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Results of Preliminary Studies of Complimentary Arrays

Introduction

Studies of complimentary arrays are directed towards an investigation of the following.

(i) the minimum number of antennas needed to get an acceptable u-v plane coverage and the antenna configurations which achieve this coverage.

(ii) the minimum tracking time needed to get an acceptable coverage, and

(iii) the optimum array configurations for a 36-element VLA which will achieve the best possible coverage.

Various configurations with 18 elements have been tried. Sampling statistics and u-v plane coverage plots have been obtained for declinations of 30°, 0° and -15° and for tracking times of ± 6, ± 4, and ± 2 hours. The basic configuration used is a symmetrical Wye with one arm rotated 5° from North-South.

Although a judicious selection of antenna positions reduces the percentage of holes considerably, it is concluded that no choice of configurations using 18 elements will bring the hole percentage down to the acceptable limit of 15 %. Future studies will be done with 24 and 27 elements.

The Computer Program

The computer program accepts as input the relative locations of elements for the two configurations, the number of elements, the declination and tracking time. It computes the element coordinates for the two configurations, termed

ARRAY1 and ARRAY2. Plots of the u-v plane coverage and sampling statistics are printed out. These include the performance of the two configurations individually as well as their combined performance. A comparison of the two is also obtained.

The model number is a seven digit number in which the first two digits indicate the number of elements, the third digit indicates the tracking time, the fourth digit indicates the declination (0 for 0°, 1 for 30° and 2 for -15°) and the last three digits are used for sequencing. Thus model 1842005 means an 18-element array tracking for ±4 hours at a declination of -15°. The configuration is given by the label 005.

Choice of Configuration

The first model chosen was one in which the superposition of ARRAY1 and ARRAY2 led to the supplemented Wye. This choice was based on Hogg's studies of element configurations. Succeeding configurations were based of the results of the previous ones. In each, an attempt was made to overcome the shortcomings of the previous one. After six such models, model 007 was chosen arbitrarily. In this model ARRAY1 was tapered and ARRAY2 was having the longer baselines. Repetition of baseline lengths was avoided. Models 008 - 010 involved a different concept. ARRAY1 had only the North arm and the South-East arm and ARRAY2 had only the North arm and the South-West arm. Relative element locations were the same in both. The computer program was suitably modified for this.Three types of element locations were tried for these models -- uniform, supplemented and tapered.

The configurations used are shown in Figs. 1 and 2.

Results

A. Percentage of Holes:

Table 1 shows the percentage of holes for the different models. The best

performance for any declination is that of Model 006 in which the percentages of holes at declinations of 30° and 0° and -15° are 20.80, 40.76 and 29.14, respectively. A slightly better performance at a declination of 0° is given by model 003 (38.88 percent holes).

Models 008 --010 do not appear to offer any advantage. This is because in removing one arm, a large number of baseline orientations is lost.

The superiority of model 006 lies in the fact that in addition to providing both short and long baselines, it avoids repetitive baselines. With six elements on an arm, there are a maximum of 30 possible different baselines along the arm. Model 006 provides 27 different baseline lengths whereas model 003, the next best model, provides only 12. This is a significant factor to be borne in mind in deciding the antenna locations.

Tracking times of less than ± 6 hours lead to very large number of holes.

B. Measure of Complimentarity

Table 2 shows a measure of complimentarity of the two arrays of each model. It records for each declination, the percentage of unsampled cells as well as the percentage of cells sampled by both ARRAY1 and ARRAY2. In an ideally complimentary pair of arrays, the cells sampled by one will not be sampled by the other and vice versa. The number of twice-sampled cells, expressed as a percentage of the total number of sampled cells, therefore, gives an idea of how much the arrays chosen depart from the ideal.

It is seen that the models which result in the smallest number of holes do not necessarily have highly complimentary pair of antennas. For example, model 006 has the smallest number of holes at 30° and -15° declinations. However, of the cells sampled by this model, 37.78 % are twice-sampled at 30° declination and 43.82 % at -15° declination. It is clear from Table 2 that these figures are rather high.

