NATIONAL RADIO ASTRONOMY OBSERVATORY

VLA SCIENTIFIC MEMORANDUM #5

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COMPLEMENTARY ARRAY CONFIGURATIONS FOR THE VLA

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A. INTRODUCTION

One of the most important parameters in the design of the VLA is the array configuration to be used during observations. The configuration determines the locations of the observing stations. The observing stations in turn determine the locations of the repeater stations and other electronics layout. A detailed study of the array configuration has been made in the VLA Proposal. This study recommends that the VLA consist of 36 antennas located on the arms of an equiangular wye having one arm rotated 5° from the north-south direction. In the optimum configuration, the elements lie symmetrically on the three arms of the wye. The elements are uniformly spaced except near the center where the spacing of the first three elements is reduced to one-third of the spacing elsewhere. This configuration, designated as the supplemented wye, leads to the best coverages in the u-v plane for all declinations. To use the VLA with the supplemented wye configuration for resolutions of 1", 3", 9", and 27", one needs 33 observing stations for each arm, thus requiring 99 observing stations in all.

In this report we study the use of complementary array configurations for the VLA. When the same source is observed with two different element configurations and the two transfer functions are superposed, the result is equivalent to a single observation with a larger number of interferometer pairs. Consequently there is a reduction in the number of holes in the transfer function and a neater beam (i.e. one having fewer and smaller sidelobes) is obtained. Likewise, a given sidelobe level can be achieved with fewer elements by the complementary array technique. In either case, of course, the cost is the additional observing time. This report considers these two aspects of the complementary arrays, namely,

(i) the minimum number of elements required to achieve an acceptable beam,

(ii) the best beam that can be achieved with 36 elements when used in two complementary array configurations.

In either case optimum element locations have been determined on the arms of an equiangular symmetrical wye with one arm rotated 5° from the north-south.

B. COMPUTER PROGRAM

Two computer programs have been used. One computes the transfer functions for the two configurations individually as well as the superposed transfer function and counts the number of unsampled cells in the transfer function. Each model is designated by a seven digit number which includes information about the number of elements, declination and tracking time. The ratio of field of view to beamwidth is taken as 124 and sampling is done at an interval such that grating lobes are separated from the main beam by 20'. The other program accepts the transfer function as the input and computes the Fourier transform to produce the beam. In computing the Fourier transform, equal weighting is given to all the sampled cells and an overall gaussian taper, decreasing to 15 dbs at the edges, is superposed on the whole transfer function. The program computes the beamwidth, the gain, and the mean and maximum sidelobes in the field of view. For this purpose the field of view is divided into five zones which are square annuli centered at the beam center. A plot of the beam can be obtained on the Calcomp Plotter.

C. MINIMUM NUMBER OF ELEMENTS

Two approaches have been used to find the array configurations which have the highest degree of complementarity as indicated by the number of holes in the superposed transfer function. In one, the distribution of holes in the transfer function of a given element arrangement is studied. Another arrangement is then chosen which will, hopefully, fill the holes of this transfer function. In the other approach, pairs of configurations are tried to see which gives the lowest number of holes in the superposed transfer function.

To use the first approach, it is necessary to have a good idea of the location of the track of a given baseline in the u-v plane. For this purpose a computer program was written which gave the transfer function of a given baseline (that is, of a given length and orientation with respect to east-west) for specified declination and tracking time. Plots were obtained for declinations of 30°, 0°, and -15° and tracking time of eight hours for a set of baseline lengths and orientations. These plots were used to find out the element locations that will fill a given region of holes.

This approach proved ineffective, due to the limited number of elements. Although one knows exactly what element locations will fill the holes of a given transfer function, these locations are far too many to be filled by the number of elements available. Most of the optimization has, therefore, been done using the second approach.

(1) Arrays with 18 elements:

A number of models were tried using 18 elements. One gets 153 correlators with 18 elements, so that in the two configurations there are 306 correlators available as compared to the 630 available with 36 elements. The performance is, naturally, not as good as that of the 36 element VLA. If the source is tracked for 12 hours with each configuration, the rms sidelobes do not exceed -20 dbs throughout the field of view. However, at declination 0° the maximum sidelobe near the edge of the field of view is -11.2 db, and the maximum sidelobe close to the beam is -12.5 db. The performance of the best model with 18 elements is summarized in Table V. The transfer functions and beams for declinations of 30°, 0°, and -15° for this model are shown in Figures 1 - 4. Only the central portions of the beam in right ascension and declination are shown. The successive cross sections have been staggered for the sake of clarity. The observing stations for this model are shown in Table I. A total of 44 observing stations per arm will be needed.

