NATIONAL RADIO ASTRONOMY OBSERVATORY
SOCORRO, NEW MEXICO
VERY LARGE ARRAY PROJECT

VLA ELECTRONICS MEMORANDUM NO. 167

THE EFFECTS OF WAVEGUIDE VELOCITY DISPERSION AND THE DISCRETE DELAY STEPS ON THE ACCURACY OF MEASUREMENT OF THE VISIBILITY PHASE

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In VLA Electronics Memorandum No. 164 J. Granlund examined a number of factors which bear upon the phase stability or phase measurement accuracy of the VLA. He suggested that consideration be given to installing phase equalizers for the long waveguide runs and to decreasing the minimum step size in the delay system. The present memorandum examines these suggestions.

1.0 PHASE EQUALIZATION OF THE WAVEGUIDE

The calculations in VLA Electronics Memorandum No. 164 show that over a 50 MHz IF bandwidth the velocity dispersion in the 60 mm diameter waveguide results in departures from phase linearity which vary up to $2{\cdot}5^{\circ}$ at the band edges. This is roughly equivalent to an rms phase error of $1{\cdot}25^{\circ}$ over the band. In combining the signals received from two antennas the effects of the dispersion will tend to cancel rather than add, so the rms over a 50 MHz band for the longest waveguide run represents the worst case. This corresponds to 21 km at channel 7[1] frequency, for which the rms phase deviation is slightly larger than the figures quoted above, about 2°. A 2° rms phase deviation results in a reduction in the correlation by a factor of $\cos 2^{\circ} = 0.9994$, which is negligible.

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[1]The frequency channel for the longest runs is selected to minimize the total attenuation, and channel 7 is about optimum when the effects of the 20 mm bends are taken into account.

The velocity dispersion of the waveguide is only one mechanism by which small deviations from phase linearity are introduced across the IF bands. Filters, amplifiers and other components all contribute, and these components have sufficient individual variation that their effects are not identical from one antenna to another. The largest departures from phase linearity are expected to be those resulting from reflections and unwanted modes within the waveguide. All of these effects are tolerable so long as the combination of them does not decrease the signal correlation by more than a few percent. The main affect of the phase irregularities is to make it necessary to recalibrate the instrumental phase on a calibration source every time that the final IF stages are tuned in frequency by an interval that is not small compared with the frequency scale of the irregularities. For spectral line observations such calibration must include measurements in the spectral line mode, so that the responses of the spectral channels are individually calibrated.

In conclusion, the effect of an equalizer for the waveguide phase characterisitcs would be to increase the correlation by no more than 0.1%. It would not eliminate the need to repeat phase calibration when making changes in the observing frequency. The effect of the velocity dispersion in the waveguide is, of course, important over bandwidths of several hundred megahertz, and because of this only one sideband is used in the waveguide transmission system.

2.0 THE EFFECT OF THE MINIMUM DELAY STEP

VLA Electronics Memorandum No. 164 also points out that the minimum increment of 625 p.s. in the compensating delays results in a phase change of $5{\cdot}6^{\circ}$ at the 25 MHz center frequency of the final IF amplifiers when used at full bandwidth. At the 40 MHz center frequency presently used with reduced bandwidths, the step is 9°. Since, at the lower frequency bands, these steps are larger than the 1° per GHz phase criterion[2] it was suggested that the choice of 625 p.s. minimum step size should be reviewed.

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[2]The 1° per GHz criterion is a goal for the VLA receiving system originally recommended by S. Weinreb. I interpret it as applying to the rms error in the phase of the visibility measured by any antenna pair. The component of this error resulting from instrumental causes should not exceed 1° per GHz of the observing frequency. The criterion was mainly intended as a guide to the required local oscillator stability over periods between calibration observations.

