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EFFECTS OF SKEWED SAMPLING TIMES ON THE COMPLEX, THREE-LEVEL CORRELATOR

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1.0 INTRODUCTION

In the VLA[1], received signals are converted to baseband, split into "sine" and "cosine" parts in a 90° phase difference network, quantized to three levels, and sampled at a rate of twice the bandwidth or greater. They then pass through digital delay lines to an array of multipliers, where essentially all possible (3 level) products are formed at each sample time, and the products are each accumulated for a predetermined number of samples. In particular, the product of two "cosine" signals (or two "sine" signals) yields the real part of a complex correlation, and that of a "cosine" and a "sine" (or "sine" and "cosine") yields the imaginary part. This is illustrated in Figure 1.

For each input signal, then, four sampling switches are required, since two bits are needed to encode the three levels of the "sine" part and the "cosine" part. Design considerations in the quantizers (which are high-speed voltage comparators) and the samplers (planned to be edge-triggered flip-flops) make it possible that the effective sampling times of the four samplers may be unequal (principally because of propagation time variations in the comparators). In this report, we examine the effects of such skewed sampling times.

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[1]This paragraph describes the VLA correlator in continuum mode. In the spectrometer modes, operation is somewhat different.

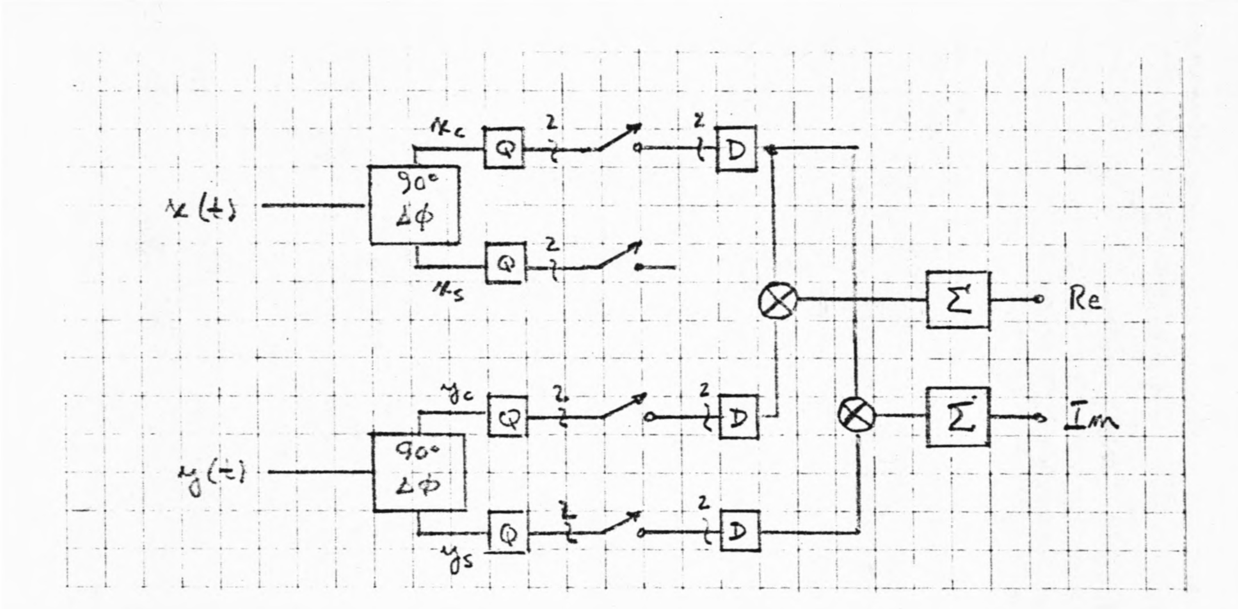

Figure 1: Simplified block diagram of correlator.

We shall also allow the quantization thresholds to depart from their optimum values, but otherwise the circuitry will be assumed to be ideal.

Without loss of generality, two types of sampling skew will be considered separately. First, quadrature timing skew, in which the sine and cosine parts are sampled at different times but the two bits of each part are sampled simultaneously; and second, sign timing skew, in which the two bits of one part are sampled at different times.

