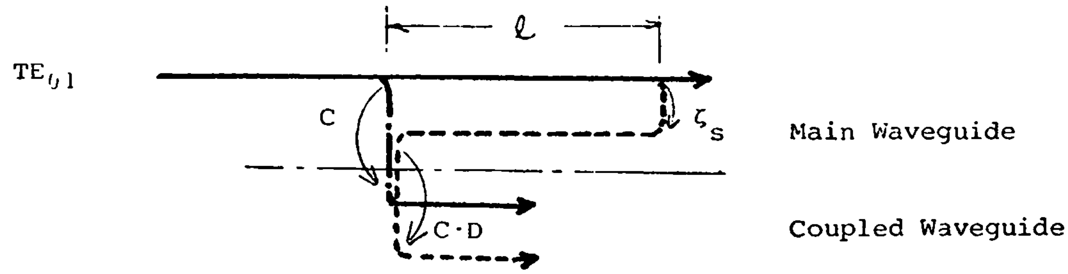


FIGURE 8-1

the reflection effect occurs. The ripple amplitude, period and contribution can be obtained by converting some symbols in the foregoing section as follows:

$\zeta_s{}' \rightarrow D$; directivity of coupler

$\zeta_s \rightarrow \zeta_s$

This shows the importance of coupler and to suppress the amplitude of this kind of ripples less than 0.1 dB

$$20 \log \left| \zeta_s \cdot e^{-2\alpha 01 \ell} \cdot D \right| \leq -44 \text{ dB}$$

9. TE_{01} MODE REFLECTION EFFECT CAUSED BY COUPLERS

When there are many TE_{01} mode reflection sources such as couplers along waveguide line, shown in Figure 9-1, the attenuation characteristics of ith coupler can be expressed as follows (by centeral limit theorem).

