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M E M O R A N D U M (VLA Scientific Memorandum No. 1)

To: VLA Design Group

From: E. J. Blum

Subj: Side Lobes of the VLA

The VLA proposal examines the effects and the level of side lobes in several chapters, but some related questions are not studied. I do not intend to exhaust the subject, but I would like to present here a comprehensive view with the idea of VLA dynamic range in mind.

The VLA will be able to detect objects down to a flux threshold of 10^{-4} fu. On the other hand, some strong radio sources exist with flux around 10^{3} fu. The quiet sun radiates 10^{5} fu. from its whole surface, and when disturbed relatively narrow regions may have also flux up to 10^{5} fu. So, we have to think about dynamic ranges of 70 dB during the night and much more during the daytime, with 90 dB possible.

VLA side lobes come from different physical processes, and we may classify them, not too arbitrarily in the following way:

I. Diffraction side lobes

II. Near side lobes due to holes (incomplete coverage of UV plane)

III. Side lobes due to phase or amplitude errors

IV. Far side lobes due to incomplete coverage of UV plane

Categories I and III apply to the array as well as to individual dishes. The last category being closely related to II and III.

I. Diffraction Side Lobes

Diffraction side lobes are produced by an ideal array of perfect dishes. In principle their effect may always be supressed by a proper mathematical or physical processing.

(a) Dish diffraction side lobes may be reduced by tapered illumination, and also by filtering the data or convolving it with a synthetic lobe. The measurement over a field of several beam widths is necessary to reach a reasonable reliability.

(b) Array diffraction side lobes are reduced by tapered illumination. During data processing the weight of spatial harmonics is decreased according to their length.

As diffraction side lobes may be corrected and decrease rapidly with distance from the main beam, we will not consider them in the following analysis.

II. Holes Side Lobes

As dishes are full apertures, they do not produce such side lobes. The array is the only source, and the magnitude of the side lobes has been evaluated in the VLA proposal (Appendix F, p. 6.20 ∝ Sq.). Their rms value is for $\frac{\pi N^2}{8} = 22{,}000$ and n = 3,300 (15% holes_, $\sqrt{\bar{B}^2} = \frac{1}{500}$ or 27 dB. Table 6.2 gives values from computed models ranging from 16 to 25 dB. If the pattern of holes side lobes is precisely known, a restoration might be done.

The restored field will look smoother, but the lack of information due to holes cannot be replaced.

Another way to make the estimate is to consider the vector resulting from the addition of random unit vectors, each coming from one hole, and representing the lack of information introduced by holes. The sum vector is $\sqrt{n}$, compared with main lobe amplitude $\frac{\pi N^2}{8} - n$, so rms value of holes side lobe is $\frac{\sqrt{n}}{\frac{\pi N^2}{8} - n}$ in good agreement, within a factor $\sqrt{2}$, to appendix F result. Finally the holes side lobe field may be taken with a rms value 25 to 27 dB.

III. Side Lobes Due to Phase and Amplitude Errors

(a) Dishes. We follow Ruze in Jasik (Antenna Engineering Handbook, p. 2.37)

$$\bar{P}(\phi) = P_o(\phi) + S(\phi)\,\frac{c^2 \pi \bar{\delta}^2}{\eta_A}\,\exp - \frac{\pi^2 c^2}{\lambda^2} \sin^2 \phi \qquad (1)$$

where

$\bar{P}(\phi)$ is the power pattern for average system

$P_o(\phi)$ is the power pattern for system without error

$s(\phi)$ slowly varying function

c correlation interval

η_A effective area of dish

$\bar{\delta}^2$ mean square error radian squared

Assuming $c/\lambda = 1$, with $\lambda = 0.1$ m, surface errors of 3 mm rms or $\bar{\delta}^2 = 0.04$, effective area 1/2 500 m^2 (Chapt. 11 of VLA Proposal), the rms value of the side lobes is:

53 dB close to main lobe

60 dB at $\approx$ 20°

70 dB at $\approx$ 40° (assuming $S(\phi) = 1$)

When used in an array, dishes errors side lobes are added randomly if the dishes inaccuracies are random. This is probably not quite true, chiefly for deformations which are due to structure common to all dishes. So, we think it will be safe not to add any extra attenuation to values above, and even to decrease slightly these values to take care of possible large scale deformations ($c > \lambda$).

