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DIRECTIONAL FREQUENCY OFFSETS IN ROUND-TRIP PHASE-MEASURING SCHEMES, WITH APPLICATION TO THE IRAM ARRAY AND THE VLA.

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I. Introduction

Systems for establishing reference frequencies at distant antennas in radio interferometers or synthesis arrays include, in almost all cases, a round-trip measurement in which a signal is returned from the antennas for phase comparison at the master-oscillator station. This enables a correction to be made for changes in the electrical length of the transmission line to the antenna, caused by changes in temperature or pressure. In general, the signal frequencies used in the two directions should be as closely equal as possible to reduce errors resulting from reflections. However, a small frequency offset is often required to enable the signals to be separated, and the purpose of this memorandum is to investigate the tolerance on the frequency difference.

In schemes which make use of a modulated reflector (Swarup and Yang, 1961) no offset is necessary, but the signal transverses the cable in both directions without amplification and the cable length is more severly limited by attenuation than in some other schemes. In systems in which the signals in opposite directions are separated by time multiplexing, as in the VLA, again no frequency offset is necessary to separate them, although in the VLA a small offset is used for reasons described in Section IV.

Note that in all cases the absolute phase of the oscillator at the antenna is not required, and the system is designed to compensate for changes in the phase of the remote oscillator relative to the master.

II. A Round-Trip Phase-Reference System With Frequency Offset

A schematic diagram of the system to be considered is shown in Figure 1. Frequencies f_1 and f_2 are several gigahertz or hundreds of megahertz, and (f_1-f_2) is of the order of 1MHz. This particular scheme is based on one suggested by S. Weinreb for the IRAM interferometer.

First assume that there are no reflections at any point in the cable. At the antenna the phases of f_1 and (f_1-f_2) relative to their phases at the central location are $2\pi f_1 L/v$ and $2\pi(f_1-f_2)L/v$ where v is the velocity in the cable. The phase of the f_2 oscillator is constrained to equal the difference of these phases, i.e. $2\pi f_2 L/v$. The phase change in f_2 in travelling back to the central location is $2\pi f_2 L$, and thus the measured round-trip phase is $4\pi f_2 L/v$ (modulo 2π, of course). Now suppose that the length of the line changes by a small fraction, β. The phase of the oscillator f_2 at the antenna relative to the master oscillator changes to $2\pi f_2 L(1+\beta)v$. The required correction to the f_2 oscillator is just half the change in the measured round-trip phase. Note that the main effect of velocity dispersion in the line is to introduce small phase terms which vary only by negligible second-order amounts as the line-length changes, and can be ignored.

Next consider what happens if reflections occur at points A and B, spaced along the line by a distance of ℓ. The complex voltage reflection coeffecients are ρ_A and ρ_B and their values will be considered to be the same at frequencies f_1 and f_2. Signals f_1 and f_2, after traversing the cable, now include components which have been reflected once at A and once again at B. The coefficients ρ_A and ρ_B are sufficiently small that signal components arising from more than one reflection at each point can be neglected. For the frequency f_1 arriving at the antenna, the amplitude of the reflected component relative to the unreflected one is

$$A = |\rho_A||\rho_B|[10^{-(2\ell\alpha/10)}]^{\frac{1}{2}} = |\rho_A||\rho_B|10^{-(\ell\alpha/10)} \quad (1)$$

where α is the attenuation coefficient of the cable. Similarly the phase of the reflected component relative to the unreflected one is, modulo 2π,

$$\Theta_1 = 4\pi\ell f_1/v + \arg(\rho_A) + \arg(\rho_B). \quad (2)$$

Figure 2 shows a phasor representation of the reflected and unreflected components. The reflected component causes the resultant phase to be deflected through an angle ϕ_1 given by

$$\phi_1 \cong \tan\phi_1 = \frac{A\sin\Theta_1}{1 + A\cos\Theta_1} \quad (3)$$

Similarly, the phase of the frequency f_2 is deflected through an angle ϕ_2, given by equations equivalent to (2) and (3) with f_1, Θ_1, and ϕ_1 replaced by f_2, Θ_2, and ϕ_2, respectively.

With the reflections at A and B the round-trip phase for line length L is

$$4\pi f_2 L v^{-1} + \phi_1 + \phi_2 \quad (4)$$

When the line length increases to $L(1 + \beta)$ the angles ϕ_1 and ϕ_2 vary in a non-linear manner with ℓ and become $(\phi_1 + \delta\phi_1)$ and $(\phi_2 + \delta\phi_2)$ respectively. The round-trip phase then becomes

$$4\pi\ f_2\ L(1+\beta) + \phi_1 + \delta\phi_1 + \phi_2 + \delta\phi_2 \tag{5}$$

(The effect of the reflection on the phase of the signal at frequency (f_1-f_2) has been neglected in (4) and (5) since its rate of change with line-length is very small.) The applied correction for the increase in line length is half the measured change in round-trip phase:

$$2\pi\ f_2\ \beta\ L + \tfrac{1}{2}(\delta\phi_1 + \delta\phi_2) \tag{6}$$

However, the change in the phase of f_2 at the antenna, i.e. the exact correction, is

$$2\pi\ f_2\ \beta\ L + \delta\phi_1 \tag{7}$$

Consequently there is a phase error equal to

$$\delta\phi_1 - \tfrac{1}{2}(\delta\phi_1 + \delta\phi_2) = \tfrac{1}{2}(\delta\phi_1 - \delta\phi_2)$$

If f_1 and f_2 were equal, the phase error would be zero. It should therefore be possible to specify a maximum allowable frequency difference in terms of the maximum tolerable error.

