# The Origin of the $180^{\circ}$ Phase Shift in HTRP Polarization Data 

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## I. Introduction.

A simplified analysis of the HTRP multiplying polarimeter (HTRP Memo 107) predicted that the phase offsets of the signals RL (Stokes U) and LR (Stokesm) should be equal. However, actual polarization data have shown that the phase offsets are equal only in signals originating in the lower sideband (LSB) of the MkIII videoconverters. The phase offsets of RL and LR in the upper sideband (USB) of all videoconverters differ by 180 degrees (e.g. Figure 1). This memorandum describes how the phase shift originates in the image rejection mixer of a MkIII videoconverter.

Since a $180^{\circ}$ phase shift is equivalent to an inversion, the USB polarization position angles are erroneously calculated as $-\arctan (U / Q)$ instead of $\arctan (U / Q)$. As a rough approximation, the true polarization position angle of the USB signal may be calculated by multiplying $U / Q$ by negative one before computing the arctangent. A more accurate calculation of position angle would incorporate the $180^{\circ}$ phase shift into an instrumental term as shown in HTRP Memo 107.

## II. Signal Analysis.

Consider a MkIII videoconverter input which is right circularly-polarized and composed of two frequency components.

$$
\begin{equation*}
R=\cos \left(\omega_{U} t+\phi_{R}\right)+\cos \left(\omega_{L} t+\phi_{R}\right) \tag{1}
\end{equation*}
$$

The frequency component designated by $\omega_{U}$ represents the frequency components destined for the videoconverter USB, and the frequency component designated by $\omega_{L}$ represents the frequency components destined for the videoconverter LSB. To simplify the signal analysis, each component is assumed to have unity amplitude and a phase of $\phi_{R}$. The phase term accounts for source intrinsic position angle and phase delay in the signal path. If the image rejection mixer of the videoconverter is configured for a local oscillator frequency of $\omega_{L O}$, then by definition $\omega_{L}<\omega_{L O}<\omega_{U}$. Using this relationship and an input signal defined by equation (1), one can easily show in the analysis of an image rejection mixer (e.g. Thompson, Moran, and Swenson, 1986, or HTRP Memo 110) that the outputs of the videoconverter are

$$
\begin{align*}
& R_{U S B}=\cos \left[\left(\omega_{U}-\omega_{L O}\right) t+\phi_{R}\right]  \tag{2}\\
& R_{L S B}=\cos \left[\left(\omega_{L O}-\omega_{L}\right) t-\phi_{R}\right] \tag{3}
\end{align*}
$$

Similarly, the left circularly-polarized outputs of a separate videoconverter are

$$
\begin{equation*}
L_{U S B}=\cos \left[\left(\omega_{U}-\omega_{L O}\right) t+\phi_{L}\right] \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
L_{L S B}=\cos \left[\left(\omega_{L O}-\omega_{L}\right) t-\phi_{L}\right] \tag{5}
\end{equation*}
$$

Equations (2) through (5) show a leading phase in each USB signal and a lagging phase in each LSB signal. As shown below, the leading phase in the USB signal causes a $180^{\circ}$ phase shift at the HTRP multiplying polarimeter.

The USB LR signal generated by the polarimeter is equivalent to the product of $L_{U S B}$ and $R_{U S B}$. The USB RL signal is equivalent to the product of $L_{U S B}$ and $R_{U S B}$ shifted by $90^{\circ}$.

$$
\begin{gather*}
L R_{U S B}=R_{U S B} \bullet L_{U S B}=\frac{1}{2} \cos \left(\phi_{R}-\phi_{L}\right)=\frac{1}{2} \cos \psi  \tag{6}\\
R L_{U S B}=R_{U S B}(+\pi / 2) \bullet L_{U S B}=-\frac{1}{2} \sin \psi=\frac{1}{2} \sin (\psi \pm \pi) \tag{7}
\end{gather*}
$$

The signals produced by the polarimeter in the LSB are

$$
\begin{align*}
L R_{L S B} & =\frac{1}{2} \cos \psi  \tag{8}\\
R L_{L S B} & =\frac{1}{2} \sin \psi \tag{9}
\end{align*}
$$

The angle $\psi=\phi_{R}-\phi_{L}$ is the difference in path lengths of the right and left circularlypolarized IF's, and contains the source intrinsic polarization angle. Equations (6) through (9) show that the phase offsets in the signals produced by a pair of videoconverters are the same except for the phase of $R L_{U S B}$, which contains an additional $180^{\circ}$ offset.

## III. Results.

Figure 1 displays the measured RL and LR in the upper and lower sidebands of a pair of videoconverters for an observation of the pulsar PSR0329+54 at transit. RL and LR have been normalized to the total intensity, but have not been completely calibrated for gain. These signals vary sinusoidally with parallactic angle as the linear polarization vector rotates in the plane of the sky (HTRP Memo 107). The data points were selected from a region in the pulsar pulse profile where the percent linear polarization was high, about $65 \%$. The equations associated with each set of data are the non-linear least-squares best fits to the data. The phase offsets in the LSB data are approximately equal as predicted by equations (8) and (9). The small difference ( $108-105=3^{\circ}$ ) in phase between RL and LR may be attributed to the difference in signal path lengths between the RL and LR detector circuits (HTRP Memo 109). The phase offsets in the USB data differ by approximately $180^{\circ}$ as predicted by equations (6) and (7). The difference in phase ( $120-108=12^{\circ}$ ) between USB and LSB signals was not predicted by the preceding analysis. This apparent discrepancy arises from the assumption that the phases $\phi_{R}$ and $\phi_{L}$ are independent of frequency, which is generally not true. One can show that the phase offsets in the USB and LSB signals differ by writing equation (1) in terms of a frequency dependent phase, and repeating the signal analysis. For example,

$$
\begin{equation*}
R=\cos \left(\omega_{U} t+\phi_{R}\left(\omega_{U}\right)\right)+\cos \left(\omega_{L} t+\phi_{R}\left(\omega_{L}\right)\right) \tag{10}
\end{equation*}
$$

## IV. Acknowledgment.

I would like to thank Dr. Richard Fisher for an insightful discussion on image rejection mixers.

## V. References.

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Thompson, A. R., Moran, J. M., and Swenson, G. W., 1986, Interferometry and Synthesis in Radio Astronomy, John Wiley and Sons, New York.

Figure 1: Linear Polorization of PSRO329+54 at Transit