Chow has suggested that if ARRAY1 has p fraction of holes and ARRAY2 has q fraction of holes, then the following gives a measure of complimentarity of these two arrays when used as a complimentary pair.

(i) If the two arrays are truly complimentary, then the total fraction of holes will be (p + q - 1) or zero, whichever is greater.

(ii) If the two arrays are truly uncomplimentary then the total fraction of holes will be p or q, whichever is smaller.

(iii) If the two arrays are completely uncorrelated in design, then the total fraction of holes will be pq. This is the random case.

Table 3 compares the result obtained with Chow's analysis (all figures are percentages rather than fractions). It is seen that in almost all cases, the results fall somewhere between the "random" and "truly uncomplimentary" cases of Chow. This is a poor reflection on the choice of arrays indeed. No attempt is made here to explain this.

C. Integration Time

Table 4 lists the percentage of cells which have an integration time of ten minutes or more. A comparison with the percentage of holes shows that models which have smaller numbr of holes do not necessarily have a smaller number of cells having long integration times. In other words, reduction of holes does not automatically reduce the integration times of the sampled cells. Optimisation attempted so far is only with respect to the number of holes, and not with respect to the integra-times of sampled cells.

Conclusions

Computations using 18-element complimentary arrays lead to the following conclusions:

(1) 18 elements are insufficient to produce an acceptable u-v plane coverage

even with tracking times of as much as 12 hours. This is not surprising since we have only 153 correlators compared to the 630 available with a 36-element VLA.

(2) The performance varies considerably with declination. The same model will not be optimum for all declinations. But it appears that two models should suffice -- one for large positive or negative declinations and one for declinations close to 0°.

(3) In choosing the two antenna configurations for any model, repetitive baselines should be avoided. The best coverage is obtained when there is the maximum number of distict baseline lengths.

(4) Choice of configurations which are not symmetrical (for example, having elements only along two arms of the Wye) does not appear to be promising but needs further consideration.

(5) A very high degree of complimentarity is difficult to achieve. This will be necessarily so in any system using a limited number of elements and restricting their locations to the arms of the Wye. There is also no satisfactory measure of the degree of complimentarity.

More exhaustive studies are proposed to be done with 24 and 27 elements.