The considerations on which the 625 p.s. choice was based are described in VLA Electronics Memorandum No. 109. That memorandum shows that a step size of twice 625 p.s. would be tolerable from the viewpoint of preserving signal correlation. The important figure to consider with regard to the phase is the rms value of the difference between the induced errors for any two antennas. This is equal to $2\pi f_0 \tau_0/\sqrt{6}$ where f_0 is the center frequency of the signals passing through the delay and τ_0 is the minimum delay increment. For f_0 = 25 MHz the rms phase error is $2{\cdot}3^{\circ}$. Thus it exceeds the 1° per GHz criterion for the 1·35-1·72 GHz band, and would make a significant contribution at 4·5-5·0 GHz. There are, however, three considerations that either do or can substantially mitigate the delay-induced phase errors, and these are described below.

2.1 The IF Center Frequency

The IF center frequency of 40 MHz presently used for bandwidths less than 50 MHz is a feature of the prototype IF system which was designed for continuum observations only. The new IF system for both spectral line and continuum observing will use much lower center frequencies; see VLA Technical Report No. 29. For 25 MHz bandwidth the center frequency will be 12·5 MHz and for 12·5 MHz bandwidth, 6·25 MHz. It is rather unlikely that the full 50 MHz bandwidth will be used for 1·35-1·72 GHz observations because of the heavy spectrum usage in the frequency range. The 25 MHz bandwidth will thus cut the phase errors down to $1{\cdot}3^{\circ}$ rms and the 12·5 MHz bandwidth to $0{\cdot}65^{\circ}$ rms. The $1{\cdot}3^{\circ}$ rms error takes up the full error budget for the 1·35-1·72 GHz band, so the problem is not fully solved and a further decrease by a factor of at least 3 is desirable.

2.2 The Effect of Averaging

Thus far we have considered only the effect of the delay steps on the instantaneous values of the visibility phase. In mapping, the visibility data are averaged over cells of the (u,v)-plane, and this averaging has a powerful reduction effect on the phase fluctuations. Figure 1 illustrates the behavior of the phase errors. For each antenna the required compensating delay relative to a phase reference point is calculated, and the delay settings are chosen to provide the best-fit staircase function with step height τ_0. The delay-error waveform of an individual antenna can be described as a sawtooth waveform having a slowly varying period, and it clearly averages to something very close to zero over an integral number of cycles. For any hour angle and declination, the waveform period depends upon the position of the antenna relative to the delay reference point for the array. The quantity of importance in the present context is the overall delay error for an antenna pair, which is the sum (or difference) of the error waveforms for two individual antennas. An example of such a waveform is shown in the bottom curve in Figure 1; it is neither strictly periodic nor random in nature. For the present purpose it is most convenient to consider the effect of averaging of such a waveform in terms of the time interval during which the difference in the space transmission delays for the two antennas changes by τ_0. It will be assumed that there are two independent values of delay error during such an interval, and that if there are n intervals during the time that the spacing vector for the antenna pair crosses a cell in the (u,v)-plane, the rms delay-induced phase error is reduced by a factor of $(2n)^{-\frac{1}{2}}$. This assumption is conservative since it certainly underestimates the reduction resulting from averaging[3]. The number n depends upon the

[3]The sawtooth error waveforms of the individual antennas average over a period of time in such a way that complete cycles essentially cancel, leaving only the effect of any residual fraction of a cycle. The average of the combined error waveform for an antenna pair results from the sum of two such residuals. Note that it is possible to reduce the residuals by moving the delay reference point further from the antennas, thereby increasing the frequencies of the sawtooth error waveforms. In this report it is assumed that the delay reference point is near the array center and has not been located to take advantage of this affect.

baseline parameters and the hour angle and declination of the observation, as well as the cell size. Determination of the precise effect of the averaging in any particular observation calls for a computer analysis, but an approximate investigation of the general case is attempted in Appendix 1. From this the following conclusions can be drawn.

The number n is proportional to the cosine of the declination, so there is no averaging for directions near the pole. For directions south of 60° declination, however, this effect is not very important.

