NATIONAL RADIO ASTRONOMY OBSERVATORY
SOCORRO, NEW MEXICO
VERY LARGE ARRAY PROGRAM

VLA ELECTRONICS MEMORANDUM NO. 186

CORRELATOR ERROR DUE TO GAIN CALIBRATION SIGNAL

L. R. D'Addario

August 17, 1979

On alternate waveguide transmission cycles, a known amount of broadband noise is added at the input of each receiver in order to monitor variations in gain by means of a synchronous detector located downstream in the signal processing. It is planned that the added noise will be up to 10% of the total noise power. Since the correlator integrates over many waveguide cycles, the correlation coefficient of its input signals is not constant during an integration, but is smaller when the calibration noise is on. In this report, the errors which this causes in estimates of the correlated power are investigated.

We can use an ALC loop to control the baseband power supplied to the three-level quantizers, or equivalently to control the quantizer threshold voltages, in any of several ways. Consider the following cases.

Case I.

ALC rapidly on the current signal, reducing the gain when the calibrator noise is on (CAL ON). The threshold-to-rms input voltage ratio can then be kept always at the value which gives best signal-to-noise ratio (SNR), but the true correlation coefficient ρ and the three-level correlation r_I will both be lower during CAL ON than during CAL OFF.

Case II.

ALC according to the level of the CAL OFF signal only. The thresholds will then be lower than the best SNR value during CAL ON (which increases r_I for a given ρ), but ρ will be lower then. Thus r_I will change by a smaller relative amount than ρ between CAL OFF and CAL ON.

Case III.

ALC according to the average signal level. The thresholds will then be low during CAL ON and high during CAL OFF. r_I will behave qualitatively as in Case II.

Following the notation of Schwab (1978), we have for any fixed ρ

$$r_I(v_5, v_6, \rho) = 2[L(v_5, v_6, \rho) - L(v_5, v_6, -\rho)] \tag{1}$$

where L is the bivariate normal integral and v_5, v_6 are the threshold-to-rms input voltage ratios of the two quantizers, assuming no dc offset in either quantizer's thresholds. Letting unprimed variables refer to CAL OFF and primed to CAL ON, we have

$$\bar{r}_I = \frac{1}{2}\,[r_I(v_5, v_6, \rho) + r_I(v_5', v_6', \rho')] \tag{2}$$

for the actual response averaged over an integral number of CAL cycles.

The three cases correspond to:

$$\text{I} \qquad \begin{aligned} \rho' &= \rho/\alpha\beta \\ v_5' &= v_5 \\ v_6' &= v_6 \end{aligned} \tag{3a}$$

$$\text{II, III} \quad \begin{aligned} \rho' &= \rho/\alpha\beta \\ v_5' &= v_5/\alpha \\ v_6' &= v_6/\beta \end{aligned} \tag{3b}$$

where α,β are ratios of the rms CAL OFF signal voltage to the rms CAL ON voltage for the two channels. Note that in Case II v_5, v_6 is held constant, whereas in Case III an average of v_5 and v_5', v_6 and v_6' is held constant.

Our objective is to determine ρ from measurements of $\bar{r}_I$. Assume that v_5, v_6, α, and β are known accurately; this is a fairly reasonable assumption for the VLA, because α and β are measured by synchronous detectors and v_5 and v_6 will either be controlled by digital feedback or monitored through the self-multipliers (autocorrelators). Then the only unknown in (2) is ρ, so that in principal a solution is possible. A formula to invert $\bar{r}_I(\rho)$ may be somewhat complicated, however, since it must account for four variable parameters. But since Schwab (1978) has given accurate formulas for inverting r_I over suitable ranges of v_5 and v_6, we may hope to use these with a simple correction for $\alpha\neq 1$, $\beta\neq 1$ and obtain sufficiently accurate results.

Therefore, consider defining estimates $\hat{\rho}$ of ρ as follows:

$$\text{I} \qquad \bar{r}_I = r_I(v_5,\ v_6,\ \tfrac{1}{2}(1+\tfrac{1}{\alpha\beta})\hat{\rho}) \tag{4a}$$

$$\text{II, III} \qquad \bar{r}_I = r_I(\tfrac{1}{2}(1+\tfrac{1}{\alpha})v_5,\ \tfrac{1}{2}(1+\tfrac{1}{\beta})v_6,\ \tfrac{1}{2}(1+\tfrac{1}{\alpha\beta})\hat{\rho}). \tag{4b}$$

That is, in Case I we approximate $\bar{r}_I$ as $r_I(\bar{\rho})$ where $\bar{\rho}$ is the average of ρ and ρ'; and in Cases II and III we make the additional approximation that an "effective" value for the first threshold is the average of v_5 and v_5', and similarly for the second threshold.