The difference between the phase angles ϕ_1 and ϕ_2 is given by

$$\phi_1-\phi_2 = \frac{\partial\phi_1}{\partial f_1}\ (f_1-f_2)$$

$$= \frac{4\pi\ell v^{-1}A\ \cos\theta_1(1+A\ \cos\ \theta_1)+4\pi\ell v^{-1}A^2\sin^2\theta_1}{(1 + A\ \cos\ \theta_1)^2}\ (f_2-f_1) \tag{8}$$

Since A <<1, terms in A^2 can be omitted from the numerator in (8) and the denominator is approximately unity. Thus

$$\phi_1 - \phi_2 = 4\pi \ell v^{-1} A \cos \Theta (f_2 - f_1) \tag{9}$$

The variation of $\phi_1 - \phi_2$ with line-length is given by

$$\begin{aligned}(\delta\phi_1 - \delta\phi_2) &= \frac{\partial}{\partial\ell} (\phi_1 - \phi_2)\beta\ell \\ &= [-(4\pi v^{-1})^2 \ell f_1 A \sin \Theta + 4\pi v^{-1} \cos \Theta(A - 0.1\, \ell A\alpha \log_e 10)]\mathrm{x} \\ &\qquad (f_2 - f_1)\beta\ell \\ &= 4\pi v^{-1} A[\cos\Theta - 0.1\, \ell\alpha(\log_e 10)\cos \Theta - 4\pi v^{-1}\ell f_1 \sin \Theta]\mathrm{x} \\ &\qquad (f_2 - f_1)\beta\ell\end{aligned} \tag{10}$$

With practical parameter values such as $v = 2.7 \times 10^8 \mathrm{m\ s^{-1}}$, $\ell = 100$ m, $\alpha = 4 \times 10^{-2}$ dB $\mathrm{m^{-1}}$ and $f_1 = 2.5$ GHz, the terms in the square brackets in (10) have maximum values of 1, 0.9 and 1.1×10^4 from left to right respectively. If the two smaller terms are neglected, the magnitude of the phase error becomes

$$\tfrac{1}{2}(\delta\phi_1 - \delta\phi_2) = 8\pi^2 v^{-2} |\rho_A| |\rho_B| \beta\ell^2 10^{-(\alpha\ell/10)} f_1(f_1 - f_2) \sin \Theta \tag{11}$$

The factor $\ell^2 10^{-(\alpha\ell/10)}$ has a maximum value at

$$\begin{aligned}2\ell 10^{-(\alpha\ell/10)} - 0.1\alpha\ell^2(\log_e 10)10^{-(\alpha\ell/10)} &= 0 \\ \ell = 20(\alpha \log_e 10)^{-1}&\end{aligned} \tag{12}$$

This maximum occurs because for small values of ℓ the change in the angle Θ with frequency or cable expansion is small, and for large values of ℓ the reflected component is greatly attenuated. The maximum value is equal to

$$[\ell^2 10^{-(\alpha\ell/10)}]_{max} = 10.21\alpha^{-2} \qquad (13)$$

Curves of $\ell^2 10^{(\alpha\ell/10)}$ are plotted in Figure 3 for various values of α that correspond to good quality cables. It is evident that reducing the attenuation in a cable increases the error in the round-trip phase correction, and one should consider using a cable with as much attenuation as can be tolerated from signal-level considerations.

The principal cause of reflections in a cable that is used with a movable antenna are the connectors that are inserted at the antenna stations. Unless the antenna is at the closest station, there will be one or more interconnecting loops of flexible cable when an unused station is bypassed. If there are n connectors in the cable there are $N = n(n-1)/2$ pairs between which reflections can occur. The phasors of the corresponding reflected components will combine randomly, and for the overall error in the round-trip phase correction (11) becomes

$$\delta\phi_{rt} = 5.66\ \pi^2 v^{-2} |\rho|^2 \beta\ f_1(f_1 - f_2)\ F(\alpha,\ell) \qquad (14)$$

where

$$F(\alpha,\ell) = \sqrt{\sum_{i=1}^{N} \sum_{k=i}^{N} \ell_{ik}^4 10^{-(2\alpha\ell_{ik}/10)}}\ , \qquad (15)$$

the rms value has been used for sin Θ, and the reflection coefficients are all approximated by a single value, ρ.

