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AN ANALYSIS OF UNIFILAR AND BIFILAR STRIPPING OF HELIX WAVEGUIDE

J. W. Archer

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When making measurements of the mechanical alignment of the 60 mm diameter helix waveguide used in the VLA transmission system, it has sometimes been found that the "mouse" has snagged one or both helix windings and stripped wire from a short length of guide. The investigation described here evaluates the effects on waveguide performance of this kind of non-uniformity.

The waveguide wall structure may be represented[1] by an anisotropic impedance sheath, with impedances Z_η and Z_ζ defined, as shown in Figure 1, for directions parallel and normal to the helix windings:

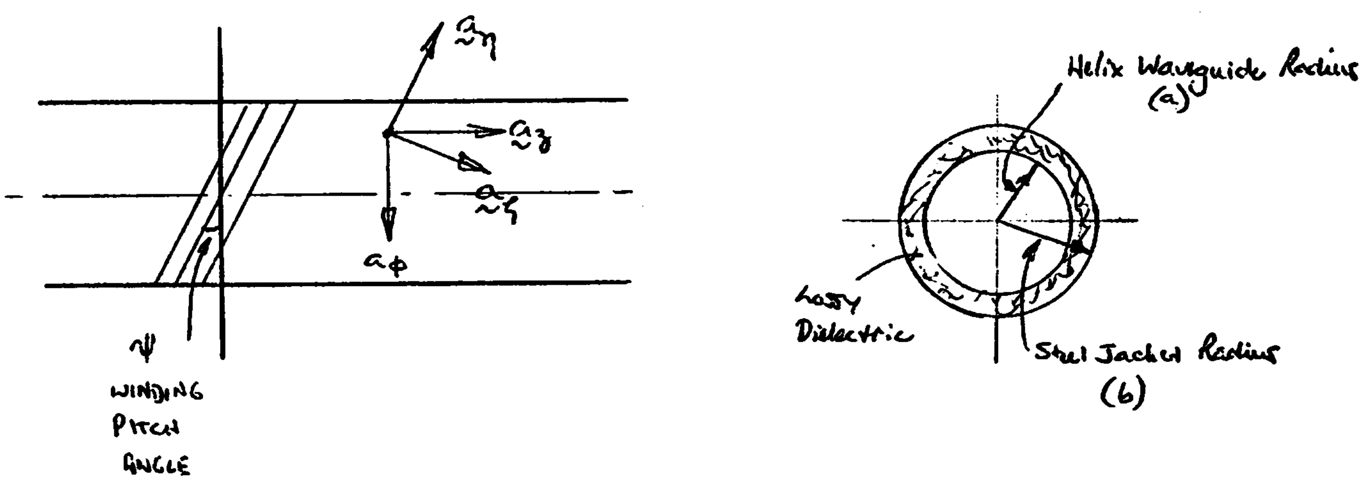

FIGURE 1: DEVELOPED VIEW OF WAVEGUIDE

For small ψ, the surface impedance components in the more conventional ϕ, z directions are[1]:

$$Z_z = Z_\zeta(1-\psi^2) + Z_\eta(\psi^2) \sim Z_\zeta$$

$$Z_\phi = Z_\eta(1-\psi^2) + Z_\zeta(\psi^2) \sim Z_\eta$$

where approximations are valid if

$$\psi << 1.$$

The field components in a cylindrical coordinate system are then subject to the boundary conditions, at $r = a$

$$\frac{E_\phi}{H_z} = Z_\phi \qquad \frac{E_z}{H_\phi} = -Z_z$$

The solution of Maxwell's equations in the cylindrical coordinates of the waveguide are, in general, hybrid modes, except for the circular symmetrical modes of order $p = 0$.

<u>TE_{on} modes</u>

$$H_\phi = E_r = E_z = 0$$

$$E_\phi = \frac{-j\omega\mu}{K_m} A_m J_o'(K_m r)$$

$$H_r = \frac{j\beta_m}{K_m} A_m J_o'(K_m r)$$

$$H_z = A_m J_o(K_m r)$$

where K_m is the m^{th} root of

$$\frac{-j\omega\mu}{K_m} \frac{J_o'(K_m a)}{J_o(K_m a)} = Z_\phi$$

TM$_{on}$ modes $\quad E_\phi = H_r = H_z = 0$

$$E_r = \frac{-j\beta_m}{K_m} B_m J_o' \; (K_m r)$$

$$H\phi = \frac{-j\omega\varepsilon}{K_m} B_m J_o' \; (K_m r)$$

$$E_z = B_m J_o \; (K_m r)$$

where K_m is the m^{th} root of

$$\frac{jK_m}{\omega\varepsilon} \frac{J_o \; (K_m a)}{J_o' \; (K_m a)} = -Z_z$$

EH$_{1m}$, HE$_{1m}$ modes (Prop. constant $\gamma = k \cos\theta$, when $k = \frac{2\pi}{\lambda_0}$)

$$E_z = A_m J_1 \; (K_m r) \cos\phi$$

$$H_z = \frac{B_m}{Z_o} J_1 \; (K_m r) \sin\phi$$

$$E_\phi = \frac{j}{\sin\theta} \left[B_m J_1' \; (K_m r) + A_m \cos\theta \frac{J_1 \; (K_m r)}{K_m r} \right] \sin\phi$$