For any antenna pair, n is approximately proportional to u, and if the observing declination is within 60° of the equator n will have a value of about 50 at the hour angle for which u is a maximum. Over the (u,v)-plane the effect of averaging is, in the mean, to reduce the rms value of the delay-induced phase fluctuations by a factor of at least five.

The reduction in the rms phase errors will produce an equal reduction in the resulting rms errors in the map. However, because the remaining phase errors are concentrated around the v axis in the (u,v)-plane, the corresponding errors in a map will appear as phase errors in Fourier components with periodicities in the north-south direction. Inducing a small phase error ϕ in a Fourier component is equivalent to adding a quadrature component of relative amplitude sin ϕ. Since we are concerned with rms errors of about $1{\cdot}3^{\circ}$ before averaging, the rms amplitude of the unwanted quadrature components would be about 2%. It seems rather unlikely that such 2% north-south sidelobes would be noticed in most cases, considering that phase errors of at least equal amplitude will be present as a result of phase instabilities in the equipment and the atmosphere.

2.3 Computer Correction for Delay-Induced Phase Errors

If mapping near the pole becomes important, or if the north-south sidelobe effects become troublesome, it is possible to apply a correction for the delay-induced phase-errors in the computer. In setting the delays the exact values required are computed and the nearest combination of bits then picked. The errors are therefore known and corrections could be applied through the fringe rotators, as in the case of 'round-trip' phase corrections, or to the visibility data. Another possible scheme would be to choose the delay settings such that the errors average out to a very small residual over some chosen time period. This would involve using the next-to-nearest bit combination as well as the nearest to obtain phase errors of opposite sign. The delay errors would still be small enough to produce neglible decorrelation.

The conclusion from the above considerations is that the effect of the 625 p.s. delay step should not be significant. If it is, however, there are ways to reduce it which do not entail reducing the step size and thus avoid a major redesign of the samplers.

3.0 VARIATION OF WAVEGUIDE PATH LENGTH AND THE ROUND-TRIP PHASE CORRECTION

In the discussion of the round-trip phase correction in VLA Electronics Memorandum No. 164 it is pointed out that correction for path length changes in the waveguide, which are accomplished through phase adjustment of a local oscillator, result in a linear phase shift with frequency across the IF bands. This is, of course, to be expected when correcting for a delay change by making a phase adjustment which is constant with frequency. Two points are worth mentioning in this regard.

First, the appropriate frequency to use when computing the correction for the IF phase shift is the center of the IF band. This is because the instrumental phase constants of the array are derived from observations of calibration sources, and the visibility values obtained from such observations are effectively averaged over the IF bands with weighting proportional to the product of the voltage responses of the two channels. In the presence of a linear phase gradient, the resulting measured phase is therefore very close to that at the band center.

Second, the affect of the linear phase gradient depends upon its magnitude. Measurements of the diurnal variation of the 600 MHz round-trip phase show a typical range of 20^{o} p-p. This corresponds to a change in waveguide length of 1·4 cm, which is consistent with the expected thermal expansion in the 20 mm waveguide runs on the antennas. The corresponding change in delay is 47 p.s. which results in a phase gradient of $0{\cdot}017^{o}$ per MHz. For continuum observations over bandwidths of up to 50 MHz the phase gradient is of no consequence since it is too small to cause any measurable loss in correlation. For spectral line observations the effect of a linear phase gradient could be serious, but the total IF bandwidth for such observations is unlikely to be more than 12·5 MHz, and the resulting phase errors, even at the band edges, are negligible.

ADDENDUM

The discussion, in Section 2.2 and the Appendix, of the effects of averaging over the (u,v)-cells can be applied also to the question of how a synthesis array rejects interference from a source that is stationary with respect to the antennas. The fringe frequency compensation introduced into the local oscillator system causes the correlator outputs from a stationary source to vary sinusoidally at the natural fringe rate. During a cell crossing time there are $n\tau_0 f$ fringes, and since complete fringe cycles average to zero the output is reduced, on average, by a factor $\frac{1}{2n\tau_0 f}$. As in the case of the delay errors, averaging is least effective in reducing unwanted effects in areas of the (u,v)-plane near the v-axis and areas of the sky near the poles.