III. Application to the IRAM Synthesis Array

In the IRAM system three antennas will initially be used at distances up to 350 m from the master oscillator, and as a later development the distance of one of them may be increased to 1 km. The observing frequency will be near 100 GHz and the goal for the phase accuracy of the 100 GHz local oscillator is 1°. The phase error at 100 GHz resulting from reflections is $\delta\phi_{rt}$ from (14) multiplied by 100 GHz/f_1, which is independent of f_1. However, phase errors at 100 GHz resulting from noise in the loop, errors in phase detectors, etc. increase with the multiplication factor 100 GHz/f_1. For this reason, and to leave the band 1-2 GHz free for IF transmission in the cable, the frequencies f_1 and f_2 should not be too low. On the other hand they should not be so high that the total attenuation in 1 km of cable exceeds 60-70 dB, and they should be well below cutoff of high-order modes in coaxial cable, which start at 2.9 GHz for the 1 5/8 inch size. Thus frequencies in the range 2.0-2.5 GHz are certainly close to optimum for f_1 and f_2. A multiplication factor of at least 40 is required to reach 100 GHz, and a goal of 1° phase accuracy in the local oscillator requires that $\delta\phi_{rt} < (40x57)^{-1}$. To keep the electronics at the antennas as simple as possible, at least for the initial implementation, tuning is to be accomplished by varying f_1 and f_2 by about 3% to allow continuous coverage from one harmonic to the next at the local oscillator frequency.

The natural frequency of the phase lock loop at the antennas places some constraints on various parameters. With a tunable oscillator at the antennas the natural frequency of the loop will probably need to be as high as 100 kHz since residual frequency modulation on such an oscillator would be much greater than that on an oscillator derived from a high-stability crystal. The effective noise bandwidth of the loop, B_n, is approximately three times the natural frequency, the exact factor depending upon the loop damping (e.g. Gardner, 1966). One requirement to achieve a phase accuracy of 1/40° is a signal-to-noise ratio in the loop exceeding $(40x57)^2$ in power, or 67 dB. Thus a lower limit on the level, p_1, of the signal f_1 at the antenna is set by

$$p_1 > (40x57)^2(F-1)kTB_n \qquad (16)$$

where F is the noise figure of the mixer at the antenna-end of the cable, k is Boltzmann's constant and T = 300 K. For F = 10 and B_n = 300 kHz, $p_1 > 6x10^{-8}$. If the power level into the cable at the master oscillator is no more than 10^{-1}W, the cable attenuation must not exceed 62 dB. For a 1 km distance α must be less than $6.2x10^{-2}$ dB m^{-1}. With this value we can now calculate the maximum allowable offset frequency (f_1-f_2). It will be assumed that the cable to the most distant antenna station in any direction contains about 8 intermediate stations, i.e. 8 connecting loops and 16 connectors. There should therefore be 120 pairs of connectors, and we shall arbitarily estimate that the connector spacings ℓ_{jk} are such that the overall effect of the reflections is equivalent to that for 1/3 as many pairs with the spacing that maximizes $\ell_{jk}^2\, 10^{-(\alpha\ell_{jk}/10)}$. Thus we insert the following values on (14):

$$v = 2.7\times10^{8}\ \mathrm{m\ s^{-1}} \qquad \rho = 0.05$$

$$\beta = 10^{-5} \qquad f_1 = 2.3\times10^{9}\ \mathrm{Hz}$$

$$F(\alpha,\ell) = \sqrt{40}\times 10.21\alpha^{-2} \qquad \alpha = 6\times10^{-2}\ \mathrm{dB\ m^{-1}}$$

$$\delta\phi_{rt} = (40\times57)^{-1}$$

The value of β is based on the assumption that no more than 1° C temperature change occurs between calibration observations, or during whatever time interval is critical. The above values yield (f_1-f_2) = 550 kHz. For the 350 m cable lengths a value of α as high as 0.17 dB m^{-1} could be used, with which (f_1-f_2) could be as high as 4.4 MHz. In order to avoid sidebands on the oscillator at the antenna resulting from modulation at the frequency offset (f_1-f_2), this offset should exceed the natural frequency of the phase-lock loop by at least one order of magnitude, i.e. it should probably not be less than 1 MHz.

A detailed evaluation of the possible performance will need to take account of the parameters of available cables and oscillators. However, the above considerations indicate that for a 350 m distance a system with a 1 MHz offset should be feasible and the cable diameter should be no greater than 7/8-inch for which α is about 6.5×10^{-2} dB m^{-1}. If the accuracy goal cannot be met for the 1 km length with the above system it would always be feasible to change to one in which a crystal oscillator is used at each antenna, as in the VLA. The natural frequency of the phase lock loop at an antenna, and consequently also the offset frequency, could then be reduced by two or more orders of magnitude. A possible crystal oscillator scheme is shown in Figure 4.

The phase stability goal of 1° at 100 GHz corresponds to only 8 microns of path length and it is, of course, possible that effects other than those discussed above will set the limit on the overall phase accuracy.