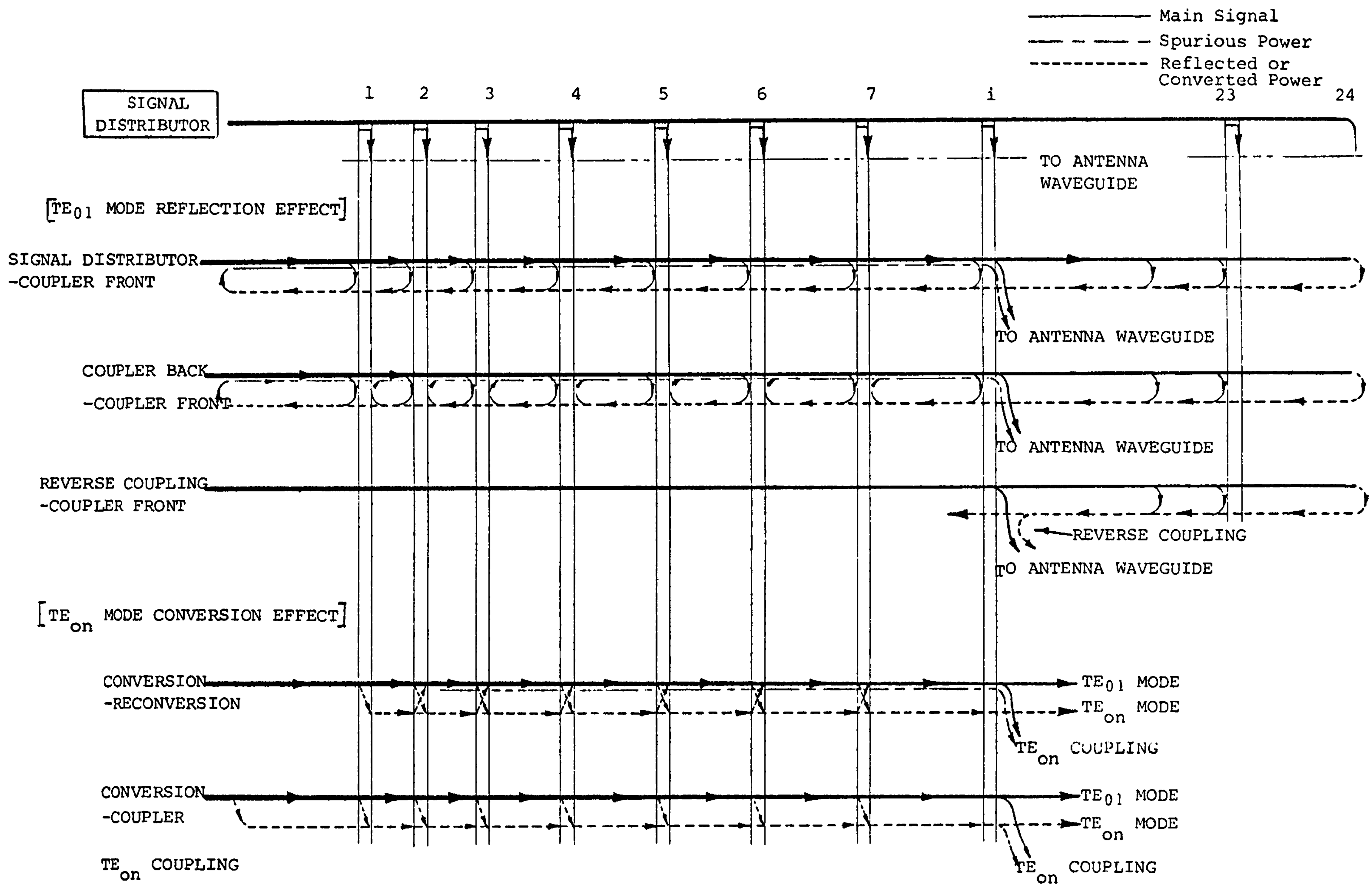


FIGURE 9-1: MULTI-REFLECTION AND MODE CONVERSION EFFECT

$$A = e^{-(\alpha+j\beta)(\ell_0+\ell_i)} e^{-(r_0+r_1+r_2+r_3)} \cdot C_i$$

where

$$r_0 = (1 - a_1 - a_2 - \text{---} - a_i)$$

$$r_1 = \sum_{j=1}^{N_s} \{\zeta_s \zeta_{fj}\ e^{-2\alpha(\ell_0+\ell_j)-2\alpha 0j} \cdot e^{-2j\beta(\ell_0+\ell_j)}\}$$

$$r_2 = \sum_{j=1}^{i-1} \sum_{k=j+1}^{N_s} \{\zeta_{bj}\zeta_{fk}\ e^{-2\alpha(\ell_k-\ell_j)-2\alpha jk} e^{-2j\beta(\ell_k-\ell_j)}\}$$

$$r_3 = \sum_{j=i+1}^{N_s} \{D_i \zeta_{fj}\ e^{-2\alpha(\ell_j-\ell_i)-2\alpha ij}\ e^{-2j\beta(\ell_j-\ell_i)}\}$$

ai; insertion loss of i th coupler

αjk; the sum of insertion loss of couplers between j th and k th couplers

$$(= \sum_{m=j+1}^{k-1} a_m)$$

ζ_s; reflection coefficient in circular port of signal distributor

ζ_{fj}; reflection coefficient in front main circular port of j th coupler

ζ_{bj}; reflection coefficient in back main circular port of j th coupler

D_i; directivity of i th coupler

C_i; coupling of i th coupler

ℓ_0; the distance between control building and the center of the "Y"

ℓ_i; the distance between the center of the "Y" and i th antenna station

α; attenuation constant of TE_{01} mode

β; phase constant of TE_{01} mode

The terms r_1, r_2, r_3 causes ripple in attenuation and variation of attenuation σ^2 can be shown to be as follows:

$$\sigma^2 = \sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2$$

$$\sigma_1{}^2 = \frac{1}{2} \sum_{j=1}^{N_S} \{\zeta_s \zeta_{fj}\ e^{-2\alpha(\ell 0+\ell j)-2\alpha 0j}\}^2$$

$$\sigma_2{}^2 = \frac{1}{2} \sum_{j=1}^{i-1} \sum_{k=j+1}^{N_S} \{\zeta_{bj} \zeta_{fk}\ e^{-2\alpha(\ell k-\ell j)-2\alpha kj}\}^2$$

$$\sigma_3{}^2 = \frac{1}{2} \sum_{j=i+1}^{N_S} \{D_i\ \zeta_{fj}\ e^{-2\alpha(\ell_j-\ell_i)-2\alpha ij}\}^2$$