2.0 CORRELATOR RESPONSE VS DELAY

Since all samples will be digitally retimed (delayed) to be synchronous with the system clock, a sampling time error is equivalent to a delay error. It is therefore instructive to consider the correlator's response as a function of delay error. Figure 2 is a plot of the cross-correlation function of the unquantized signals for the case of inputs having ideal low-pass spectra of bandwidth W, with no sign timing skew. The real (cos x cos) and imaginary (cos x sin) channels each have cross-correlation functions with envelope A sinc (τW)

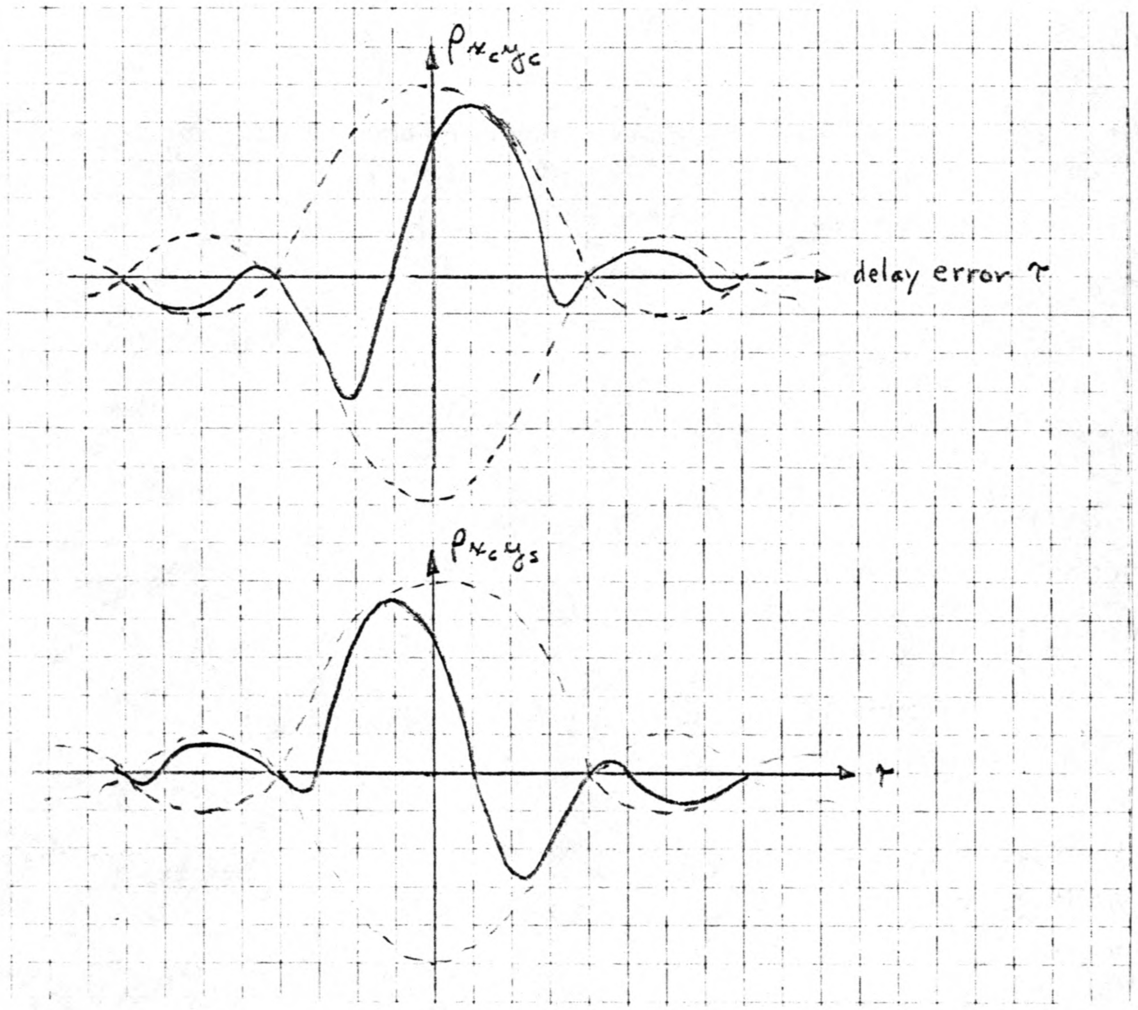

Figure 2: Correlated power vs delay error for cosine x cosine and cosine x sine channels.

multiplied by a sinusoid at frequency W/2. In the example shown, the real and imaginary parts are equal and positive at zero delay error, so that the complex result has phase 45° and amplitude A.

The response of the correlator is actually the cross-correlation of the quantized signals; but unless the correlation coefficient is large, this is simply proportional to that of the unquantized signals.