The relative error $(\rho-\hat{\rho})/\rho$ of these approximations has been computed using Schwab's sine approximation for r_I and r_I^{-1}. This formula is not very accurate above $\rho = 0.93$ (-1% error), with the error getting to -6% at $\rho = 1.0$. Nevertheless, the computations give some idea of the error introduced by the estimates in (4). The results are plotted as a function of ρ in Figure 1, for three different sets of (v_5, v_6), with $\alpha = \beta = 1.031$ (power increase of 10% for CAL ON). Similar curves were computed for unequal values of v_5 and v_6; in no case did the relative error exceed .06%.

It has been assumed that v_5 and v_6 are accurately known because they can be determined from the autocorrelator results via

$$g_5^2 = \tfrac{1}{2}[r_I(v_5, v_5, 1.0) + r_I(v_5', v_5', 1.0)]$$

$$= \begin{cases} r_I(v_5, v_5, 1.0) = \mathrm{erfc}(v_5/\sqrt{2}), & \text{Case I} \quad (5a) \\ \tfrac{1}{2}[\mathrm{erfc}(v_5/\sqrt{2}) + \mathrm{erfc}(v_5/\alpha\sqrt{2})], & \text{Cases II, III} \quad (5b) \end{cases}$$

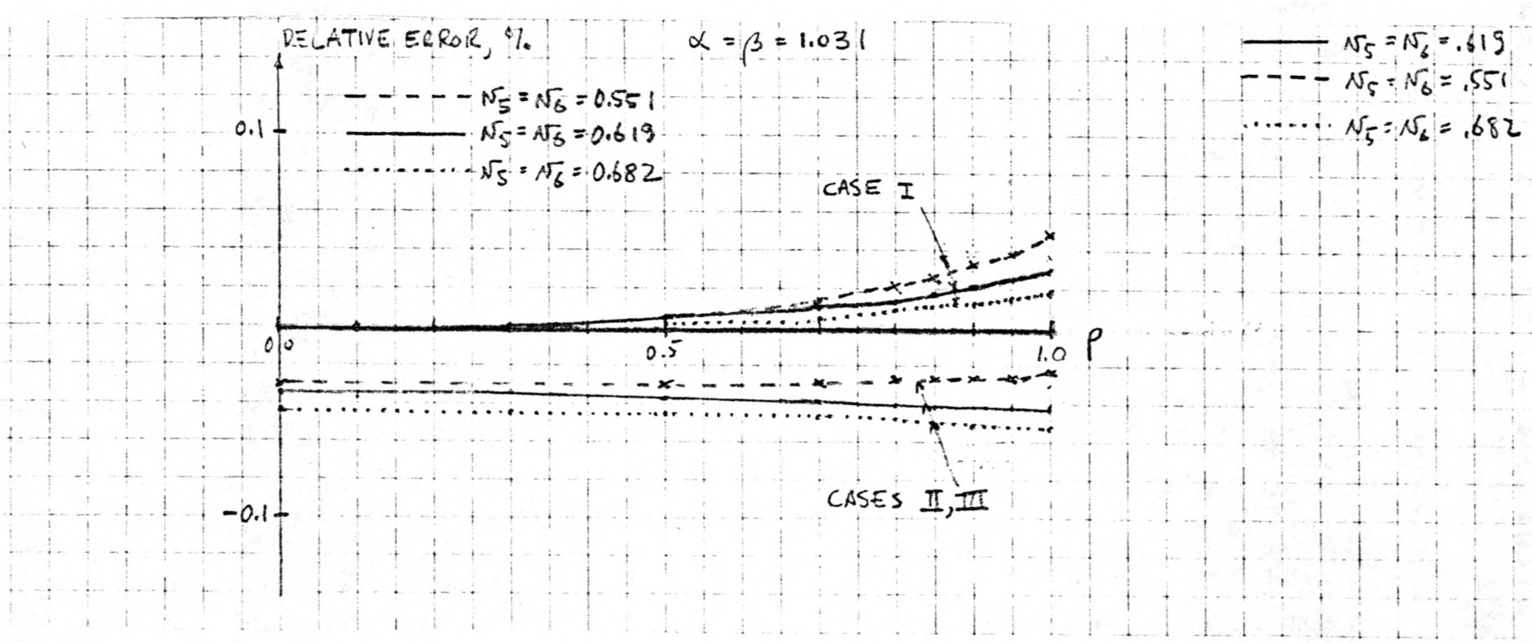

Figure 1: Relative error in estimating the CAL OFF correlation coefficient ρ using equation (4) for the three cases of ALC operation. Thresholds are assumed known.