$$H_\phi = \frac{-j}{Z_o \sin\theta} \left[A_m J_1' \; (K_m r) + B_m \cos\theta \frac{J_1 \; (K_m r)}{K_m r} \right] \cos\phi$$

$$E_r = \frac{-j}{\sin\theta} \left[B_m \frac{J_1 \; (K_m r)}{K_m r} + A_m \cos\theta \, J_1' \; (K_m r) \right] \cos\phi$$

$$H_r = \frac{-j}{Z_o \sin\theta}\left[A_m \frac{J_1(K_m r)}{K_m r} + B_m \cos\theta \, J_1'\,(K_m r)\right]\sin\phi$$

where K_m is determined by the boundary conditions

$$E_\phi = 0 \qquad \frac{E_z}{H_\phi} = -Z_z$$

Hence $$\frac{A_m}{B_m} = -\frac{K_m a}{\cos\theta}\frac{J_1'\,(K_m a)}{J_1\,(K_m a)}; \quad \frac{Z_o}{Z_z} = j\left[\frac{1}{\sin\theta}\frac{J_1'(K_m a)}{J_1\,(K_m a)} + \frac{B_m}{A_m}\frac{\cos\theta}{K_m a\sin\theta}\right]$$

Let $K_m a = \chi_m$, then

(i) $$-j\frac{Z_0}{Z_z} = \frac{ka}{(\chi_m)^3}\frac{J_1(\chi_m)}{J_1'(\chi_m)}\left[\left(\frac{\chi_m J_1'(\chi_m)}{J_1(\chi_m)}\right)^2 - 1 + \left(\frac{\chi_m}{ka}\right)^2\right]$$

For ka>>1, as is the case in the present application, this characteristic equation reduces, in the limit, to

$$\left[\frac{J_1'(\chi_m)}{J_1(\chi_m)} \cdot \chi_m\right]^2 = 1$$

The solutions correspond to the conditions

i) $A_m/B_m = 1$, χ_m is approximately the m^{th} root of $J_o(\chi_m) = 0$ (u_{0m}) HE_{1m} mod

ii) $A_m/B_m = -1$, χ_m is approximately the m^{th} root of $J_2(\chi_m) = 0$ (u_{2m}) EH_{1m} mod

By expanding the Bessel functions in equation (i) as Taylor series about the equatives u_{0m}, u_{2m}; the following asymptotic series are derived for the χ_m

HE_{1m} mode $\quad \chi_m \sim u_{0m}\left(1 + \tfrac{1}{2}j\frac{Z_0}{Z_z ka} + \text{———}\right)$ (a)

EH_{1m} mode $\quad \chi_m \sim u_{2m}\left(1 + \tfrac{1}{2}j\frac{Z_0}{Z_z ka} + \text{———}\right)$ (b)

The propagation constants for the modes are given by

$$\gamma_m^2 = k^2 - \left(\frac{\chi_m}{a}\right)^2$$

For large ka (ka>>1), the expressions for the HE_{1m} modes and EH_{1m} modes simplifies to

$\underline{HE_{1m}\text{ modes}}$

$$E_r \sim -j\frac{k}{K_m} A_m J_o(K_m r)\cos\phi$$

$$E_\phi \sim j\frac{k}{K_m} A_m J_0(K_m r)\sin\phi$$

$$H_r \sim -j\frac{k}{Z_0 K_m} A_m J_o(K_m r)\sin\phi$$

$$H_\phi \sim -j\frac{k}{Z_0 K_m} A_m J_0(K_m r)\cos\phi$$

$\underline{EH_{1m}\text{ modes}}$

$$E_r \sim j\frac{k}{K_m} A_m J_2(K_m r)\cos\phi$$

$$E_\phi \sim j\frac{k}{K_m} A_m J_2(K_m r)\sin\phi$$

$$H_r \sim j\frac{k}{Z_0 K_m} A_m J_2(K_m r)\sin\phi$$

$$H_\phi \sim j\frac{k}{Z_0 K_m} A_m J_2(K_m r)\cos\phi$$

The normalization coefficients for the modes are

$$\left.\begin{aligned}
&\underline{TE_{on}\text{ modes}} - A_m = \frac{1}{\sqrt{\pi}}\,\frac{K_m}{\sqrt{\beta_m a^2}}\,\frac{1}{J_0(\chi_m)}\\
&\underline{TM_{on}\text{ modes}} - B_m = \frac{1}{\sqrt{\pi}}\,\frac{K_m}{\sqrt{\beta_m a^2}}\,\frac{1}{J_1(\chi_m)}\\
&\underline{HE_{1m},\ EH_{1m}\text{ modes}} - A_m = \frac{1}{\sqrt{\pi}}\,\frac{K_m^2}{\beta_m^{1/2}\sqrt{\chi_m^2-1}}\,\frac{1}{J_1(\chi_m)}
\end{aligned}\right\} \iint_S \underset{\sim}{e} \times \underset{\sim}{h} \cdot \underset{\sim}{u}_z\, ds = 1$$

The surface impedances (equivalent) Z_ζ and Z_η can be approximated by

$$Z_\eta = \frac{1 + j}{\sigma\delta_s}$$

where σ is the bulk conductivity of the copper wires and δ_s, the skin depth, is given by

$$\delta_s = \left(\frac{2}{\omega\mu_0\sigma}\right)^{1/2}$$

$$\left.\begin{aligned} Z_\zeta^{(1)} &= j\,\frac{\chi_m^{\,e}}{\omega\varepsilon(1-j\tan\delta)}\tan\left[(b-a)\chi_m^{\,e}/a\right] \\ Z_\zeta^{(2)} &= -j\,\frac{1}{\omega\varepsilon_1 d}\left(\frac{d}{D-d} - \frac{\ln 4}{\pi}\right)^{-1} \end{aligned}\right\} \quad 2$$

where ε, δ are the permittivity and loss tangent of the lossy dielectric layer ($\varepsilon = \varepsilon_0\varepsilon_r$).

