NATIONAL RADIO ASTRONOMY OBSERVATORY
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VERY LARGE ARRAY PROJECT

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THE EFFECT OF MISMATCH AT THE ANTENNA MODEMS

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The circulator in the modems at each antenna only provides an adequate termination over a 1 GHz band centered on the designated transmit-receive frequency for a given unit. In this band the return loss is specified to be greater than -25 dB. Out-of-band the return loss may be less than -3 dB. The following note analyses the effect of out-of-band reflections at the modems on the phase stability and amplitude response of the waveguide transmission system.

Consider a two-antenna configuration as shown in Figure 1.

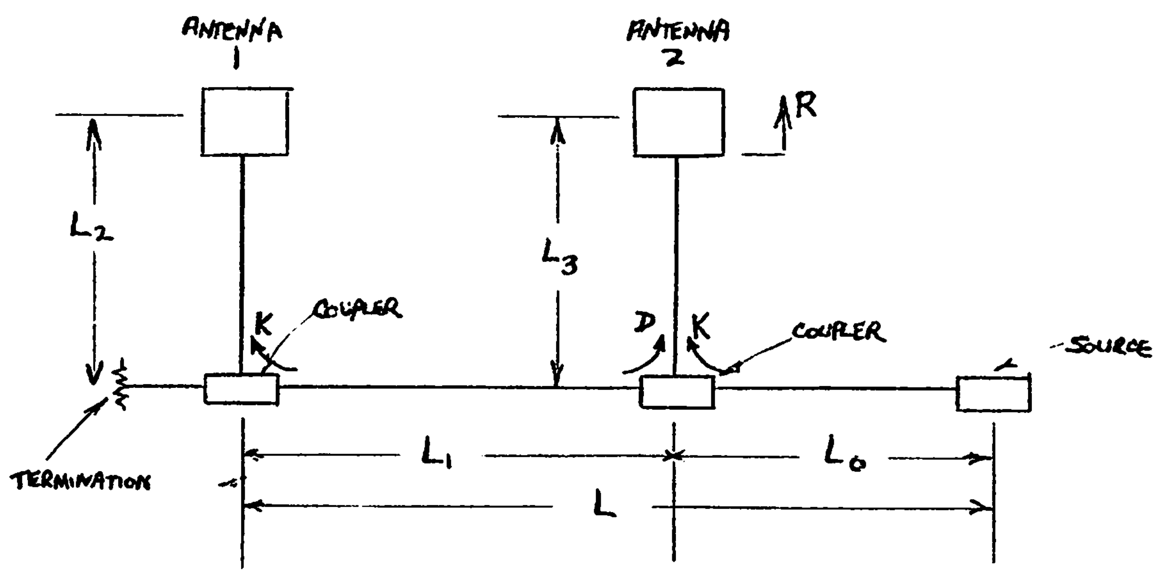

FIGURE 1

Antenna 1 is designed to transmit and receive over a certain signal band centered on frequency f_p and is assumed matched over this band. Antenna 2 presents a return loss R to signals in the designated band. Identical directional couplers are used to extract power from the main waveguide into the branch guides. The directional couplers are assumed to have forward coupling coefficient K and reverse isolation D. Their insertion loss is assumed to be negligibly small, and the return loss in the main waveguide is assumed high.

The signals received at Antenna 1, at frequency f_p, can be described in vector form by:

$$\begin{aligned} A_1 &= K \exp\left[-(\alpha_{01}+j\beta_{01})L - (\alpha_{01}{}^a+j\beta_{01}{}^a)L_2\right] \\ &\quad + RK^2D \exp\left[-(\alpha_{01}+j\beta_{01})L - (\alpha_{01}{}^a+j\beta_{01}{}^a)(L_2+2L_3)\right] \\ &= K \exp\left[-(\alpha_{01}+j\beta_{01})L - (\alpha_{01}{}^a+j\beta_{01}{}^a)L_2\right] \\ &\quad \left[1 + RKD \exp\{-(\alpha_{01}{}^a+j\beta_{01}{}^a)2L_3\}\right] \end{aligned}$$

where $\alpha_{01}+j\beta_{01}$ is the propagation constant in the main waveguide
$\alpha_{01}{}^a+j\beta_{01}{}^a$ is the propagation constant in the branch guide.