(2) Arrays with 24 elements:

With 24 elements one has 552 correlators in the two configurations. A very large number of models was tried using 24 elements. These included irregular arrays, as well as arrays with uniform spacing, tapered spacing, etc. The performance of the array is strongly influenced by the declination. Table II gives the percentage of holes for the various models tried. Model 24--023 performs best at a declination of 0° and is taken as the overall best model. In this model the two configurations consist of uniform wyes with armlengths of 2000 and 2100 meters for the 10" configuration. The observing stations needed for the 1", 3", 9", and 27" modes are given in Table III. A total of 53 stations per arm are needed. The performance of this model is summarized in Table V. The transfer functions and beams for declinations of 30°, 0°, and -15° are shown in Figures 5 - 8. It is seen that with horizon-to-horizon tracking, the maximum sidelobe anywhere in the field of view at any declination is 14 dbs below the main lobe. The RMS sidelobes everywhere are more than 20 db below the main lobe. This is an acceptable performance in view of the restrictions placed on the performance of the VLA by amplitude and phase errors and atmospheric effects.

D. BEST PERFORMANCE WITH 36 ELEMENTS

The 36 element VLA is designed to be used in the single observation mode. Its performance in this mode is acceptable at all declinations. For some specific applications, however, it may be desirable to observe with a beam having greatly reduced sidelobes. In such cases complementary array configurations can be used to synthesize a beam having negligible sidelobes.

Analysis of several combinations of configurations shows that it is possible to suppress sidelobes to values less than -22 db below the main lobe level with (RMS sidelobe level less than -28 db throughout the field of view). This performance is achieved by using horizon-to-horizon tracking with each configuration. The performance of a proposed 36 element complementary array model is summarized in Table V. The transfer functions and beams are shown in Figures 9 - 12. The two configurations of this model consist of a supplemented wye with arm length of 2100 m and a uniform wye with arm length of 2000 m in the 10" mode. The observing stations required for this model are shown in Table IV. Sixty-eight observing stations are needed on each arm for this model.

E. DISCUSSION

Several important features of complementary arrays, and of tracking arrays in general, have been brought out by this study.

The degree of complementarity of two configurations can only be defined qualitatively in the sense that the smaller the number of holes in the superposed transfer function, the higher is the degree of complementarity. A quantitative measure is not available. Chow has attempted to give a quantitative measure in a statistical sense in the following way: Suppose that under identical conditions of tracking, a fraction p of the total number of cells in the transfer function remains unsampled with one configuration and a fraction q with the other. Then if the two configurations are ideally complementary, the fraction H, of unsampled cells in the superposed transfer function will be $(p + q - 1)$ or zero, whichever is greater. On the other hand, if the two configurations are totally non-complementary, H will be equal to the smaller of p and q. The actual value of H will lie somewhere between these two extremes. A third case has been suggested in which the two arrays are completely uncorrelated in design. In this case, by statistical reasoning, H should be equal to pq. It has been suggested that in a well-designed complementary array system, H should lie somewhere between pq and $(p + q - 1)$; the closer H is to $(p + q - 1)$, the better the design. If H is equal to or greater than pq, the design is worse than random. Actually this is a misleading criterion for the degree of complementarity. The number and locations of holes in a transfer function are governed by four parameters: (i) the number of elements, (ii) the configuration used, (iii) the total tracking time and (iv) the declination of the source. In designing complementary arrays, parameters (i), (iii) and (iv) are constrained to be the same for the two configurations, and even in parameter (ii) we are constrained to place the elements on the arms of the same wye in both configurations. These constraints severely limit our ability to generate a transfer function that will fill the holes of another transfer function. It is, therefore, difficult to achieve very low values for H and a comparison of H with pq is not valid. This is borne out by the results quoted in Table VI, in which the values of H and pq are listed for several models.

Another feature brought out by this study is that the distribution of holes in a transfer function strongly depends upon the declination and tracking time. Since the sidelobe levels depend both on the number of holes and their distribution, it is better to consider the beam characteristics in optimization work, rather than consider only the percentage of holes.