where g_5^2 is the three-level autocorrelation of the first signal and erfc(·) is the complementary error function; g_6^2 is similarly defined for the second signal. Whereas (5) is monotonic in v_5 and α is known, (5) can in principle be solved for v_5. But, once again, the solution may be complicated, so we seek a simplification. In Case I (5a), the solution is $v_5 = \text{inverfc}\ (\sqrt{2}\ g_5^2)$, and this is not difficult to compute accurately. In Cases II and III, suppose we define $\hat{v}_5$ by

$$g_5^2 = \text{erfc}(\hat{v}_5/\sqrt{2})$$

or

$$\hat{v}_5 = 2\sqrt{2}\ \text{inverfc}\ (g_5^2) \tag{6}$$

and similarly for $\hat{v}_6$. We then can use these values in place of the averaged thresholds in (4b), so that

$$\bar{r}_I = r_I(\hat{v}_5, \hat{v}_6, \tfrac{1}{2}(1+\tfrac{1}{\alpha\beta})\hat{\rho}). \tag{7}$$

Figure 2 gives the relative error $(\rho\text{-}\hat{\rho})\rho$ when (7) is used, computed in the same way as Figure 1. The results are very similar to Figure 1, with the error never exceeding 0.6%.

It appears that satisfactory estimates can be obtained in all ALC schemes considered, but that the error is especially small at low correlation coefficient for Case I. Unfortunately, Case I (rapid ALC on the current signal) is not practical to implement. Present hardware implements Case II with the ALC accomplished using analog signals only, and with g_5^2 and g_6^2 being available for estimating the threshold voltage ratios. In this case, correction through equation (7) is recommended. In the future, Case II or III may be implemented by controlling the threshold voltages with digital feedback, in

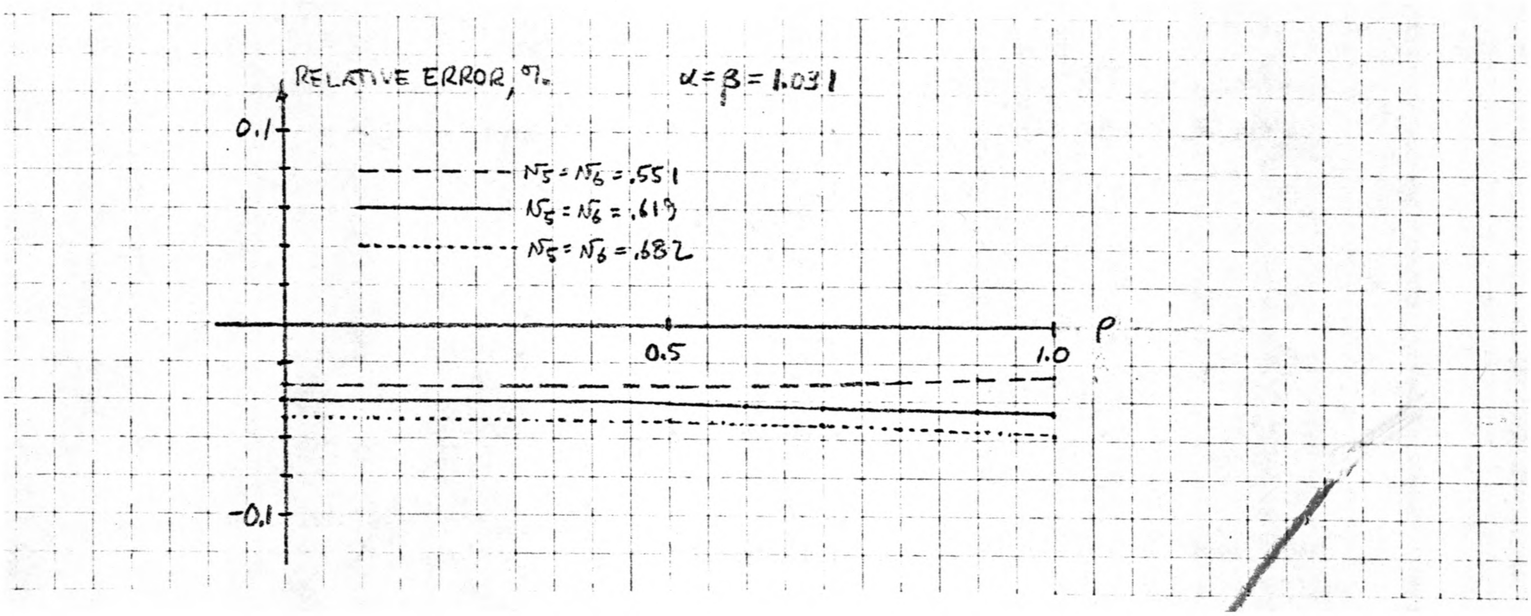

Figure 2: Relative error in estimating ρ using the measured autocorrelations and equation (7); ALC Cases II, III.

which case v_5 and v_6 may be regarded as a priori known constants. Then (4b) can be used to estimate ρ, but notice that since α and β are variable, the threshold-dependent terms cannot be regarded as constant and no simplification of the inversion is possible (a three-parameter formula is still needed).

REFERENCE

Schwab, F. W., 1978, "Quantization Correction of VLA Correlation Measurements". VLA Computer Memorandum No. 150 (issued October 1979).