It is interesting to note that the same formula, applied to West Ford dish (PIEEE 52, 589, 1966) gives 50 dB at 10° and 70 dB at 45° from the main lobe, which is very close to measured values. As the low efficiency of this antenna is probably due to surface inaccuracy, we have assumed a rms error of $\lambda/10$, with $c/\lambda = 1$ ($\lambda = 4$ cm).

(1) may be written, close to main lobe,

$$\bar{P}(\phi) = P_o(\phi) - S(\phi)\,\frac{c^2\pi\bar{\delta}^2}{\eta_A}\,\left(1 - \frac{\pi^2 c^2}{\lambda^2}\,\phi^2\right)$$

So, in this zone there will be an increase of side lobes when increasing c. For instance, at an angle of 3 main beams: $\phi = 3 \times 20'$ or $= \frac{1}{100}$ rad, if $c/\lambda = 10$ side lobe level is $\approx$ 33 dB.

On the contrary, far from main lobe increasing, c has the effect to decrease side lobe (but it may be questioned if formula (1) established on statistical basis is still valid for dish diameter $\stackrel{\sim}{\sim}$ 250 λ and $c/\lambda \stackrel{\sim}{\sim} 10$). In fact c is not really a parameter; it will be fixed mainly by the dishes construction technique.

(b) Array. Let us consider the measurement of some sky distribution $T(\theta,\phi)$ by a correlation interferometer, each antenna giving a voltage output $\gamma(\theta-\theta_o,\phi-\phi_o)$ from the direction θ,ϕ; θ_o,ϕ_o being some reference direction.

We have $u_{\theta_o \phi_o} = \iint \gamma_1(\theta-\theta_o,\phi-\phi_o)\ \gamma_2\ (\theta-\theta_o,\phi-\phi_o)\ T\ (\theta,\phi)\ d\theta d\phi$. Where $u_{\theta_o\phi_o}$ is the power output of the system pointed in the direction $\theta_o\phi_o$. With a correlation array, with N pairs

$$U\ (\theta_o\phi_o) = \sum_N u_{\theta_o\phi_o} = \iint T\ (\theta,\phi) \sum_N \gamma_1\gamma_2\ d\theta d\phi$$

Looking to Fourier transforms:

with $w\ (u,v) = TF\ (\Sigma\ \gamma_1\gamma_2)$

$\beta\ (u,v) = TF\ (T\ (\theta,\phi))$.

We have: $B\ (u,v) = w\ (u,v)\ \beta\ (u,v)$ and

$$U\ (\theta_o\phi_o) = \iint B\ (u,v)\ e^{2\pi i\ (u\theta_o + v\phi_o)}\ du\ dv$$

To study side lobes, we choose $T\ (\theta,\phi) = \delta$, or $\beta = 1$.

$$U(\theta_o\phi_o) = \iint w\ (u,v)\ e^{2\pi i\ (u\theta_o + v\phi_o)}\ du\ dv$$

Now the transfer function may be separated into three parts, w_1, w_2, iw_3. w_1 is the specified transfer function, w_2 corresponds to amplitude errors in the function, iw_3 to phase errors.

As we know $U(\theta_o\phi_o)$ is a real quantity, we may write:

$$U(\theta_o\phi_o) = \iint w_1 \; (u,v) \; \cos 2\pi \; (u\theta_o + v\phi_o) \; du \; dv \; +$$

$$+ \iint w_2 \; (u,v) \; \cos 2\pi \; (u\theta_o + v\theta_o) \; du \; dv$$

$$- \iint w_3 \; \sin 2\pi \; (u\theta_o + v\phi_o) \; du \; dv$$

The normalized output for a perfect array ($w_2 = w_3 = 0$), in its main lobe ($\theta_o = \phi_o = 0$), is $U_o = \iint w_1$. And the side lobe pattern is given by the variation of $U(\theta_o\phi_o)$.