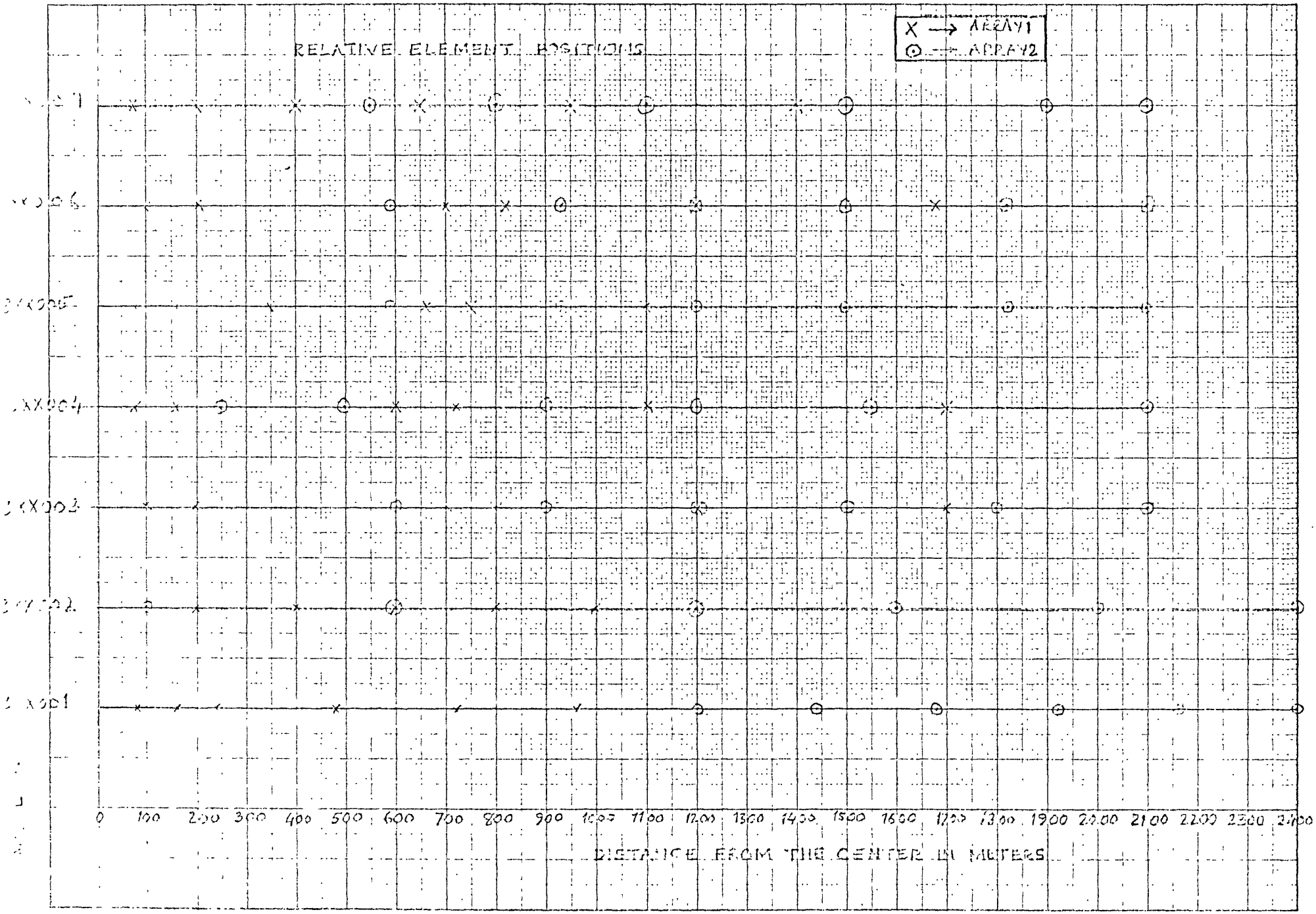

Fig. 1

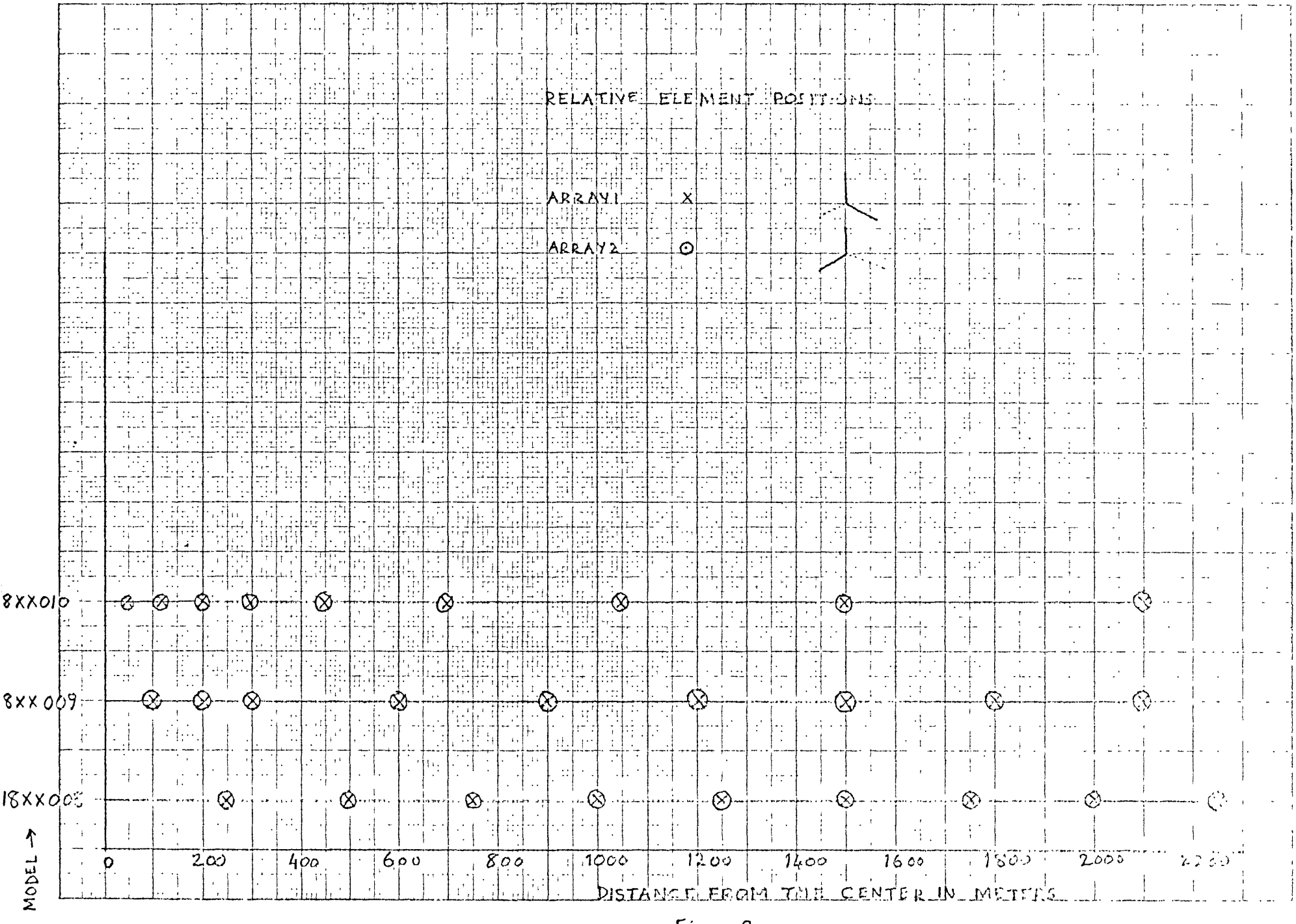

Fig. 2

TABLE 1

PERCENTAGE OF HOLES

MODEL		$\delta = 30°$			$\delta = 0°$			$\delta = -15°$	
	± 5 Hrs	± 4 Hrs	± 2 Hrs	± 6 Hrs	± 4 Hrs	± 2 Hrs	± 5.3 Hrs	± 4 Hrs	± 2 Hrs
18xx001	43.37	64.08	82.43	52.68	64.65	84.91	29.40	48.86	75.13
18xx002	27.66	43.10	68.52	46.95	55.90	76.45	32.08	43.46	67.54
18xx003	23.43	38.78	67.05	38.88	50.41	73.73	29.74	40.91	65.82
18xx004	23.64	35.38	61.93	47.35	53.95	73.07	34.98	43.35	64.62
18xx005	28.27	43.02	69.07	45.54	55.15	76.40	34.21	45.22	68.34
18xx006	20.80	35.73	65.00	40.76	51.21	73.02	29.14	39.74	64.45
18xx007	24.53	39.38	66.63	48.48	56.27	75.07	32.94	42.65	65.83
18xx008	41.66	59.72	77.09	50.21	57.08	73.38	43.11	50.23	67.98
18xx009	35.31	49.79	70.25	51.87	58.30	74.56	45.34	52.58	69.26
18xx010	37.11	49.38	68.91	62.38	65.23	77.31	51.14	58.89	73.78

TABLE 2