$\chi_m^{\,e}$ is the m^{th} eigenvalue for the modes which must exist in the exterior region between helix and jacket. These modes are required to satisfy the boundary conditions at the helix wall[3]. When the boundary conditions are satisfied the impedance sheath approximation is reasonable.

ε_1 is the dielectric constant of the wire insulation

d is the diameter of the copper helix wires

D is the wire separation between centers

The impedance Z_ζ is thus given by

$$\frac{1}{Z_\zeta} = \frac{1}{Z_\zeta^{(1)}} + \frac{1}{Z_\zeta^{(2)}}$$

Doubling the wire spacing, as in the case of stripping out one helix wire will result in

$$Z_\zeta^{(2)} = -j\,\frac{1}{\omega\varepsilon_1 d}\left(\frac{d}{2D-d} - \frac{\ln 4}{\pi}\right)^{-1}$$

Removing the wire completely gives[4]

$$Z_\zeta^{(2)} = \infty$$

$$Z_z \sim Z_\zeta'$$

$$Z_\phi \sim \frac{\varepsilon_r}{\varepsilon_r - 1} Z_z$$

Having defined the waveguide structure and propagation characteristics for modes of azimuthal orders p=0, 1, the coupling between modes at a step discontinuous change in the impedances Z_ϕ and Z_z will be investigated.

Consider a single mode (TE_{01} mode) to be incident from the left in normal helix waveguide on the plane z=0. To the right of the z=0 plane (z>0), the surface impedance undergoes a step discontinuous change due to the stripping of one or both helix wires from the waveguide. The stripping is assumed semi infinite in extent.

The transverse fields in the waveguide $\underset{\sim}{e}_t$, $\underset{\sim}{h}_t$ will be represented in terms of the modes which can propagate in the waveguide on either side of the z=0 junction.

For z<0
$$\underset{\sim}{e}_t = \underset{\sim}{e}_1 e^{-j\beta_1 z} + \sum_{i=1}^{a} r_i \underset{\sim}{e}_i e^{+j\beta_i z}$$
$$\underset{\sim}{h}_t = \underset{\sim}{h}_1 e^{-j\beta_1 z} - \sum_{i=1}^{a} r_i \underset{\sim}{h}_i e^{+j\beta_i z}$$

where $\underset{\sim}{e}_i$, $\underset{\sim}{h}_i$ are the transverse field components of the i^{th} mode in the waveguide for z<0.

Similarly, for z>0

$$\underset{\sim}{e}_t = \sum_{i=1}^{\infty} t_i \underset{\sim}{e}_i' e^{-j\beta_i' z}$$

$$\underset{\sim}{h}_t = \sum_{i=1}^{\infty} t_i \underset{\sim}{h}_i' e^{-j\beta_i' z}$$

where $\underset{\sim}{e}_i'$, $\underset{\sim}{h}_i'$ are the transverse fields components of the i^{th} mode in the waveguide to the right of the junction z>0.

Assume a normalization of the form

$$(\underset{\sim}{e}_i, \underset{\sim}{h}_n) = \iint_S (\underset{\sim}{e}_i \times \underset{\sim}{h}^*_n) \cdot \underset{\sim}{u}_z ds = \delta_{in}$$

and $(\underset{\sim}{e}_i', \underset{\sim}{h}_n') = \delta_{in}$.

For continuity of the fields at z=0

$$\underset{\sim}{e}_1 + \sum_{i=1}^{\infty} r_i \underset{\sim}{e}_i = \sum_{i=1}^{\infty} t_i \underset{\sim}{e}_i'$$

$$\underset{\sim}{h}_1 - \sum_{i=1}^{\infty} r_i \underset{\sim}{h}_i = \sum_{i=1}^{\infty} t_i \underset{\sim}{h}_i'$$

Forming cross products and applying the orthogonality conditions, there results the system of equations defined by

$$\left.\begin{aligned}(1+r_1)(\underset{\sim}{e}_1, \underset{\sim}{h}_i') + r_i(\underset{\sim}{e}_i, \underset{\sim}{h}_i') + \sum_{n\neq 1,i} r_n(\underset{\sim}{e}_n, \underset{\sim}{h}_i') &= t_i \\ (1-r_1)(\underset{\sim}{e}_i', \underset{\sim}{h}_1) - r_i(\underset{\sim}{e}_i', \underset{\sim}{h}_i) - \sum_{n\neq 1,i} r_n(\underset{\sim}{e}_i', \underset{\sim}{h}_n) &= t_i\end{aligned}\right\}\text{Equation (X)}$$

Now $(\underset{\sim}{e}_1, \underset{\sim}{h}_2) = \iint_S (\underset{\sim}{e}_1 \times \underset{\sim}{h}^*_2) \cdot \underset{\sim}{u}_z ds = \int_o^a \int_o^{2\pi} \left[E_{1r}H_{2\phi} - E_{1\phi}H_{2r}\right] r dr d\phi$

Consider the case where the longitudinal impedance only changes. Further, assume that the circumferential impedance is negligible

i.e. $\psi \sim 0$, $Z_\eta \sim 0$, Z_ζ changes to $Z_\zeta + \Delta Z_\zeta$

Therefore, Z_z changes to $Z_\zeta + \Delta Z_\zeta$ from Z_ζ.