The first term of $U(\theta_o\phi_o)$ corresponds to diffraction and holes side lobes we have already estimated. We are interested now in the two following terms, which take respectively into account amplitude and phase error effects on the transfer function. We know that amplitude errors are relatively small -- a few percent on each spatial harmonic -- phase errors are greater. For one peculiar set of u,v, we have a value of w_3 which is εw_1, ε in radians with $|\varepsilon| \approx 0{,}1$, so we may neglect the amplitude error term. We will neglect also the effect of phase error on amplitude (decrease of w_1 by $1 - \varepsilon^2$). So, we want now to evaluate

$$\bar{E} = \iint w_3\ (u,v)\ \sin 2\pi\ (u\theta_o + v\phi_o)\ du\ dv, \text{ compared to } \iint w_1;$$ w_3 is a random function which may be positive or negative for each set of u,v. .For a given value of θ_o ϕ_o, the product by $\sin 2\pi\ (u\theta_o + v\phi_o)$ may be considered as a frequency change, which is not affecting the statistical properties of w_3. (This is true only if θ_o,ϕ_o are not too small, i.e., not too close to main lobe.) Then:

$$\bar{E} \approx \frac{2}{\pi} \iint w_3\ (u,v)\ du\ dv.$$

The factor $\frac{2}{\pi}$ comes from the mean value of sin.

Let us consider first phase errors not varying with time: imperfect phasing and positioning; these errors are permanently applied to the 630 fundamental harmonics of the VLA. During synthesis, harmonics move in UV plane, but always with the same error. Therefore we have only 630 errors. Moreover these errors are not independent. They come from linear combination of 36 independent errors given by the 36 antennas.*

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* It may be noted that the One mile telescope has about the same number of independant pairs (∿ 60 different positions).

If all harmonics are equal (no taper), and if errors have the same mean value $\sqrt{\overline{\varepsilon^2}}$ on all UV plane, we have

$$\sum\sum w_3 = \frac{2\varepsilon}{\pi} \sqrt{n} \text{ and } \iint w_1 = n, \text{ with } n = 36$$

or a mean side lobe level of $\frac{2\varepsilon}{\pi\sqrt{n}}$.

In fact, there will be some taper, and errors will increase certainly with spatial frequency. The preceding level has to be increased, chiefly because of taper; the variation of errors with spatial frequency may be taken into account by a proper choice of ε. For side lobes coming from time varying phase errors ε_1 (due to propagation through atmosphere), if we suppose that the time scale of variation is of the same order of magnitude as the mean sampling time of a cell (12 mn), a similar approach gives a side lobe level of $\frac{2\varepsilon_1}{\pi\sqrt{N}}$ with N = 22,000, negligible compared with preceding values.

If we come back now to the general case, i.e., close to the main lobe: $\overline{E} = \iint w_3 \sin 2\pi\,(u\theta_o + v\phi_o)\, du\, dv$, one can see that for small values of $\theta_o\phi_o$ the side lobe level decreases.

Finally in taking $\frac{2\varepsilon}{\pi\sqrt{36}}$ as an estimate of side lobe level, we have:

20 dB assuming $\varepsilon = 0.1$ (6° rms)

23 dB " $\varepsilon = 0.05$ (3° rms)

Considering the effect of taper, it seems that 18 to 20 dB rms is a conservative value. However, if several calibrations of the system made on precisely known fields, are possible during one synthesized field observation, then preceding side lobes value may be slightly increased.