IV. Application to the VLA

The round-trip phase measuring scheme for the VLA is outlined in Figure 5. The local oscillator frequencies travel in the waveguide as sidebands at 1200 MHz and 1800 MHz on a carrier of frequency f_o that lies between 27 and 52 GHz. The 600 MHz difference between these frequencies is used as the reference frequency. Because the signals in the waveguide travel in different directions at different times, there is no need, in principle, for any frequency offset. However, in the mixing process in which 600 MHz is obtained from 1200 and 1800 MHz, a component can be formed as (2x1200-1800) MHz in addition to the one formed from the direct frequency difference. This component is approximately 45 dB below the (1800-1200) MHz component, and as the relative phases of the 1200 and 1800 MHz signals vary, its phase will vary in a different way from that of the (1800-1200) MHz component, and produce an error in the measured phase of the latter. To prevent this, a small offset, Δf, is added to the frequency of the carrier going out from the central location to the antennas. The frequency of the (1800-1200) MHz component remains unchanged, but that of the (2x1200-1800) MHz component becomes 600 MHz + Δf. If Δf is a few tens of Hertz or more, the unwanted component is rejected by the phase lock loop at the antenna, for which the natural frequency is about 1.5 Hz. For signals returning from the antenna, the offset is introduced at the receiving mixer. The unwanted component produces an output at frequency Δf at the round-trip phase detector which is filtered out by the output time constant.

The effect of reflections in the waveguide, as considered in Section II., imposes an upper limit on Δf. In the VLA the round-trip phase is measured at 600 MHz, and the result is equal to the difference that would be obtained between round-trip phases for the 1200 MHz and 1800 MHz signals if these were measured explicitely. Near the low-end of the waveguide frequency range, at 30 GHz, a change of 600 MHz in frequency produces a change of 1 part in $3x10^3$ in the velocity in the main waveguide, for which the cutoff frequency of the TE_{01} mode is 3.83 GHz. At 50 GHz, near the high end of the band, the change is 1 part in $1.4x10^4$. The spacings between reflections in the waveguide can easily be as much as 10^6 wavelengths at the waveguide frequency, so the phases of the reflected components of the 1200 and 1800 MHz sidebands vary by many complete rotations. The 600 MHz round-trip phase measurement is therefore, in effect, the difference between two measurements for which the errors are totally independent. Thus the error in the round-trip phase correction is obtained by taking $\sqrt{2}$ times the expression in (14).

To perform a worst-case analysis for the VLA waveguide we use the minimum attenuation coefficient of $\alpha = 10^{-3}$ dB m^{-1}, and the corresponding curve of $\ell^2 10^{-(\alpha\ell/10)}$ is given in Figure 6. There are 24 antenna stations on each arm, and therefore 276 pairs of stations. At the 21 innermost stations on each arm there is a 9° sector coupler in the waveguide, for which the coupling to the antenna is -20 dB and the return loss in the main line from either direction is less than -40 dB. At the 22nd station the coupler is a 36° sector type with 14 dB coupling and return loss similar to the 9° couplers. At the 23rd station there is a 6 dB beam-splitter coupler with about -25 dB return loss in the main line, and at the end of the arm there is a 6 dB attenuator followed by a modem, so for channels other than that of the end antenna the end-station return loss is about 12 dB. In Figure 6 the -3 dB points on the curve occur 3.5 and 18 km, and a rough estimate indicates that about 100 of the inter-station distances fall within this range. Thus the factor $F(\alpha,\ell)$ is about 10^8 as a result of reflections at the first 22 stations on an arm. The directivity of the couplers should significantly reduce the effects of reflections between points that are further out along the arm than the antenna concerned. Thus the higher reflection coefficients of the two end stations would affect only the end antenna, but for its own frequency channel the end antenna should be quite well matched. Thus the estimate of 10^8 for $F(\alpha,\ell)$ should be fairly good for the A array. For the D array the shorter spacings and smaller number of stations involved will reduce the factor to about 10^6. Other parameter values used in (14) for the VLA are:

$v = 3\times10^8$ m s^{-1} $\qquad\qquad$ $\rho = 0.01$

$\beta = 10^{-5}$ $\qquad\qquad$ $f_1 = 5\times10^{10}$ Hz

For the A array these values give $\delta\phi_{rt} = 4.4\times10^{-6}(f_1-f_2)$. Then if the round-trip phase error is not to exceed 0.1°, (f_1-f_2) should not exceed 400 Hz. A 1 kHz crystal oscillator has been built into the master local oscillator system to provide the offset, and in view of the rather rough estimate of $F(\alpha,\ell)$ given above, and the small contribution to the overall phase stability that 0.1° at 600 MHz represents, the 1 kHz offset is considered to be satisfactory.

References

Gardner, F. M., Phaselock Techniques, John Wiley, New York, 1966.

Swarup, G. and Yang, K. S., Proc. I.R.E. Trans. Ant. and Prop., AP-9, 75, 1961.