In this case, $Z_\phi \sim 0$ and the eigenvalues for the TE_{on} modes are simply the roots (real valued) of

$$J_1(\chi_m) = 0.$$

The TE_{01} mode propagation is independent of changes in Z_z. Equations (X) can be written in matrix form as

$$\underset{\approx}{G} \cdot \underset{\sim}{R} = \underset{\sim}{C}$$

where

$$\underset{\sim}{R} = \{r_i\}$$

$$\underset{\approx}{G} = \{g_{ij}\} = \{(\underset{\sim}{e}_j, \underset{\sim}{h}_i') + (\underset{\sim}{e}_i', \underset{\sim}{h}_j)\}$$

$$\underset{\sim}{C} = \{c_i\} = \{(\underset{\sim}{e}_1 \cdot \underset{\sim}{h}_i') - (\underset{\sim}{e}_i', \underset{\sim}{h}_i)\}$$

If c_i=0 for all i, then (i) no solution exists if $\underset{\approx}{G}$ is singular - this will not be the case if the orthogonalization of the modes has been correctly implemented, (ii) r_i=0 for all i. Thus, for an incident TE_{01} mode the possibility of much coupling is determined by the coefficients $\{c_i\}$.

For coupling from TE_{om} to TE_{on} modes

$$(\underset{\sim}{e}_m, \underset{\sim}{h}_n') = (\underset{\sim}{e}_n', \underset{\sim}{h}_m) = \begin{cases} 0 \text{ if } m \neq n \\ \dfrac{-\pi\omega\mu\beta_m a^2}{K_m^2} A_m^2 J_1'^2(\chi_m) \text{ if } m = n. \end{cases}$$

For coupling from TE_{om} to TM_{on} modes

$$(\underset{\sim}{e}_m, \underset{\sim}{h}_n') = (\underset{\sim}{e}_n', \underset{\sim}{h}_m) = 0 \text{ because } E_r = H_\phi = 0 \text{ for } TE_{om} \text{ modes}$$
$$H_r = E_\phi = 0 \text{ for } TM_{on} \text{ modes}$$

For coupling from TE_{om} to HE_{pm}, EH_{pm} modes ($p \geq 1$)

$$(\underset{\sim}{e}_n, \underset{\sim}{h}_n') = (\underset{\sim}{e}_n', \underset{\sim}{h}_m) = 0$$

since the azimuthal integral reduces to zero for all $p \geq 1$

$$\int_0^a \int_0^{2\pi} F(r) \begin{matrix}\cos \\ \sin\end{matrix} (p\phi) \, r dr d\phi = 0.$$

Hence, at a change in longitudinal impedance Z_z, for the case where $Z_\phi = 0$ and a TE_{01} mode is incident from $z<0$

$$\int c_i \int = 0 \quad \text{for all modes}$$

and no mode interaction occurs.

Consider now the case where Z_ϕ is non-zero and changes step discontinuously at the junction.

Typically, for the lossy backing layer[2]

$$\text{at 50 GHz} \qquad \varepsilon/\varepsilon_0 \sim 4-j1$$

$$Z_\zeta' \sim 120 + j90\Omega$$

For close wound wires in the helix (D~1·1d), $Z_\zeta^{(2)} \sim -j332\Omega$, for unifilar stripping (D'~2d), $Z_\zeta^{(2)} \sim -j5680\Omega$. Therefore, for close wound helix, $Z_\zeta \sim 150 + j20\Omega$; for unifilar stripped helix, $Z_\zeta \sim 123 + j68\Omega$.

Single-wire stripping, therefore, alters the Z_ζ component of the surface impedance. However, provided

$$\delta_s \ll d \ll a$$
$$D' + d \ll \lambda_g$$

as is this case here, Z_η is essentially unchanged ($Z_\eta \sim 0.58(1+j)\Omega$ at 50 GHz). The pitch angle is also unaltered by unifilar stripping, remaining constant for 60 mm diameter waveguide, at

$$\psi = 3{\cdot}769 \times 10^{-3} \text{ radians.}$$

It is clear, therefore, that single-wire stripping affects significantly the impedance component Z_z. $|Z_\phi|$ is changed by less than 0.1%.

Where all wire is removed from the guide to the right of the z=0 plane, the discontinuity can be modelled by a waveguide of constant diameter, but with both Z_ϕ and Z_z components of surface impedance changing at the discontinuity. In the region z<0, typically,

$$Z_\phi \sim 0$$

$$Z_z \sim (150 + j20)\Omega$$

in the region z>0, typically,

$$Z_\phi \sim (160 + j93)\Omega$$

$$Z_z \sim (120 + j70)\Omega$$

The coefficients $\int C_i \int$ are now non-zero for coupling between TE_{om}-TE_{on} modes, but remain zero for coupling between TE_{01} modes and TM_{on}, EH_{pn}, HE_{pn} modes.