IV. Far Side Lobes

Far side lobes are due to array discontinuous coverage; the preceding analysis of holes and errors side lobes have been made under the assumption of narrow band. This assumption is nolonger valid far from main beam, as the pattern is smeared by the bandwidth. To estimate side lobes far from main beam, we will again consider the array as formed by small antennas; dish influence being treated separately. Then to remove the smearing effect we will split the bandwidth in n channels of frequency, each channel corresponding to an instrument without smearing. Each instrument gives in any direction of space, far from main lobe, a pattern obtained by random sum of 22,000 cells, though of rms amplitude $(22{,}000)^{-1/2}$ compared with main lobe. If there are some grating effects, i.e., some regularity in the summation, amplitude may be increased. Now we have n instruments at n different frequencies, so we have to add quadratically their contribution again and the total rms level of far side lobes is $(22{,}000 \times n)^{-1/2}$. n varies with distance of main lobe, between 1 and 3000 for the VLA configuration 20' x 10", 100 MHz bandwidth.

(Length of one arm in wavelengths: 20.000 - coherence condition $\frac{b}{F} \sim \frac{1}{4} \frac{1}{20.000}$ $b = \frac{2700}{80.000}$ MHz, $\frac{B}{b} = n_{max} = \frac{80.000}{27.000} \times 100 = 3000$). At 10° of main lobe the side lobe level is then 33 dB; and 39 dB at 90°.

V. Total Side Lobe Pattern

For small n the preceding estimate has no meaning, for the random assumption is nolonger valid, and only the sum of holes S.L. and errors S.L.

are to be considered in the main lobe region. For large n, on the contrary this estimate takes into account holes and errors S.L. which are no longer separable. For moderate n values, transition between dish main beam field, and far field, we may also consider several systems with bandwidth narrow enough to have no smearing effect, each of these systems with side lobes due to holes, and phase errors, and add them randomly. With this crude estimate, we can draw two figures to summarize the whole preceding analysis. (In the case of array field equal to dish main lobe.) Fig. 1 indicates the side lobe, 2° and more from main lobe, Fig. 2 close to main lobe. As all values are rms, the total side lobe contents have peaks which may sometime reach -5 dB (shaded zone).

It is clear from the figures that the dishes side lobes give the main protection against strong source radiation, when holes and phase errors cause rather high S.L. level. In the far zone any reduction of bandwidth gives an increase in S.L. up to the holes and phase errors side lobe level value, for very narrow bandwidth.

During the day, the sun, at 20° or less from the main lobe, may give unwanted signals. During the night there will be probably no difficulty anywhere. However, as the dish protection is the most efficient, it seems important to specify the dish far side lobe level. For instance, if in some regions side lobes of dishes are only 45 dB, daylight observations may have trouble in any solar position, and by night measurements at less than 5 or 10° of a strong source may be disturbed. The same situation may occur within 2° of main lobe if dish diffraction side lobes are not corrected (Fig. 2, assuming 20 dB and 26 dB and so on for diffraction side lobes: total side lobe, dashed curve).

It may be questioned if the criterion we have taken: rms side lobe level, measured by its average power, is a good one in a correlation system which gives positive and negative side lobes, and perhaps all preceding values, at least for the array, are slightly pessimistic.

VI. Conclusion

In this paper we have studied various aspects of side lobe levels for the VLA as it is described in the VLA proposal. We have derived some characteristic values of side lobe levels from various approaches. As some of these approaches are rather crude we would not guarantee a high accuracy for these values. Nevertheless, several points seem quite clear:

1) In the field of dish beam width (20' x 20') observations with dynamic range greater than 16-20 dB have to be made with great care.

2) If a strong source is within 2° of the main lobe, dish diffraction side lobes are to be corrected.

3) If the sun is within 20° of the main lobe there may be trouble.

4) A reduction of the bandwidth of the array would cause a noticeable increase of the side lobe level.

5) Dish errors side lobes, due to structure and common to all dishes are to be more than 50 dB.