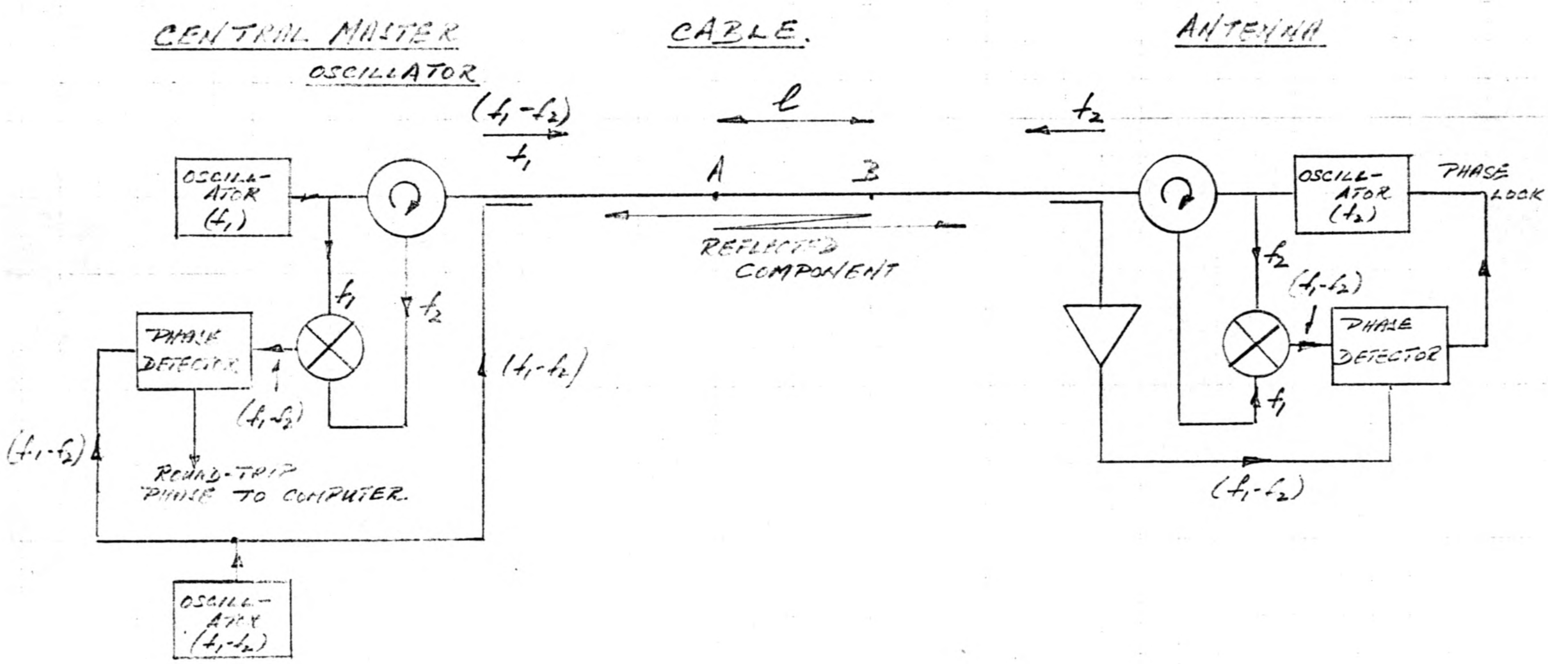

Figure 1. Phase-lock scheme for oscillator f_2 at antenna, with round-trip measurement of phase.

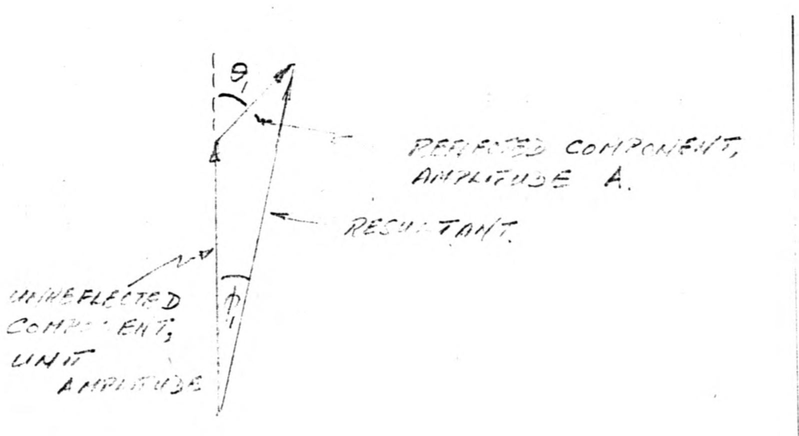

Figure 2. Phasor diagram of component at frequency f_1 transmitted by the cable.