3.0 QUADRATURE TIMING SKEW

With Figure 2 in mind, the effects of quadrature timing skew can be determined directly. Letting $r_R(\tau)$ be the real channel response as a function of delay error τ, and similarly $r_I(\tau)$ for the imaginary channel, and attributing half of the timing skew $\Delta\tau$ to each channel, the errors are

$$\varepsilon_R = r_R(0) - r_R(\Delta\tau/2) \approx \frac{dr_R}{d\tau}\Big|_0 \frac{\Delta\tau}{2}, \tag{1a}$$

$$\varepsilon_I = r_I(0) - r_I(-\Delta\tau/2)_w \approx -\frac{dr_I}{d\tau}\Big|_0 \frac{\Delta\tau}{2}. \tag{1b}$$

For ideal low-pass spectra,

$$r_R(\tau) = A \operatorname{sinc}(\tau W) \cos(\pi\tau W + \phi) \tag{2a}$$

and

$$r_I(\tau) = A \operatorname{sinc}(\tau W) \sin(\pi\tau W + \phi) \tag{2b}$$

so

$$\varepsilon_R \approx -A\pi W \sin(\phi) \frac{\Delta\tau}{2} \tag{3a}$$

$$\varepsilon_I \approx -A\pi W \cos(\phi) \frac{\Delta\tau}{2} . \tag{3b}$$

Note that the error depends on ϕ, the actual phase of the complex correlation. If we desire to keep the errors less than some fraction δ of A for all ϕ, then we need

$$|\Delta\tau| < \frac{2\delta}{\pi W} = 0.13 \text{ nsec for } W = 50 \text{ MHz and } \delta = 1\%. \tag{4}$$

4.0 SIGN TIMING SKEW

In the VLA, the two bits which encode the three-level sample indicate respectively that the sample was above the positive quantizing threshold or below the negative one; hence, the name "sign timing skew" can be applied to the time difference $\Delta\tau$ between the sampling times of the two bits.

As pointed out to the author by A. R. Thompson (private communication), if the positive bit is TRUE, then it is very improbable that the negative bit will be TRUE, and vice versa, unless $\Delta\tau$ is very large. Thus, we may regard the effective sampling time as that of the positive bit when the sample is above the positive threshold, and as that of the negative bit when below the negative threshold. When between thresholds (both bits FALSE), the effective sampling time is unimportant because the sample makes a zero contribution to the correlation sum.

Figure 3 is a plot of the threshold voltages of two ideal quantizers in the instantaneous (x,y)-plane, where x(t) and y(t) are the

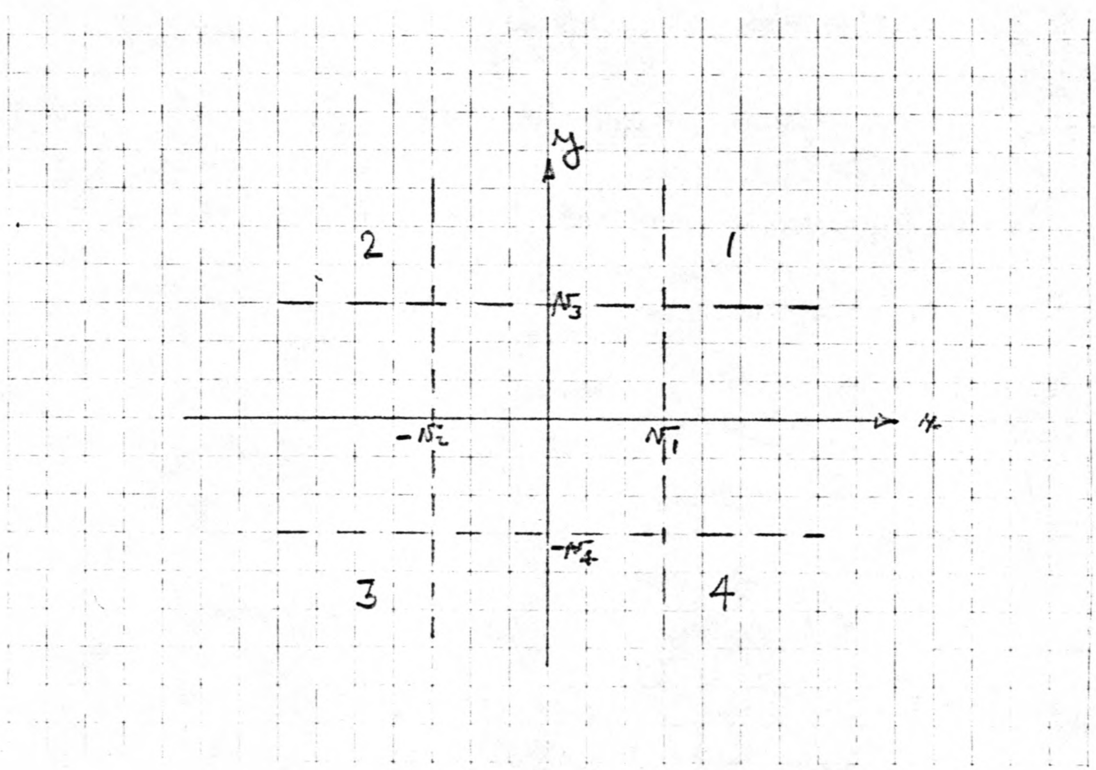

Figure 3: Quantization threshold diagram.