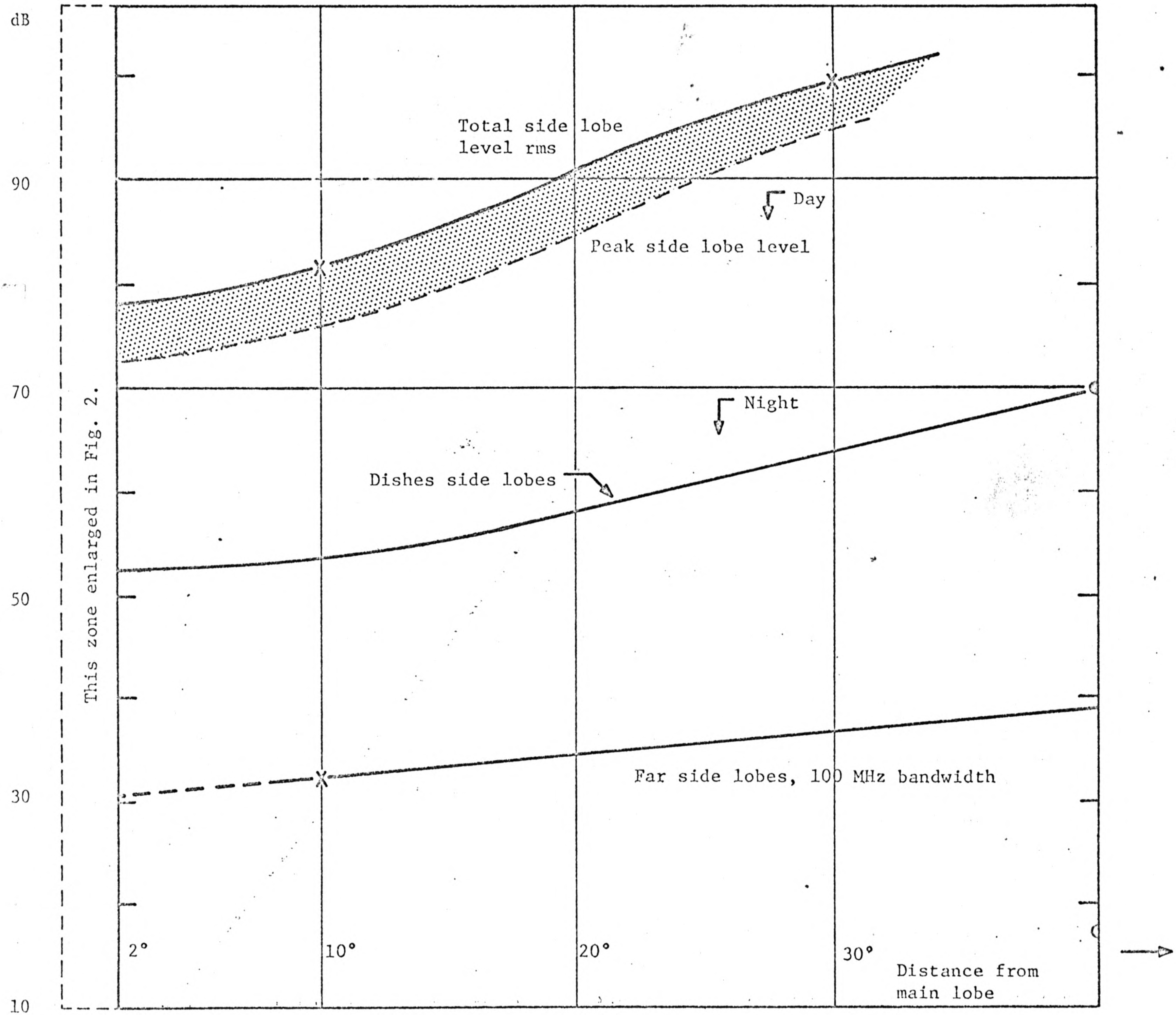
dB
90
70
50
30
10
This zone enlarged in Fig. 2.
Total side lobe
level rms
Day
Peak side lobe level
Night
Dishes side lobes
Far side lobes, 100 MHz bandwidth
2°
10°
20°
30°
Distance from
main lobe