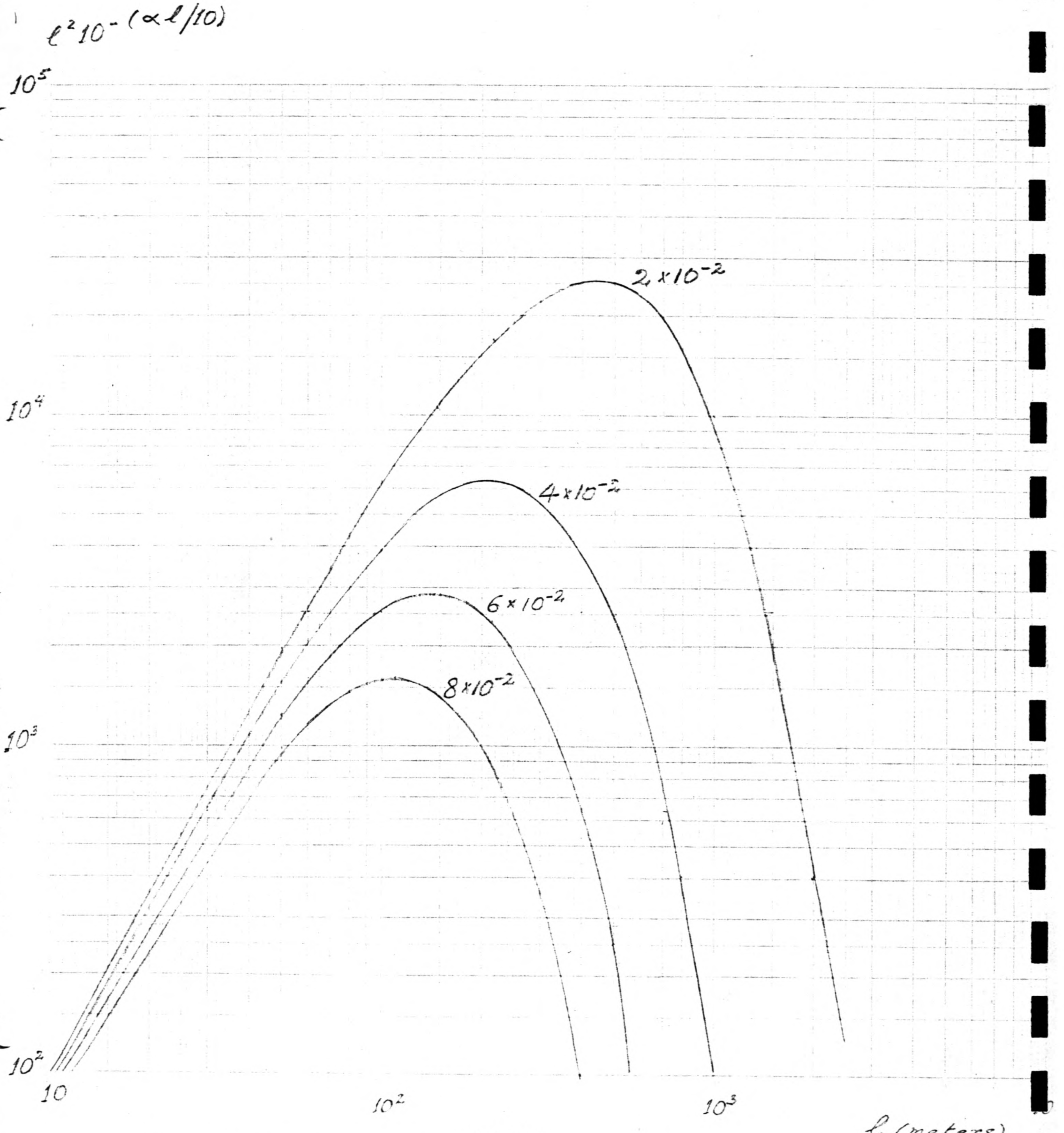

Figure 3. The function $\ell^2 10^{-(\alpha\ell/10)}$ plotted for four values of the transmission-line attenuation, α dB m^{-1}.