When the couplers have the identical performance as follows:

$$\zeta_{b1} = \zeta_{b2} = \cdots = \zeta_{bi} = \qquad = \zeta_{b0}$$

$$\zeta_{f1} = \zeta_{f2} = \qquad = \zeta_{f0}$$

$$D_1 = D_2 = \qquad = D_0$$

$\sigma_1{}^2$, $\sigma_2{}^2$, $\sigma_3{}^2$ can be simplified to be as follows:

$$\sigma_1{}^2 = \frac{1}{2} (\zeta_s \zeta_{f0})^2 \sum_{j=1}^{N_S} \{e^{-2\alpha(\ell 0+\ell j)-2\alpha 0j}\}^2 = \frac{1}{2} (\zeta_s \zeta_{f0})^2\ R_1{}^2$$

$$\sigma_2{}^2 = \frac{1}{2} (\zeta_{b0} \zeta_{f0})^2 \sum_{j=1}^{i-1} \sum_{k=j+1}^{N_S} \{e^{-2\alpha(\ell k-\ell j)-2\alpha kj}\}^2 = \frac{1}{2} (\zeta_{b0} \zeta_{f0})^2\ R_2{}^2$$

$$\sigma_3{}^2 = \frac{1}{2} (D_0 \zeta_{f0})^2 \sum_{j=i+1}^{N_S} \{e^{-2\alpha(\ell_j-\ell_i)-2\alpha ij}\}^2 = \frac{1}{2} (D_0 \zeta_{f0})^2\ R_3{}^2$$

Based on the following assumptions:

$a_1 = a_{20} = 0.0114$ (=0.1 dB)

$a_{21} = 0.025$ (13 dB coupler)

$a_{22} = 0.051$ (10 dB coupler)

$a_{23} = 0.135$ (6 dB coupler)

$a_{24} = 0$

$\ell_0 = 450$ m

ℓ_i; corresponds to antenna locations on the West Arm

α_{01}; 1.0 ~ 3.0 dB/km

i; 1 ~ 24

R_1^2, R_2^2, R_3^2 were calculated for every antenna station of 1st to 24th on the West Arm and tabulated in Figures 9-2 and 9-3.

R_1^2 is constant for every antenna station. R_2^2 increases when the antenna station becomes far away from the center, but R_3^2 is smaller near the center.

Requirement for getting $4\sigma_1 \leq 0.1$ dB under $\alpha01 = 1.2$ dB/km is:

$$\zeta_s \cdot \zeta_{f0} \leq 1.74 \times 10^{-3} \quad (-55.2 \text{ dB})$$

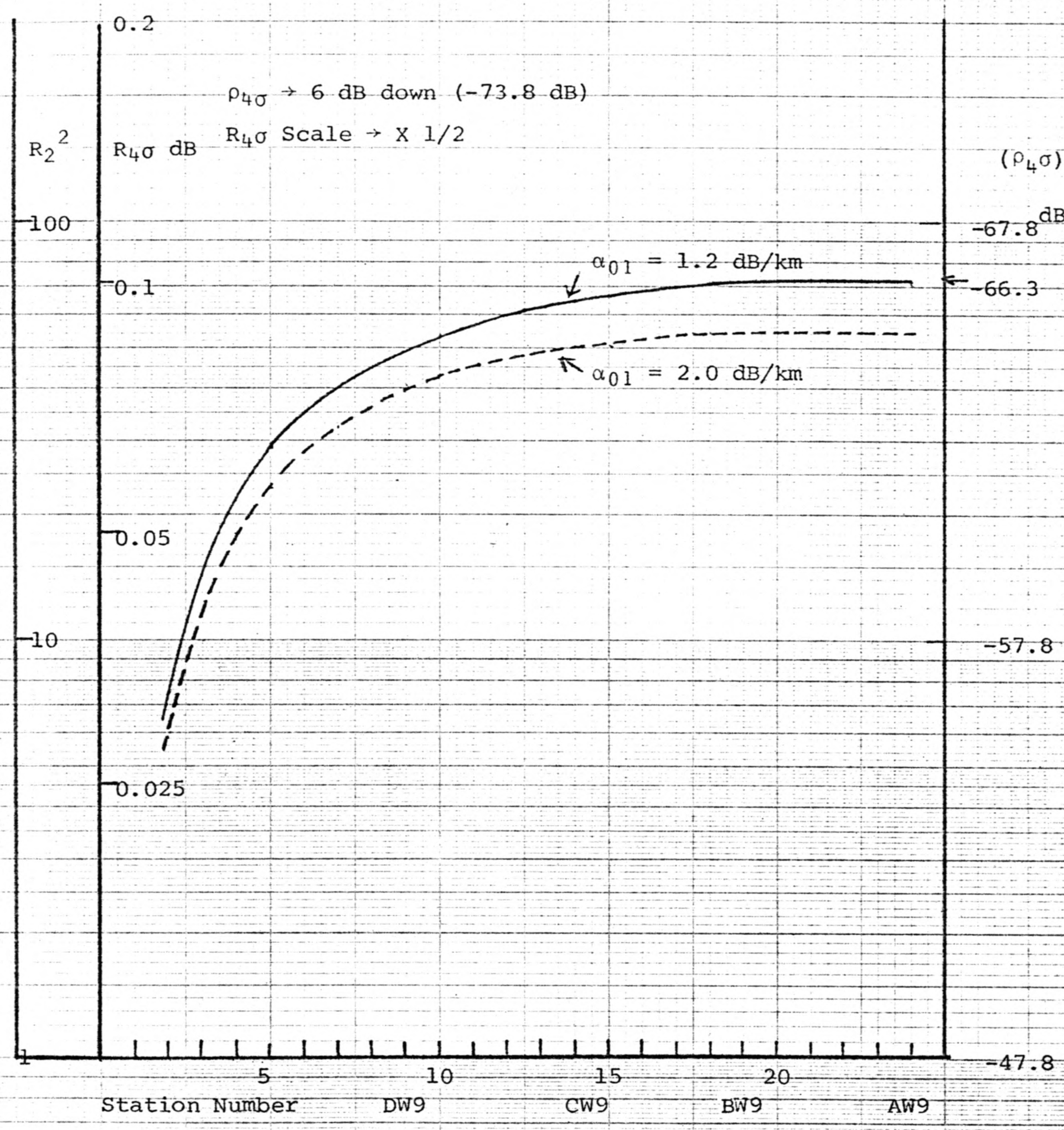
Requirement for getting $4\sigma_2 \leq 0.1$ dB at 24th station under $\alpha01 = 1.2$ dB/km is

$$\zeta_{f0} \cdot \zeta_{b0} \leq 4.80 \times 10^{-4} \quad (-66.4 \text{ dB})$$

Requirement for getting $4\sigma_3 \leq 0.1$ dB at the first station under $\alpha01 = 1.2$ dB/km is

$$\zeta_{f0} \cdot D_0 \leq 1.42 \times 10^{-3} \quad (-56.9 \text{ dB})$$

Requirements at other stations are listed in Table 9-1.



$\rho_{4\sigma}$; the requirement for $\zeta_b \cdot \zeta_{f0}$ to get $4\sigma_2 \leq .1$ dB