MEASURE OF COMPLIMENTARITY

MODEL		$\delta = 30°$			$\delta = 0°$			$\delta = -15°$	
	Unsampled	Twice Sampled		Unsampled	Twice Sampled		Unsampled	Twice Sampled	
		A	B		A	B		A	B
185X001	43.37	6.01	10.61	52.68	6.33	13.38	29.40	10.03	14.21
186X002	27.65	26.95	37.25	46.95	15.74	29.67	32.08	23.52	34.63
186X003	23.43	28.18	36.80	38.88	18.28	29.91	29.74	30.45	43.34
186X004	23.64	31.31	41.00	47.35	16.16	30.69	34.98	28.18	43 34
186X005	28.27	18.31	25.52	45.54	11.04	20.27	34.21	17.83	27.10
186X006	20.80	29.92	37.78	40.76	19.09	32.22	29.14	31.05	43.82
186X007	24.53	25.67	34.03	48.48	13.93	27.04	32.94	23.71	35.36
186X008	41.66	11.64	19.95	50.21	16.50	33.13	43.11	22.58	39.69
186X009	25.31	20.47	31.64	51.87	18.19	37.80	45.34	20.48	37.47
186X010	37.11	22.46	35.71	62.38	14.12	37.53	51.14	17.17	35.14

A : Twice-Sampled cells as a percentage of total cells

B : Twice-Sampled cells as a percentage of sampled cells.