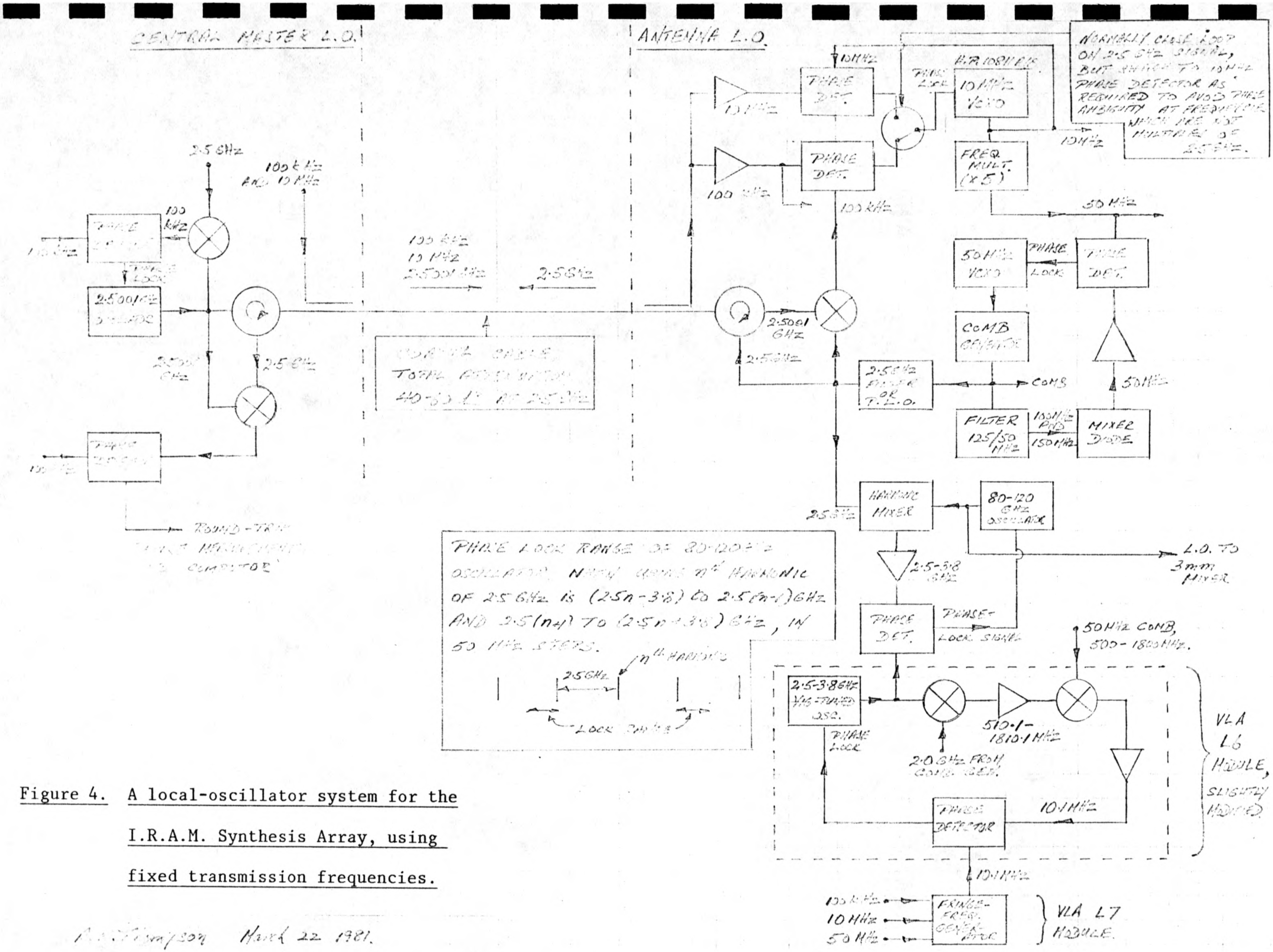

Figure 4. A local-oscillator system for the I.R.A.M. Synthesis Array, using fixed transmission frequencies.