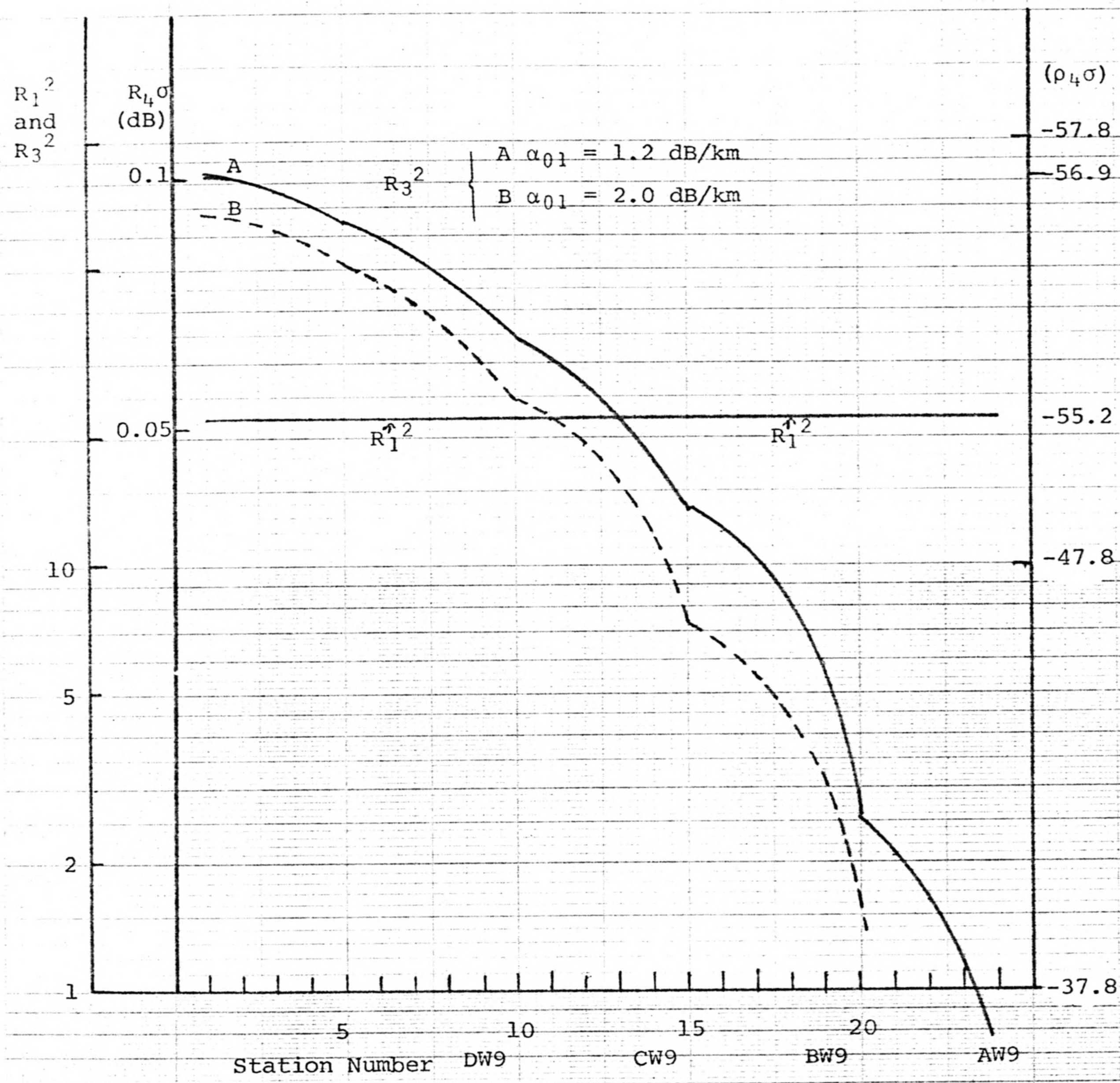
$R_{4\sigma}$; 4σ(dB) under $\rho_{4\sigma} = -66.3$ dB at 24th station

$R_2^{\,2}$; $\sum_{j=1}^{i-1} \sum_{k=j+1}^{N_s} \{e^{-2\alpha_{01}(\ell_k-\ell_j)-2\alpha k_j}\}^2$ i; station number

$\sigma_2^{\,2} = \frac{1}{2}(\zeta_{b0}\zeta_{f0})^2 R_2^{\,2}$

$R_{4\sigma} = 17.3 \times 2\sigma$

FIGURE 9-2: TE_{01} MODE MULTI REFLECTION EFFECT BETWEEN COUPLERS



$\rho_4\sigma$; the requirement for $\zeta_s\zeta_{fo}$ and $D_o\zeta_{fo}$ to get $4\sigma_1 \leq 0.1$ dB and $4\sigma_3 \leq .1$ dB

$R_4\sigma$; 4σ(dB) under $\rho_4\sigma = -56.9$ dB

FIGURE 9-3: TE_{01} MODE MULTI REFLECTION EFFECT BETWEEN COUPLERS AND SIGNAL DISTRIBUTOR AND REVERSE COUPLING

Station No.	1	2		5		10		15		20		23	24
Signal Distributor - coupler front	-55.2 dB (-75.0)	-55.2		-55.2		-55.2		-55.2		-55.2		-55.2 (-75.0)	-55.2
Coupler Back - coupler front	-56.6 dB (-85.0)	-56.2		-62.3		-65.1		-66.0		-66.3		-66.3 (-85.0)	-66.3
Directivity - coupler front	-56.9 dB (-55.0)			-55.9		-53.3		-49.1		-41.8		-44.0 (-70.0)	--
TE_{02} Generation	--	-22.0 dB		-28.7 (-32.2)		-31.6 (-38.3)		-32.9 (-41.4)		-33.3		-33.4	-33.4 (43.1)
TE_{02} Coupling													