baseband signals. If the sample pair falls in the regions marked 1 or 3, a count of +1 is accumulated by the correlator; for regions 2 or 4, -1 is accumulated; and elsewhere, 0. Let P_i be the probability that the samples are in region i; then after N samples, the accumulated count is

$$r = N(P_1 + P_3 - P_2 - P_4). \tag{5}$$

The probabilities are merely integrals of the bivariate normal probability density function, and they depend on the threshold voltages and the correlation coefficient of the signals.

Now suppose that the x sampler has a sign timing skew, causing a delay error $\tau = \tau_+$ for positive samples and $\tau = \tau_-$ for negative ones. Then the correlator response can be determined by calculating P_1 and P_4 at the correlation corresponding to τ_+, and P_2 and P_3 at τ_-.

It should be apparent that the result will depend on the actual threshold voltages. Consider first the case of equal thresholds, $v_1 = v_2$ and $v_3 = v_4$ (as defined in Figure 3). Then if the + and - delay errors were equal, we would have $P_1 = P_3$ and $P_2 = P_4$; in particular, if both had $\tau = \tau_+$, the correlator count could be written

$$r_+ = 2N\ (P_{1+} - P_{4+}) \tag{6}$$

and if both had $\tau = \tau_-$ it would be

$$r_- = 2N\ (P_{3-} - P_{2-}) \tag{7}$$

where the +, - subscripts indicate evaluation at τ_+, τ_-. Apparently, then, the actual response is the average of the ideal responses at the two delay errors:

$$r = N\,(P_{1+} + P_{3-} - P_{2-} - P_{4+}) = \tfrac{1}{2}\,(r_+ + r_-). \tag{8}$$

As in Section 3.0, if we let $\tau_+ = \Delta\tau/2$ and $\tau_- = -\Delta\tau/2$, we find

$$\varepsilon_R = r_R(0) - 1/2[r_R(\Delta\tau/2) + r_R(-\Delta\tau/2)] \approx -1/2\,\frac{d^2 r_R}{d\tau^2}\Big|_0 \left(\frac{\Delta\tau}{2}\right)^2 \tag{9a}$$

$$\varepsilon_I = r_I(0) - 1/2[r_I(\Delta\tau/2) + r_I(-\Delta\tau/2)] \approx -1/2\,\frac{d^2 \tau_R}{d\tau^2}\Big|_0 \left(\frac{\Delta\tau}{2}\right)^2. \tag{9b}$$

For ideal low-pass spectra,

$$\varepsilon_R \approx +1/8(2\pi^2 - 2/3)\ W^2\ A \cos\phi\ (\Delta\tau)^2 \tag{10a}$$

$$\varepsilon_I \approx 1/8(2\pi^2 - 2/3)\ W^2\ A \sin\phi\ (\Delta\tau)^2 \tag{10b}$$

$$|\Delta\tau| < \frac{1}{W}\sqrt{\frac{8\delta}{2\pi^2 - 2/3}} = 1.3 \text{ nsec for } W = 50 \text{ MHz and } \delta = 1\%. \tag{11}$$

The case of unequal thresholds is more complicated. The VLA quantizers attempt to minimize the effect of unequal thresholds by phase switching: the sign of one of the baseband signals is reversed during half of the samples, and simultaneously the + and - output bits are interchanged. The results are expressed by

$$\frac{2r}{N} = [P_1(\rho_+) - P_1(-\rho_+)] + [P_3(\rho_-) - P_3(-\rho_-)] \tag{12}$$

$$- [P_2(\rho_-) - P_2(-\rho_-)] - [P_4(\rho_+) - P_4(-\rho_4)].$$

[If the thresholds are equal, $P_1(-\rho) = P_4(\rho)$ and $P_2(-\rho) = P_3(\rho)$ so that the above reduces to equation (8)]. With unequal thresholds, even with phase switching, it is no longer true that the actual result is the simple average of the ideal results at the two delay errors; it is instead a weighted average of the two, with more weight being given to the sign whose threshold is smaller.