If a change in radius occurs at the discontinuity (concentric step) then additional coupling to spurious TE_{on} modes will occur. However, provided the step is concentric, no coupling to TM_{on}, EH_{pn} or HE_{pn} modes is possible, since for these modes

$$\int C_i \int = 0 \quad \text{for all } i$$

A change in impedance alone will be considered here.

For the TE_{om} to TE_{on} mode coupling, the characteristic equation defining the modes is

$$\frac{-j\omega\mu}{K_m} \; \frac{J_0'(\chi_m)}{J_0(\chi_m)} = Z_\phi$$

In the region, $z<0$, $Z_\phi \sim 0$, and so for modes in this region χ_m is simply the n^{th} root of the equation $J_1(\chi_m)=0$. For $z>0$, the p_i be a root of $J_1(p_i)=0$ and assume $\chi_i \sim p_i + \Delta p_i$, where

$$\left|\frac{\Delta p_i}{p_i}\right| << 1.$$

This assumption is valid here since $\frac{Z_\phi}{\omega\mu} << 1$.

Then expanding the Bessel function in a Taylor series about p_i

$$J_0'(\chi_i) = J_0'(p_i+\Delta p_i) \sim -\Delta p_i J_0(p_i)$$

Hence $$\Delta p_i \sim -\frac{jK_i Z_\phi}{\omega\mu} = \frac{-jp_i(Z_\phi/\omega\mu)}{1+j(Z_\phi/\omega\mu)}$$

Now $$(\underset{\sim}{e}_m,\underset{\sim}{h}_n') = \frac{-A_m A_n' 2\pi\omega\mu\beta_n'}{K_m K_n'} \left[\frac{1}{(K_m^2-K_n'^2)}\{K_n' a J_1(K_m a) J_1'(K_n' a) - K_m a J_1'(K_m a) J_1(K_n' a)\}\right]$$

and since $J_1(K_m a) = J_1(\chi_m) = 0$ in $z<0$

$$(\underset{\sim}{e}_m,\underset{\sim}{h}_n') \sim \frac{2\pi\omega\mu a A_m A_n'}{K_n'(K_m^2-K_n'^2)} J_1'(\chi_m) J_1(\chi_n')$$

where $\beta_n'^2 + k_0^2 = K_n'^2$

Furthermore, $J_1(\chi_n') = \Delta p_n J_0(\chi_n)$

where χ_n is the n^{th} root of $J_1(\chi_n) = 0$.

So $$(\underset{\sim}{e}_m,\underset{\sim}{h}_n') = \frac{2\pi\omega\mu\beta_n' a^4 A_m A_n'}{(\chi_n+\Delta p_n)\{\chi_m^2-(\chi_n+\Delta p_n)^2\}} J_1'(\chi_m) J_0(\chi_n)\ \Delta p_n$$

$$\sim \frac{2\pi\omega\mu\beta_n' a^4 A_m A_n'}{\chi_n(\chi_m^2-\chi_n^2) + \Delta p_n(\chi_m^2-3\chi_n^2)} J_1'(\chi_m) J_0(\chi_n)\ \Delta p_n$$

where $\beta_n'^2 \sim -(\chi_n^2+2\Delta p_n\chi_n)\frac{1}{a^2} + k_0^2$

$$\Delta p_n \sim \frac{-j\chi_n(\frac{Z_\phi}{\omega\mu})}{1+j(\frac{Z_\phi}{\omega\mu})}$$

Similarly,

$$(\underset{\sim}{e}_m',\underset{\sim}{h}_n) = \frac{-2\pi\omega\mu\beta_n a^4 A_m' A_n}{\chi_m(\chi_m^2-\chi_n^2)+\Delta p_m(3\chi_m^2-\chi_n^2)} J_1'(\chi_n)J_0(\chi_m)\ \Delta p_m$$

where $\beta_n^2 = -(\frac{\chi_n}{a})^2 + k_0^2$

$$\Delta p_m \sim \frac{-j\chi_m(\frac{Z_\phi}{\omega\mu})}{1+j(\frac{Z_\phi}{\omega\mu})}$$

For $m \neq n$, $(\underset{\sim}{e}_m,\underset{\sim}{h}_n')$, $(\underset{\sim}{e}_m',\underset{\sim}{h}_n)$ are both of order $\Delta p_{m,n}$ and are small compared with $(\underset{\sim}{e}_m,\underset{\sim}{h}_m')$, $(\underset{\sim}{e}_m',\underset{\sim}{h}_m)$ provided

$$\left|\frac{\Delta p_{m,n}}{p_{m,n}}\right| << 1$$

Under these conditions, since the r_i should also be of order Δp_i, the mode coupling equations (X) can be simplified by the approximations to the solutions

$$r_i \sim \frac{(\underset{\sim}{e}_1,\underset{\sim}{h}_i')-(\underset{\sim}{e}_i',\underset{\sim}{h}_1)}{(\underset{\sim}{e}_i,\underset{\sim}{h}_i')+(\underset{\sim}{e}_i',\underset{\sim}{h}_i)}$$

$$t_i \sim \frac{(\underset{\sim}{e}_1,\underset{\sim}{h}_i')+(\underset{\sim}{e}_i',\underset{\sim}{h}_1)}{2}$$