APPENDIX

Variation of the Number of Delay Steps in Crossing a (u,v) Cell

We wish to calculate n, the number of times the delay difference for a pair of antennas changes by τ_0 during the time that the spacing vector for an antenna pair crosses one cell in the (u,v)-plane. The following is an approximate treatment.

Let Δq be the average path length across a cell in the (u,v)-plane and let Δt be the time to traverse it. The spacing vector has components of motion $\frac{du}{dt}\Delta t$ and $\frac{dv}{dt}\Delta t$ during time Δt, and

$$\Delta t = \Delta q \Big/ \sqrt{\left(\frac{du}{dt}\right)^2 + \left(\frac{dv}{dt}\right)^2} \qquad \text{(A1)}$$

Also during Δt the difference in the space path delays for the two antennas changes by

$$n\tau_0 = \omega_0 u \cos \delta \, \Delta t / f \qquad \text{(A2)}$$

where ω_0 is the angular rotation of the earth, δ is the declination of the observation and f is the observing frequency. By dividing (A2) by τ_0 and substituting for Δt from (A1), one obtains

$$n = \left| \frac{\omega_0 \, u \, \Delta q \cos \delta}{f \ \tau_0 \sqrt{\left(\frac{du}{dt}\right)^2 + \left(\frac{dv}{dt}\right)^2}} \right| \qquad \text{(A3)}$$

where the absolute value is taken to eliminate the sign of the delay change.

The baseline components for an antenna pair are:

B_x in the direction $H = 0,\ \delta = 0$

B_y in the direction $H = +6$ hours, $\delta = 0$

B_z in the direction $\delta = +90^{\circ}$

where H is the hour angle; see, for example, Hogg et. al. 1969.

In terms of these components[4]

$$u = B_x \sin H - B_y \cos H$$

$$\frac{du}{dt} = (B_x \cos H + B_y \sin H)\ \frac{dH}{dt}$$

$$\frac{dv}{dt} = \sin \delta\ (B_x \sin H - B_y \cos H)\ \frac{dH}{dt} = u \sin \delta\ \frac{dH}{dt}$$

By substituting for u and v in (A3) and remembering that $\frac{dH}{dt} = \omega_0$ one obtains

$$n = \left| \frac{\Delta q \cos \delta\ (B_x \sin H - B_y \cos H)}{f\ \tau_0 \sqrt{(B_x \cos H + B_y \sin H)^2 + \sin^2\delta (B_x \sin H - B_y \cos H)^2}} \right| \tag{A4}$$

If $B_x = \alpha B_y$ this becomes

$$n = \left| \frac{\Delta q \cos \delta\ (\alpha \sin H - \cos H)}{f\ \tau_0 \sqrt{(\alpha \cos H + \sin H)^2 + \sin^2\delta (\alpha \sin H - \cos H)^2}} \right| \tag{A5}$$

This shows that n is independent of ω_0 and B_z, as would be expected, and for a given orientation n is also independent of the length of the baseline.

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[4]Note that B_x, B_y, B_z, u, and v are measured in wavelengths at the observing frequency.

When n is sufficiently large the delay-induced phase errors are satisfactorily reduced by the averaging, so we are mainly concerned with the conditions under which n becomes small. The expression for n consists of $(\frac{\Delta q}{f\tau_0})$ multiplied by a trigonometric function of δ, α and H. First consider the minimum value of $(\frac{\Delta q}{f\tau_0})$. The mean value of Δq is shown in VLA Electronic Memorandum No. 129 to be 0·79 Δu for cells of equal dimensions in the u and v directions. The minimum practical value of Δu corresponds to a map field equal to the full width of a single antenna beam, so

$$\frac{1}{\Delta u} \approx \frac{2\lambda}{25\ \mathrm{m}}$$

where λ is the observing wavelength. This leads to a minimum value for $(\frac{\Delta q}{f\tau_0})$ of 53 wavelengths, independent of f. Thus when the other factors in (A5) are about unity, the averaging will reduce the delay-induced phase errors by at least one order of magnitude.