It is clear that the relationship for the return signals (at the same frequency) from Antenna 1 to the source is identical in form to the foregoing expression.

The maximum peak-peak ripple amplitude (E) in the received signal as a function of transmitting frequency is given by:

$$E = 20\left[\log_{10}\{1+RKD\exp(-2\alpha_{01}{}^aL_3)\}-\log_{10}\{1-RKD\exp(-2\alpha_{01}{}^aL_3)\}\right]$$

The ripple period in terms of frequency is:

$$P_r = c/2L_3 \text{ (Hz)}$$

where c is the velocity of light.

Let β_{01}' be the phase constant at a frequency $f_p' = f_p + \Delta f$, $\Delta f/f_p \ll 1$. Since the return signals are, in reality, offset slightly in frequency from the outgoing signals, the round trip phase measurement will possess an error given by:

$$\phi_e = (\beta_{01}-\beta_{01}')L + (\beta_{01}{}^a-\beta_{01}{}^{a\prime})L_2 +$$

$$\sin^{-1}\left[\frac{K' \exp(-2\alpha_{01}{}^a L_3)\sin(2\beta_{01}{}^a L_3)}{\sqrt{1+2K'\exp(2\alpha_{01}{}^a L_3)+2K'\exp(4\alpha_{01}{}^a L_3)}}\right] -$$

$$\sin^{-1}\left[\frac{K' \exp(-2\alpha_{01}{}^{a\prime} L_3)\sin(2\beta_{01}{}^{a\prime} L_3)}{\sqrt{1+2K'\exp(2\alpha_{01}{}^{a\prime} L_3)+2K'\exp(4\alpha_{01}{}^{a\prime} L_3)}}\right]$$

where $K' = KRD$.

If $\alpha_{01}{}^a = \alpha_{01}{}^{a\prime}$ and $K' \exp(-2\alpha_{01}{}^a L_3) \ll 1$, then

$$\phi_e \sim (\beta_{01}-\beta_{01}')L + (\beta_{01}{}^a-\beta_{01}{}^{a\prime})L_2 + K' \exp(-2\alpha_{01}{}^a L_3)\left[\sin(2\beta_{01}{}^a L_3)-\sin(2\beta_{01}{}^{a\prime} L_3)\right]$$

Typically, $R = -3$ dB, $K = -20$ dB, $D = -30$ dB

$\alpha_{01}{}^a = 0{\cdot}08$ dB/meter, $\alpha_{01} = 1{\cdot}5$ dB/km

$L_2 \sim L_3 \sim 20$ meters, $L \sim 1$ km.

Then,

$RKD \sim 5{\cdot}0118 \times 10^{-6}$

$\therefore E = 4{\cdot}364 \times 10^{-5}$ dB

and

at $f_p \doteqdot 40$ GHz

$$\phi_e \sim \frac{2\pi\Delta f}{c} L + \frac{2\pi\Delta f}{c} \frac{L_2}{\sqrt{1-\left(\frac{18\cdot3}{f}\right)^2}} + 2{\cdot}512 \times 10^{-6}\left[\frac{2\pi\Delta f}{c} \frac{2L_3}{\sqrt{1-\left(\frac{18\cdot3}{f}\right)^2}}\right]$$

where L, L_2, L_3 are in meters

f is in GHz, Δf in Hz.

Therefore, if $\Delta f = 10$ kHz

$$\phi_e = 0{\cdot}21411 \text{ radians } (12{\cdot}26^{\circ}).$$

If L_2, L_3 change by 20 cm; the corresponding change in round trip phase error is:

$$\Delta\phi_e \sim 4{\cdot}7 \times 10^{-4} \text{ radians } (0{\cdot}027^{\circ})$$

The mismatch at the modem input port therefore has an insignificant effect upon the waveguide system response and system phase stability.