Therefore,

$$(\underset{\sim}{e}_i,\underset{\sim}{h}_i') = \frac{-2\pi\beta_i' a^4}{2\chi_i^2} J_1'(\chi_i)J_0(\chi_i)\ A_i A_i'$$

$$(\underset{\sim}{e}_i',\underset{\sim}{h}_i) = \frac{-2\pi\beta_i a^4}{2\chi_i^2} J_1'(\chi_i)J_0(\chi_i)\ A_i A_i'$$

$$(\underset{\sim}{e}_1,\underset{\sim}{h}_i') = \frac{2\pi\beta_i' a^4 A_1 A_i'}{\chi_i(\chi_1^2-\chi_i^2)+\Delta p_i(\chi_1^2-3\chi_i^2)} J_1'(\chi_1)J_0(\chi_i)\Delta p_i$$

$$(\underset{\sim}{e}_i',\underset{\sim}{h}_1) = \frac{-2\pi\beta_1 a^4 A_1 A_i'}{\chi_i(\chi_i^2-\chi_1^2)+\Delta p_i(3\chi_i^2-\chi_1^2)} J_1'(\chi_1)J_0(\chi_i)\Delta p_i$$

and

$$r_i = \frac{A_i'}{A_1}\frac{J_1'(\chi_1)}{J_1'(\chi_i)} \quad 2\Delta p_i \chi_i^2 \left[\frac{(\beta_i'-\beta_1)}{(\beta_i'+\beta_i)\left[\chi_i(\chi_1^2-\chi_i^2)+\Delta p_i(\chi_1^2-3\chi_i^2)\right]}\right]$$

$$t_i = -\pi a^4 A_1 A_i' \left[\frac{(\beta_i'+\beta_1)J_1'(\chi_1)J_0(\chi_i)\Delta p_i}{\chi_i(\chi_1^2-\chi_i^2)+\Delta p_i(\chi_1^2-3\chi_i^2)}\right]$$

The common term $(\omega\mu)$ has been eliminated from the above expressions.

For i=1 $$r_1 \sim \left[\frac{\beta_1'-\beta_1}{\beta_1'+\beta_1}\right]\frac{A_1'}{A_1}$$

$$t_1 \sim \frac{\pi a^4}{2\chi_1^2} A_1 A_1' \ (\beta_1'+\beta_1)\ J_1'(\chi_1)J_0(\chi_1).$$

Typically,

at 50 GHz $\dfrac{Z_\phi}{\mu\omega} = (4\cdot 05 + j2\cdot 35) \times 10^{-4}$

Thus $\chi_1' = 3\cdot 8326 - j0\cdot 00155$

$\beta_1' = 1\cdot 0393\times 10^3 - j0\cdot 00635$

$\beta_1 = 1\cdot 0394\times 10^3$

$r_1 \sim 10^{-7}, \quad t_1 \sim 1$

Attenuation of TE_{01} mode in stripped waveguide is 0·00636 nepers/m or 0·055 dB/meters at 50 GHz.

At 20 GHz, typically

$\dfrac{2_\phi}{\mu\omega}$ $(1\cdot 013 + j0\cdot 589)\times 10^{-3}$

$\chi_1 1 = 3\cdot 8339 - j3\cdot 88\times 10^{-3}$

$$\beta_1' = 3{\cdot}989\times10^2 - j0{\cdot}0414$$

$$\beta_1 = 3{\cdot}9835\times10^2$$

$$r_1 \sim 6{\cdot}918\times10^{-4}\lfloor -4{\cdot}297^\circ \qquad t_1 \sim 1.0$$

Attenuation of TE_{01} mode in stripped waveguide is 0·0414 nepers/m or 0·359 dB/meters at 20 GHz.

For i>1, provided $\left|\frac{\Delta p_i}{p_i}\right| << 1$

$$r_i \sim \frac{A_i}{A_1} \Delta p_i \frac{J_1'(\chi_1)}{J_1'(\chi_i)} \left(\frac{\beta_1}{\beta_i} - 1\right) \left(\frac{\chi_i}{\chi_i^2 - \chi_1^2}\right)$$

$$t_i \sim -\pi a^4 A_1 A_i' \Delta p_i J_1'(\chi_i) J_0(\chi_i) \left\{\frac{(\beta_i' + \beta_1)}{\chi_i(\chi_1^2 - \chi_1^2)}\right\}$$

Substituting for the normalization coefficients

$$r_i \sim \Delta p_i \left\{\frac{J_0(\chi_1)}{J_0(\chi_i)}\right\}^2 \sqrt{\frac{\beta_1}{\beta_i}} \left(\frac{\beta_1}{\beta_i} - 1\right) \frac{1}{\chi_1} \left(\frac{\chi_i^2}{\chi^2 \ \chi_1^2}\right)$$

$$= \Delta p_i \left\{\frac{J_0(\chi_1)}{J_0(\chi_i)}\right\}^2 \sqrt{\frac{\beta_1}{\beta_i}} \left(\frac{\beta_1}{\beta_i} - 1\right) \frac{1}{\chi_1} \left\{\frac{1}{1 - \left(\frac{\chi_1}{\chi_i}\right)^2}\right\}$$

$$t_i \sim -\Delta p_i \frac{\chi_i \chi_1}{\sqrt{\beta_i \beta_1}} \frac{(\beta_i + \beta_1)}{\chi_i(\chi_1^2 - \chi_i^2)}$$

$$= -\Delta p_i \left(\frac{\chi_1}{\chi_1^2 - \chi_i^2}\right) \frac{\beta_i + \beta_1}{\sqrt{\beta_i \ \beta_1}}$$