For observations at the pole the factor cos δ in (A5) becomes zero and n = 0 because the delays remain fixed at all hour angles. However, at $\delta = 60^\circ$ the reduction from the cos δ factor is only ½ and the fraction of the observable sky at the VLA site which is north of declination 60° is less than 8%.

To examine the effect of the remaining factors, Table A1 shows expressions for n derived from equation (A4) for $\delta = 0^\circ$, 30° and 60°, and for $B_x = 0$ (east-west baseline if $B_z = 0$), $B_y = 0$ (north-south baseline) and $B_x = B_y$. The quantity u_{max} in the expressions is the maximum value of u with respect to H for an antenna pair and is given by

$$u_{max} = \sqrt{B_x^2 + B_y^2}$$

Each expression in Table A1 contains the factor u/u_{max}, and thus n falls off linearly with u and becomes very small as the baseline vector crosses the v axis. The factors which include the functions of H all have minimum values of the order of unity. Their variation with H is small at $\delta = 60^{\circ}$, but as δ approaches zero they can become very large. However, since we are mainly concerned with n becoming small, the important conditions are high declinations and hour angles at which u becomes small. In general, the mean over the (u,v)-plane of the reduction due to averaging of the delay-induced phase fluctuations should be at least a factor of five.

Reference

Hogg, D.E., Macdonald, G.H., Conway, R.G., and Wade, C.M., Astron. J., *74*, 1206, 1969 (December).