TRACKING TIME : ±6 HOURS (±5.3 HOURS FOR $\delta = -15°$)

TABLE 3

COMPARISON WITH CHOW'S ANALYSIS

MODEL	p	q	pq	p+q−1	ACTUAL HOLES		p	q	pq	p+q−1	ACTUAL HOLES		p	q	pq	p+q−1	A
186X001	74.40	62.91	46.85	37.37	43.37		84.89	61.46	52.17	46.35	52.68		80.96	38.41	31.10	19.37	2
186X002	57.45	43.35	24.72	0.70	27.66		72.43	58.79	42.58	31.22	46.95		68.34	40.22	27.49	8.56	3
186X003	50.59	44.66	22.60	0	23.43		67.12	53.48	35.89	20.60	38.88		58.66	40.63	23.83	0	2
186X004	51.65	40.62	21.01	0	23.64		69.72	61.48	42.86	31.20	47.35		60.58	46.22	28.00	6.80	
186X005	52.20	41.76	28.48	9.96	28 27		80.59	53.90	43.44	34.49	45.54		76.66	39.72	30.45	16.38	
186X006	69.13	41.76	20.52	0	20.80		67.76	53.90	36.52	21.66	40.76		58.37	39.72	23.18	0	
186X007	58.32	40.54	23.64	0	24.53		72.25	59.30	42.84	31.55	48.48		68.20	41.04	27.39	9.24	
186X008	68.79	61.23	42.12	30.02	41.66		68.13	65.58	44.68	33.71	50.21		64.46	56.08	33.15	20.54	
186X009	59.49	55.36	32.93	14.85	35.31		68.49	65.19	44.65	33.68	51.87		64.68	60.18	38.92	24.86	6
186X010	58.83	55.81	32.83	14.64	37.11		75.08	73.19	54.95	48.27	62.38		68.43	65.53	44.84	33.95	

p = Percentage of holes in ARRAY1, q = percentage of holes in ARRAY2, pq = percentage of holes in "Random" case,
p+q−1 = percentage of holes in the "Truly complimentary" case; Tracking time: ±6 Hours

TABLE 4

REDUNDANCY IN COMPLIMENTARY ARRAYS

(FIGURES GIVE PERCENTAGE OF CELLS SAMPLED ≥ 10 MIS)

MODEL		$\delta = 30°$			$\delta = 0°$			$\delta = -15°$	
	ARRAY1	ARRAY2	COMBINED	ARRAY1	ARRAY2	COMBINED	ARRAY1	ARRAY2	COMBINED
186X001	13.12	2.39	15.03	11.44	2.85	13.64	[illegible]	2.52	13.95
186X002	14.13	8.37	24.17	15.36	9.34	24.24	14.57	9.57	23.52
186X003	13.58	6.97	23.45	13.27	6.78	22.24	13.55	8.25	24.56
186X004	14.00	11.33	28.41	13.10	12.55	[illegible]	13.67	12.86	26.16
186X005	14.33	7.79	22.85	12.82	9.10	20.13	12.84	9.25	21.65
186X006	13.81	7.79	25.42	13.88	9.10	24.25	13.70	9.25	25.49
186X007	14.13	8.77	24.79	11.87	11.88	22.24	13.03	10.82	23.86
186X008	8.80	7.29	16.65	6.74	7.00	15.95	9.09	8.69	20.22
186X009	11.04	11.16	23.49	11.26	10.49	21.02	11.59	11.73	23.35
186X010	11.76	11.59	24.44	11.97	10.75	22.61	10.74	10.86	20.67

Tracking time : ± 6 Hours.