A quantitative estimate of the effect can be obtained for ρ_+, $\rho_- \ll 1$. Collecting the terms in ρ_+ and ρ_- from (12),

$$\frac{2r}{N} = [P_1(\rho_+) + P_4(-\rho_+) - P_1(-\rho_4) - P_4(\rho_+)]$$

$$+ [P_3(\rho_-) + P_2(-\rho_-) - P_2(\rho_-) - P_3(-\rho_-)];$$

we can let

$$2\alpha = \frac{P_1(0) + P_4(0)}{P_3(0) + P_2(0)} \approx \frac{P_1(\rho_+) + P_4(-\rho_+)}{P_3(\rho_-) + P_2(\rho_-)} \approx \frac{P_1(-\rho_4) + P_4(\rho_+)}{P_2(\rho_-) + P_3(-\rho_-)} \ . \tag{13}$$

Then

$$r \approx \alpha r_+ + (1-\alpha) r_- . \tag{14}$$

To calculate α, note that

$$P_1(\rho) = L(v_1, v_3, \rho) \tag{15}$$

$$P_1(0) = L(v_1, v_3, 0) = \tfrac{1}{4}\ \mathrm{erfc}(v_1/\sqrt{2})\ \mathrm{erfc}(v_3/\sqrt{2}) \tag{16}$$

where L is the bivariate normal integral and erfc is the complementary error function; using similar expressions for P_2, P_3, P_4, (13) becomes

$$2\alpha = \frac{\mathrm{erfc}(v_1/\sqrt{2})[\mathrm{erfc}(v_3/\sqrt{2}) + \mathrm{erfc}(v_4/\sqrt{2})]}{\mathrm{erfc}(v_2/\sqrt{2})[\mathrm{erfc}(v_4/\sqrt{2}) + \mathrm{erfc}(v_3/\sqrt{2})]} = \frac{\mathrm{erfc}(v_1/\sqrt{2})}{\mathrm{erfc}(v_2/\sqrt{2})} \ . \tag{17}$$

Thus, for small ρ, the effect of sign timing skew in one sampler is altered by having unequal thresholds in the associated quantizer (v_1, v_2) but not by unequal thresholds in the other quantizer (v_3, v_4).

As before, let $\tau_+ = \Delta\tau/2$ and $\tau_- = -\Delta\tau/2$, so that

$$\varepsilon_R = r_R(0) - \alpha r_R(\Delta\tau/2) - (1-\alpha)r_R(-\Delta\tau/2)$$

$$\approx \frac{\Delta\tau}{2}\left[\alpha\frac{dr_R}{d\tau}\Big|_{\Delta\tau/4} - (1-\alpha)\frac{dr_R}{d\tau}\Big|_{-\Delta\tau/4}\right] \tag{18}$$

and similarly for ε_I. For ideal low-pass spectra,

$$\varepsilon_R \approx -\frac{\Delta\tau}{2} A\pi W[\alpha \sin(\pi W\Delta\tau/4 + \phi) + (1-\alpha)\sin(\pi W\Delta\tau/4 - \phi)] \tag{19}$$

Maximizing (19) over ϕ for $\Delta\tau \ll W^{-1}$ gives

$$\varepsilon_{R_{MAX}} \approx -\frac{\Delta\tau}{2} A\pi W(1-2\alpha). \tag{20}$$

If v_1 and v_2 differ by 10%, then $(1-2\alpha) \lesssim .08$. Then we require

$$|\Delta\tau| < \frac{2\delta}{\pi W(1-2\alpha)} = 1.6 \text{ nsec for } \begin{array}{l} W = 50 \text{ MHz} \\ \delta = 1\% \\ |v_1-v_2|/v_0 = 0.1 \end{array} \tag{21}$$

5.0 CONCLUSIONS

The results in (4), (11) and (21) indicate that quadrature timing skew puts the tightest constraint on the samplers, and that any tradeoff between it and sign timing skew should favor minimizing the former.

Sign timing skew errors are worsened if the corresponding quantizer has unequal thresholds. The equal-threshold error is proportional to $(\Delta\tau)^2$ [(cf. equation (10)], and the additional error to $\Delta\tau$ [equation (20)]; for the ±10% threshold tolerances presently allowed in the VLA, the effects are comparable at the 1% error level.

6.0 ACKNOWLEDGMENT

The author thanks A. R. Thompson for helpful discussions and for preliminary calculations on sign timing skew effects.