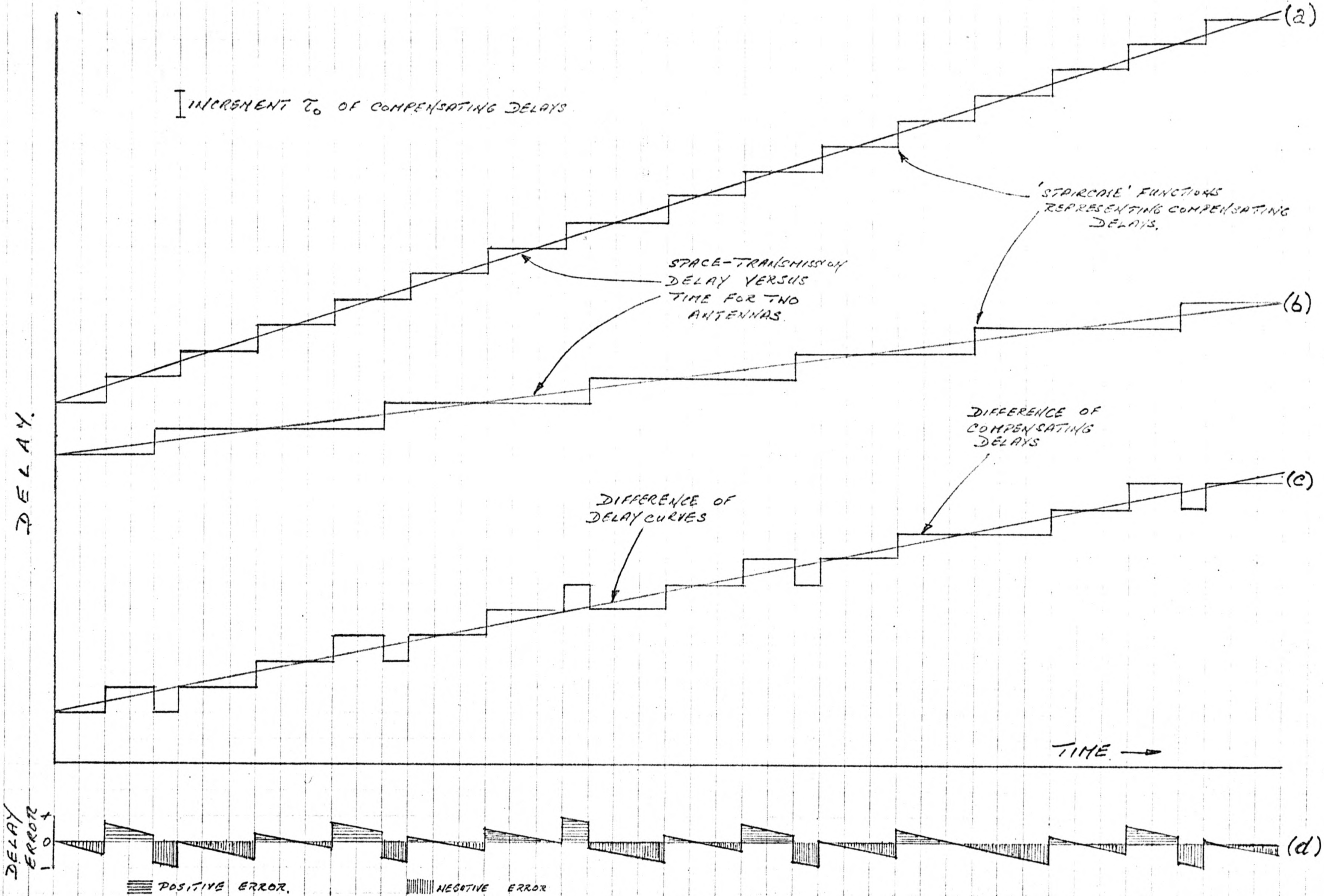

Figure 1 (a) & (b) show space transmission delay and stepped compensating delay for two antennas. The differences between the two space transmission delays and the compensating delays are shown by curves (c). The resulting delay error is shown by (d). Note that the difference between the space transmission delays changes [redacted] in curve (d) is 2·3 squares.

	$\delta = 0°$	$\delta = 30°$	$\delta = 60°$
$B_x = 0$ (east-west baseline if $B_z = 0$)	$n = \frac{\Delta q}{f\tau_0}\left\lvert \frac{u}{u_{max}} \frac{1}{\sin H}\right\rvert$	$n = \frac{\Delta q}{f\tau_0}\left\lvert \frac{u}{u_{max}} \frac{1}{\sqrt{1-\frac{3}{4}\cos^2 H}}\right\rvert$	$n = \frac{\Delta q}{f\tau_0}\left\lvert \frac{u}{u_{max}} \frac{1}{\sqrt{1-\frac{1}{4}\cos^2 H}}\right\rvert$
$B_y = 0$ (north-south baseline).	$n = \frac{\Delta q}{f\tau_0}\left\lvert \frac{u}{u_{max}} \frac{1}{\cos H}\right\rvert$	$n = \frac{\Delta q}{f\tau_0}\left\lvert \frac{u}{u_{max}} \frac{1}{\sqrt{1-\frac{3}{4}\sin^2 H}}\right\rvert$	$n = \frac{\Delta q}{f\tau_0}\left\lvert \frac{u}{u_{max}} \frac{1}{\sqrt{1-\frac{1}{4}\sin^2 H}}\right\rvert$
$B_x = B_y$	$n = \frac{\Delta q}{f\tau_0}\left\lvert \frac{u}{u_{max}} \frac{\sqrt{2}}{\sqrt{1+\sin 2H}}\right\rvert$	$n = \frac{\Delta q}{f\tau_0}\left\lvert \frac{u}{u_{max}} \frac{\sqrt{2}}{\sqrt{\frac{5}{4}+\frac{3}{4}\sin 2H}}\right\rvert$	$n = \frac{\Delta q}{f\tau_0}\left\lvert \frac{u}{u_{max}} \frac{\sqrt{2}}{\sqrt{\frac{7}{4}+\frac{1}{4}\sin 2H}}\right\rvert$

Table A1 Expressions for n for three declinations and three baseline orientations.

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