( ); requirement to suppress variation less than 0.1 dB^{P-P}

TABLE 9-1: REQUIREMENT FOR SPURIOUS SIGNAL LEVEL TO GET $4\sigma \leq 0.1$ dB

10. SPECIFICATION FOR COUPLERS

Almost all the ripple mechanisms in VLA Waveguide System has been reviewed in this report. In addition there are some waveguide components, such as couplers, adapters, and signal distributors, which cause some variation in attenuation response, but usually their frequency response is not so fast nor so complicated as the TE_{01} mode reflection effect or TE_{on} mode conversion effect. So when the performance in the narrow band of 10 MHz or so is considered, only the latter should be considered. Those are listed again below:

[Main Waveguide]

1. TE_{01} mode reflection effect caused by couplers
2. TE_{on} mode conversion effect

[Coupler]

3. TE_{on} mode coupling effect

[Antenna Waveguide]

4. TE_{01} mode reflection effect between adapter and coupler

Item 4 can be considered to be identical for all antenna stations and the effect of Item 3 becomes smaller at stations far away from the center. From those facts, it would seem reasonable to have the following assignment of variation to each factor to get the design goal of less than 0.2 dB between 2σ values in 10 MHz band through all the system.

[Stations near the center]

1., 2. } less than 0.14 dB

3. less than 0.10 dB

4. less than 0.10 dB

[Stations far away from the center]

1., 2. } less than 0.17 dB

3. less than 0.05 dB

4. less than 0.10 dB

To get these values, the design goal of 9^{o} sector coupler that seems to be the most promising and to be used at almost twenty stations should be set as shown in Table 10-1. Some more basic data to get this design goal is listed in Table 10-2.

		<Design Goal>	<Present Situation>
<Main Line>			
1.	Insertion loss	less than 0.125 dB (in ch 7 ~ 11)	≃ .15 dB
2.	Return loss front	greater than 47 dB	(40 ~ 50 dB)
	back	greater than 20 dB	ok (≃ 40 dB)
3.	TE_{02} mode generation	less than -33 dB	ok
<Coupling>			
1.	Coupling value	-20 ~ -30 dB	ok (nominal -25 dB)
2.	Coupling variation	less than 1.0 dB in any channel	ok
		less than 0.5 dB in more than 6 channels	ok
3.	Return loss	greater than 20 dB	ok
3	TE_{02} discrimination	greater than 20 dB	13 ~ 14 dB
<Directivity>		greater than 10 dB	ok (10 ~ 13 dB)

TABLE 10-1: DESIGN GOAL OF 9° Sector Coupler

Station No.	1	2	5	10	15	20	23	24
I. TE_{01} Mode Reflection Effect A. Signal Distributor - coupler front	-55.2 dB (-75.0)	-55.2	-55.2	-55.2	-55.2	-55.2	-55.2 (-75.0)	-55.2
B. Coupler Back - coupler front	-56.6 dB (-85.0)	-56.2	-62.3	-65.1	-66.0	-66.3	-66.3 (-85.0)	-66.3
C. Directivity - coupler front	-56.9 dB (-55.0)		-55.9	-53.3	-49.1	-41.8	-44.0 (-70.0)	--
II. TE_{02} Mode Effect TE_{02} Mode Conversion Effect	--	-22.0 dB	-28.7 (-32.2)	-31.6 (-38.3)	-32.9 (-41.4)	-33.3	-33.4	-33.4 (43.1)
TE_{02} Coupling								

( ); requirement to suppress variation less than 0.1 dB^{P-P}

TABLE 10-2: REQUIREMENT FOR SPURIOUS SIGNAL LEVEL TO GET $4\sigma \leq 0.1$ dB

11. SUMMARY OF ANALYSIS AND CONCLUSIONS

If there exist two mode conversion sources with the distance ℓ_c, the ripple of the following properties occurs:

(ripple period)

$$f_{px} = \frac{2\pi}{B_x \ell_c} \qquad (11\text{-}1)$$

(ripple amplitude)

$$R_{max} = 17.3\ C_c \cdot C_r{}'\ e^{-\Delta\alpha\ell_c}\ (dB^{P-P}) \qquad (11\text{-}2)$$