Fig. 1

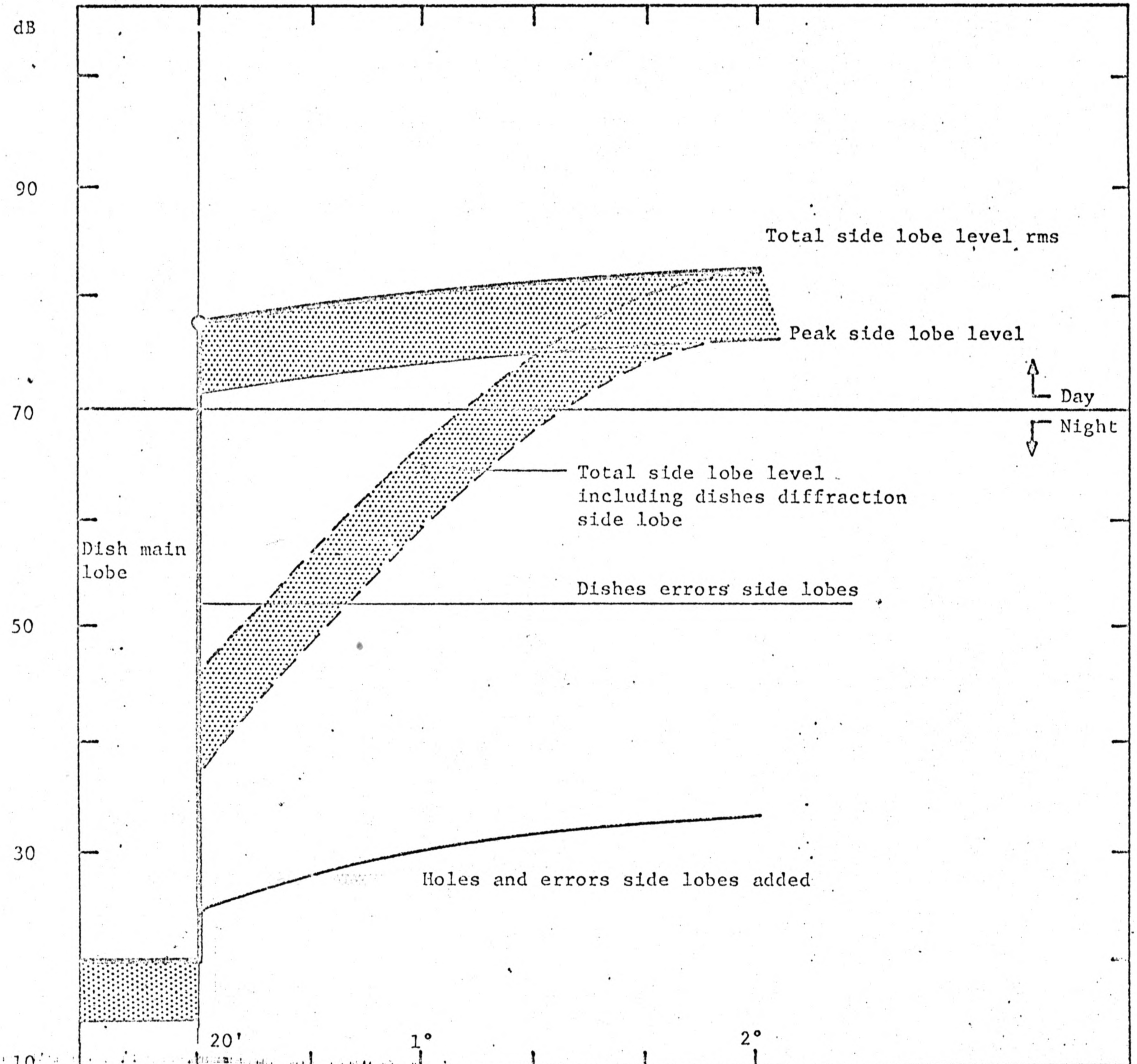
dB
90
70
50
30
10
20'
1°
2°
Total side lobe level rms
Peak side lobe level
Day
Night
Total side lobe level
including dishes diffraction
side lobe
Dish main
lobe
Dishes errors side lobes
Holes and errors side lobes added

Fig. 2