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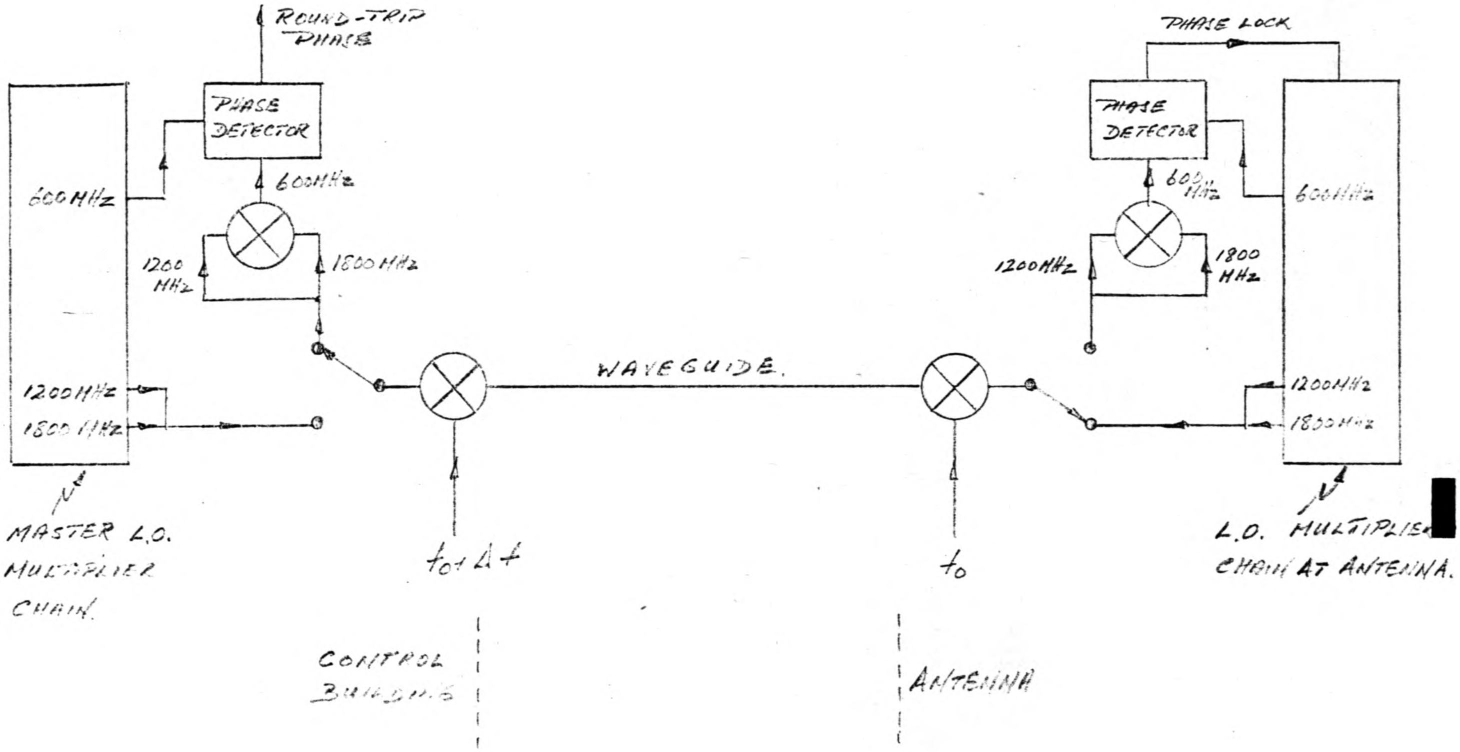
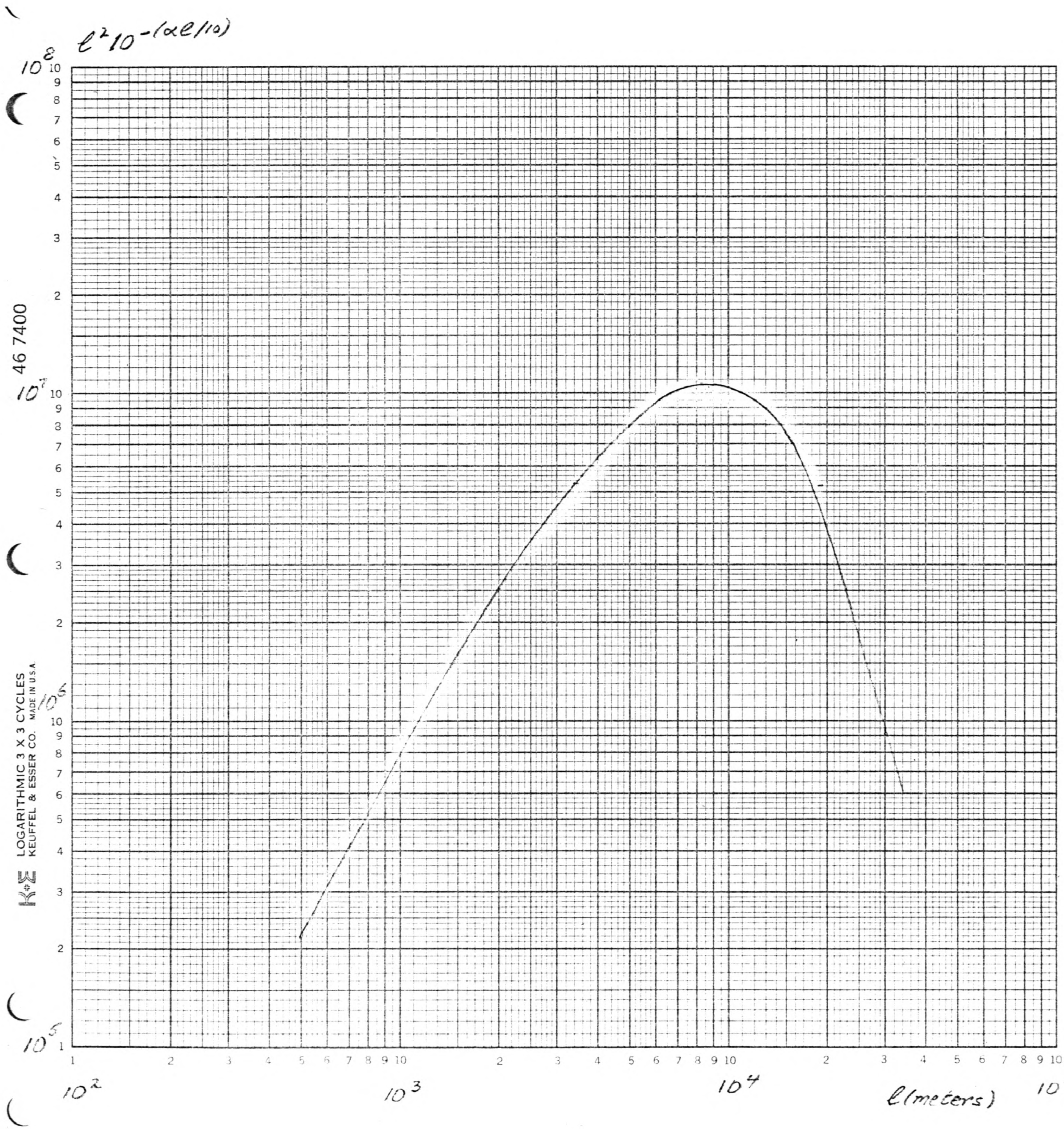


Figure 5. Simplified schematic diagram of the round-trip phase measurement in the VLA.



<u>Figure 6.</u> The function $\ell^2 10^{-(\alpha\ell/10)}$ plotted for the VLA waveguide with $\alpha = 10^{-3}$ dB m^{-1}.