Typical coupling values from these formulae are, at 50 GHz

<u>TE_{02} mode</u> $\quad r_2 \sim -90{\cdot}79$ dB$\lfloor 30{\cdot}124^0$

$t_2 \sim -62{\cdot}74$ dB $\angle 30{\cdot}124^{\circ}$

<u>TE_{03} mode</u> $r_3 \sim -74{\cdot}43$ dB $\angle 30{\cdot}124^{\circ}$

$t_3 \sim -67{\cdot}719$ dB $\angle 30{\cdot}124^{\circ}$

<u>TE_{04} mode</u> $r_4 \sim -64{\cdot}34$ dB $\angle 30{\cdot}124^{\circ}$

$t_4 \sim -70{\cdot}636$ dB $\angle 30{\cdot}124^{\circ}$

In order to confirm these results measurements have been made on short lengths of 60 mm diameter helix waveguide from which one or both helix wires have been removed. The measurements of attenuation were made over a range of frequencies from 27 GHz to 39 GHz using a swept oscillator and diode detectors connected to a data normalizer.

The system was calibrated with a standard length of normal helix guide. The test piece was then inserted in place of a normal guide section and the change in total attenuation plotted as a function of frequency. The minimum resolution was estimated to be ±0·025 dB, enabling (for a five meter long test piece) the change in total attenuation to be estimated to an accuracy of 5 dB/kilometer.

For one strand of wire stripped from the test length (5 m), the change in attenuation was not measurable, implying an increase in attenuation of less than 0·0005 dB/meter. This behavior is predicted by the foregoing analysis (See Figure 2).

For both wires stripped from the test section (1 m) the change in attenuation with frequency is as shown in Figure 3. Once again, this general behavior is predicted by the analysis. The increased loss is due primarily to TE_{on} mode loss at the walls and is not due to strong mode coupling to other TE_{on} modes. The attenuation was measured to be 0·4 dB/meter at 27 GHz, decreasing to about 0·15 dB/meter at 39 GHz.

REFERENCES

[1]Karbowiak, A.E., "Microwave Propagation in Anisotropic Waveguides", IEE Monograph No. 147R, Aug. 1955.

[2]Unger, H.G., "Normal Modes and Mode Conversion in Helix Waveguide", BSTJ, **40**, 1961, p255.

[3]Unger, H.G., ibid.
Carlin, J.W., "A Relation for the Lost Characteristics of Circular Electric and Magnetic Modes in Dielectric Lined Waveguide", BSTJ, **50**, 1975, p1639.

[4]Unger, H.G., "Helix Waveguide Theory and Application", BSTJ, **37**, 1958, p1599.

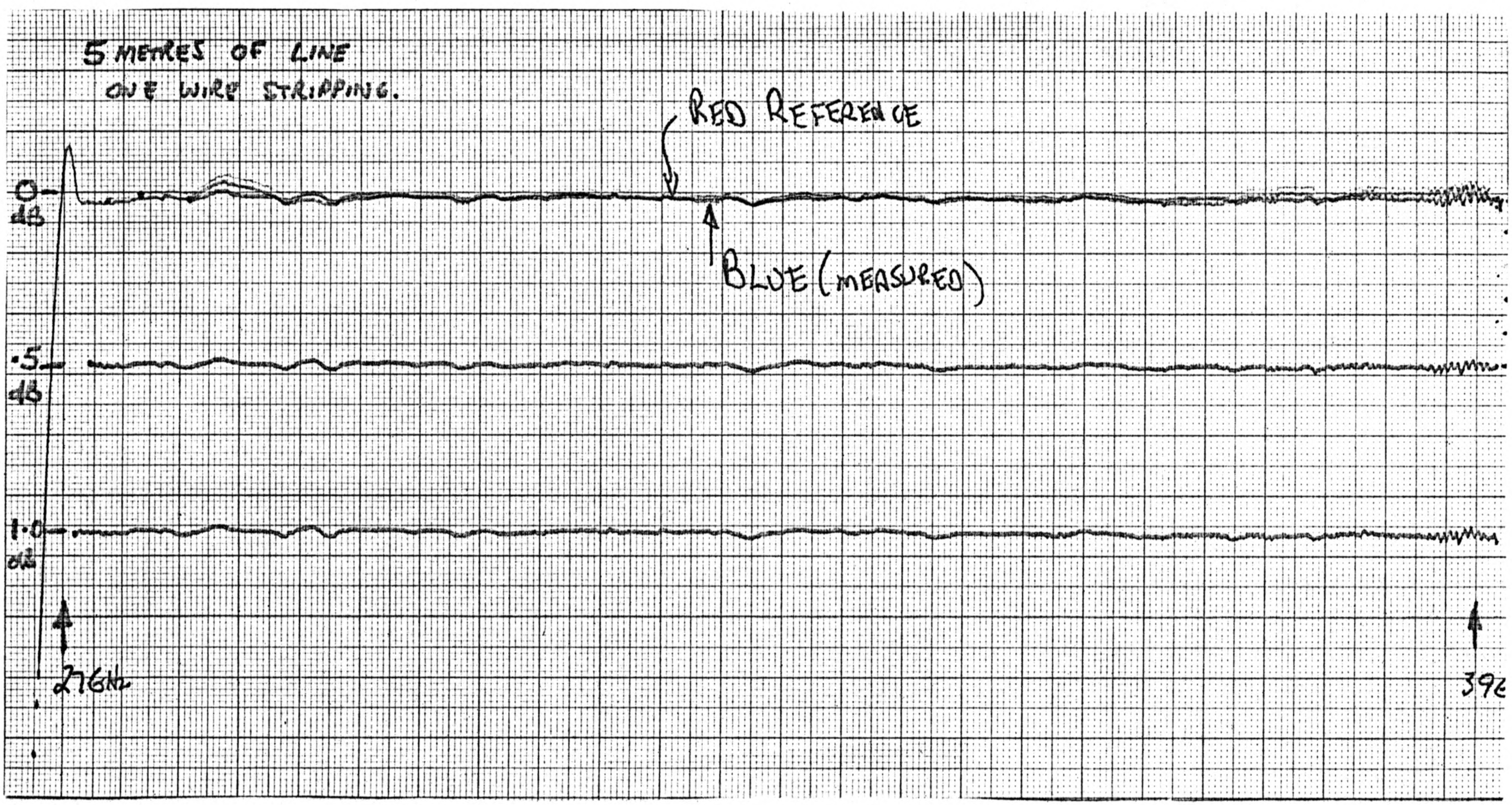
5 METRES OF LINE
ONE WIRE STRIPPING.
RED REFERENCE
BLUE (MEASURED)
0 db
.5 db
1.0 db
27GHz
39G