(effect to L.O. system)

$$\Delta\Phi_c = C_c \cdot C_r{}' e^{-\Delta\alpha\ell_c} \cdot 2\cos\frac{(\Delta\beta+\Delta\beta')\ell}{2}\sin\frac{(\Delta\beta-\Delta\beta')\ell}{2} \qquad (11\text{-}3)$$

At that time when the distance ℓ_c changes uniformly according to the waveguide line the following formula by $\Delta\ell_c$:

$$\Delta\ell = t \cdot \ell \qquad (11\text{-}4)$$

t: constant (is determined by temperature change or other factors around waveguide and expected to be less than 10^{-4} in main 60 mm waveguide line)

The phase difference between two pilot signals is affected by this mode conversion by the following amount:

$$\Delta\Delta\Phi_c \leq C_c C_r e^{-\Delta\alpha\ell_c}\ \beta_x \cdot \Delta f_p \cdot \Delta\ell_c$$
$$= \frac{R_{max}^{P-P}}{17.3} \cdot \frac{f_p}{f_{px}} \cdot t \cdot 2\pi\ (rad) \qquad (11\text{-}5)$$
$$\left\{\text{optimized case; } \frac{(\Delta\beta-\Delta\beta')\ell}{2} = \frac{\Delta f_p}{f_{px}} \cdot \pi = m\pi\right\}$$

$$\Delta\Delta\Phi_c \leq C_c C_r e^{-\Delta\alpha \ell_c} \cdot 2\Delta\beta\Delta\ell_c$$

$$= \frac{R_{max}^{P-P}}{17.3} \cdot \frac{\ell_c}{\lambda_b} \cdot t \cdot 2\pi \text{ (rad)} \quad (11\text{-}6)$$

$$\{\text{worst case; } \frac{(\Delta\beta-\Delta\beta')\ell}{2} = \frac{\Delta f_p}{f_{px}} \cdot \pi = (m+\tfrac{1}{2})\pi \text{ and } \Delta\ell_c << \frac{\pi}{2\Delta\beta} = \frac{\lambda_b}{4}\}$$

If there exist two reflection sources with distance ℓ_ζ, the ripple of the following properties occurs:

(ripple period)

$$f_p = \frac{2\pi}{2\frac{d\beta}{df}\cdot\ell_\zeta} \doteqdot \frac{150}{\ell_\zeta} \quad (\text{MHz, } \ell_i \text{ in m}) \quad (11\text{-}7)$$

(ripple amplitude)

$$R_{max}^{P-P} = 17.3\ \zeta_s\zeta_s' e^{-2\alpha_{01}\ell_s} \quad (d\beta^{P-P}) \quad (11\text{-}8)$$

(effect to L.O. system)

$$\Delta\Phi_\zeta = \zeta_s \cdot \zeta_s' e^{-2\alpha_{01}\ell_\zeta} \cdot 2\cos(\beta_{01}+\beta_{01}')\ell \cdot \sin(\beta_{01}-\beta_{01}')\ell \quad (11\text{-}9)$$

When the distance ℓ_ζ changes uniformly according Formula 11-4 the phase difference change between two pilot signals can be shown as follows:

$$\Delta\Delta\Phi_\zeta = \zeta_s \cdot \zeta_s' e^{-2\alpha_{01}\ell_\zeta} \cdot 2 \cdot (\beta_{01} - \beta_{01}')\Delta\ell_\zeta$$

$$= \frac{R_{max}^{P-P}}{17.3} \cdot \frac{\Delta f_p}{f_p} \cdot t \cdot 2\pi \text{ (rad)} \quad (11\text{-}10)$$

$$\{\text{optimized case; } (\beta_{01}-\beta_{01}')\ell = m\pi\}$$

$$= 4 \cdot \zeta_s \cdot \zeta_s' \, e^{-2\alpha_{01} \ell_\zeta}$$

$$= \frac{4 \cdot R_{max}^{P-P}}{17.3} \quad \text{(rad)} \qquad (11\text{-}11)$$

{worst case; $(\beta_{01} - \beta_{01}')\ell = (m + \frac{1}{2}\pi)$ and

$$\Delta \ell_\zeta \geq \frac{\pi}{\beta_1 + \beta_1} \doteqdot \frac{\lambda_g}{4}\}$$

From those analysis, the following can be concluded:

1) Usually, the ripple caused by TE_{01} mode reflections has finer ripple and much more unstable than the one caused by mode conversions.
2) By suitable channel assignment to each station, the instability of the L.O. system caused by mode conversion could be minimized. (Especially 5 MHz pilot signals.)
3) The ripple caused by coupler and adapter reflections could have worst effect to the L.O. system and would be most unstable ripple.
4) By having a fixed attenuation at suitable positions in the antenna waveguide route, the instability of the L.O. system caused by TE_{01} mode reflection effect between coupler and adapter could be minimized.
5) The effect of mode conversions or TE_{01} mode conversions on to the L.O. system could be bigger than that of the first order delay.
6) The ripples caused by mode conversions become less unstable in higher frequency than lower frequency. But the ones caused by TE_{01} mode reflection become more unstable in higher frequency than lower frequency.

APPENDIX A. THE EFFECT OF 1-100 KHz OFFSET IN LO SYSTEM

The correction error caused by an offset in the LO round-trip measuring system can be calculated as follows: (refer to Formulae 3-7 and 3-8)

$$\Phi_e = \Phi_t - \Phi_r$$

$$= -(\beta_{01}-\beta_{01}')\cdot\ell + C_c C_r e^{-\Delta\alpha\cdot\ell}\mathrm{Sin}\Delta\beta\ell - C_c'\cdot C_r' e^{-\Delta\alpha'\ell}\mathrm{Sin}\Delta\beta'\ell \qquad (1)$$

Φ_e; correction error

Φ_t; phase shift of TE_{01} mode at f_p

Φ_r; phase shift of TE_{01} mode at $f_p + \Delta f_p$ (offset frequency)

As Δf_p is small enough to assume $C_c = C_c'$, $C_r = C_r'$ $\Delta\alpha = \Delta\alpha'$ Φ_e becomes:

$$\Phi_e = \Phi_{e\ell} + \Phi_{ec}$$

$$= -(\beta_{01}-\beta_{01}')\cdot\ell + C_c\cdot C_r e^{-\Delta\alpha\ell}(\mathrm{Sin}'\Delta\beta\ell - \mathrm{Sin}\Delta\beta'\ell) \qquad (2)$$

$\Phi_{c\ell}$; correction error caused by the first order delay

Φ_{ec}; correction error caused by mode conversion

The first term in (2) can be calculated as follows:

$$\Phi_{e\ell} = -(\beta_{01}-\beta_{01}')\ell$$

$$= \frac{2\pi}{300}\cdot\frac{1}{\sqrt{1-(f_{c01}/f)^2}}\cdot\Delta f_p\cdot\ell \qquad (3)$$

$$\doteq \frac{2\pi}{300}\cdot\Delta f_p\cdot\ell$$

At the farthest station that is 21 Km away from the center,

$$\Phi_{e\ell} \doteq 25.5 \text{ (deg) for } \Delta f_p = 1 \text{ KHz}$$

$$\doteq 2550 \text{ (deg) for } \Delta f_p = 100 \text{ KHz}$$

The second term in (2) can be calculated as follows:

$$\Phi_{ec} = C_c \cdot C_r \cdot e^{-\Delta\alpha\ell}(\sin\Delta\beta\ell - \sin\Delta\beta\ell')$$

$$= C_c \cdot C_r \cdot e^{-\Delta\alpha\ell}(\Delta\beta - \Delta\beta') \cdot \ell \qquad (4)$$

$$= C_c \cdot C_r \cdot e^{-\Delta\alpha\ell}\ \beta_x \cdot \Delta f_p \cdot \ell$$

When TE_{02} mode is considered as spurious mode at 35 GHz:

$$\Phi_{ec} = 4.68 \times 10^{-2} \times \ell \cdot C_c C_r e^{-\Delta\alpha\ell} \cdot \text{(deg) (for } \Delta f_p = 1 \text{ KHz)}$$

$$= 4.68 \times \ell \cdot C_c \cdot C_r e^{-\Delta\alpha\ell} \text{ (deg) (for } \Delta f_p = 100 \text{ KHz)} \qquad (5)$$

(ℓ in km)

The correction error caused by an offset of 1 ~ 100 KHz goes up more than 25 degrees easily at the farthest antenna and cannot be neglected. Such error mostly depends upon the first order delay and is determined by the distance between transmitter and receiver. And it should be stable to temperature change. So once it is known, it could be corrected by another process.

ACKNOWLEDGEMENT

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