FIGURE 2

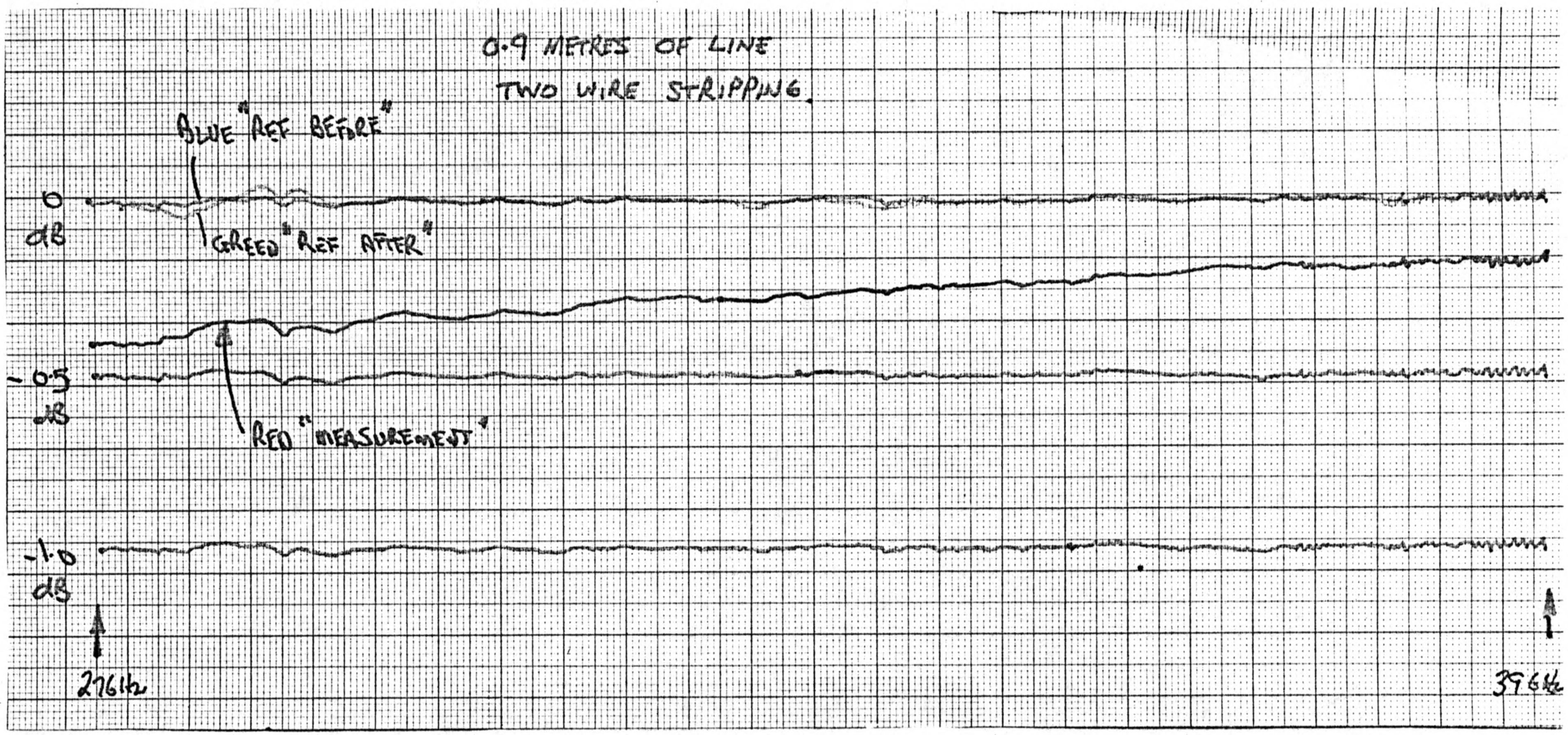
0.9 METRES OF LINE
TWO WIRE STRIPPING.
BLUE "REF BEFORE"
0
dB
GREEN "REF AFTER"
-0.5
dB
RED "MEASUREMENT"
-1.0
dB

FIGURE 3