It is also seen from this study that the best pairs of configurations depend upon the number of elements. For example, at declination 0° and with a tracking time of 8 hours, consider two pairs of configurations: (a) uniform wye, arm length 2000 m and supplemented wye, arm length 2100 m, (b) uniform wye, arm length 2000 m, and uniform wye, arm length 2100 m. The percentage of holes for 24 and 27 elements are as follows:

	(a)	(b)
24 elements	29.84	27.14
27 elements	20.74	22.15

A similar remark can also be made with regard to declination and tracking time. For a given number of elements and tracking time, the model that is best for $\delta = 30°$ is not necessarily best for $\delta = 0°$. Similarly, for a given number of elements and declination, the model that is best for 12 hours of observation time is not necessarily best for 8 hours of observation time.

The above indicates that if the VLA is to be built with a smaller number of antennas than the proposed 36, with a provision for adding more antennas at a later date, and/or the best performance is to be achieved under all conditions of observation, it is necessary to be able to set up any array configuration within the framework of the wye. This entails the capability of observing from the tracks. However, the cost of such a capability is not commensurate with the advantages gained. Therefore, only the overall best models for 18, 24, and 36 elements have been included in this report.

F. CONCLUSIONS

The study in this report leads to the following conclusions:

(i) The performance goals of the VLA can be achieved with 24 antennas by taking two observations with complementary configurations, tracking the source from horizon-to-horizon with each configuration.

(ii) A beam having maximum sidelobes less than -22 db can be obtained with a 36-element VLA when used in the complementary arrays mode.

(iii) The use of complementary arrays does not lead to a substantial reduction of observing time with each configuration. Complementary arrays are, therefore, very costly in terms of time.

TABLE I

OBSERVING STATIONS FOR COMPLEMENTARY ARRAYS WITH 18 ELEMENTS

(Distances are in meters from the center)

ARRAY 1			
Resolution in Seconds			
1"	3"	9"	27"
1000	333	111	37
2000	667	222	74
7000	2333	778	259
8200	2733	911	304
12000	4000	1333	444
16800	5600	1867	622

ARRAY 2			
Resolution in Seconds			
1"	3"	9"	27"
5900	1967	656	218
9300	3100	1033	344
12000	4000	1333	444
15000	5000	1667	556
18200	6067	2022	674
21000	7000	2333	778

Total number of observing stations per arm = 44

TABLE II

COMPLEMENTARY ARRAYS WITH 24 ELEMENTS

PERCENTAGE OF HOLES FOR DIFFERENT MODELS

MODEL	Declination 30°			Declination 0°			Declination -15°		
	$\pm 6^h$	$\pm 4^h$	$\pm 2^h$	$\pm 6^h$	$\pm 4^h$	$\pm 2^h$	$\pm 6^h$	$\pm 4^h$	$\pm 2^h$
24--011	6.59	16.92	46.67	24.08	32.51	59.40	12.86	20.58	47.72
24--012	6.66	15.33	44.98	24.84	31.32	57.81	15.98	22.35	46.97
24--013	7.18	17.75	47.08	23.59	30.49	56.65	12.41	18.98	45.99
24--014	6.21	15.36	44.72	23.96	30.91	56.30	13.63	19.60	44.10
24--015	6.38	15.75	44.47	22.01	28.57	55.37	16.37	21.71	43.43
24--016	7.88	17.68	45.26	23.68	29.84	55.95	17.96	23.56	46.80
24--017	10.36	16.99	41.51	31.87	37.09	58.24	23.33	28.90	48.08
24--018	12.34	19.79	44.96	32.36	37.95	59.79	25.38	31.38	51.58
24--019	8.78	17.45	44.69	25.22	30.83	56.57	20.85	26.91	49.16
24--020	7.38	13.50	38.96	29.20	34.64	56.17	19.09	24.21	44.58
24--021	9.34	16.76	42.61	28.69	34.79	57.43	21.25	27.13	48.75
24--022	7.31	17.45	45.46	20.77	27.91	55.74	16.84	23.14	46.88
24--023	6.94	17.83	45.74	19.63	27.14	54.99	14.81	22.05	48.00
24--024	5.90	17.10	48.05	21.69	30.00	57.89	9.39	16.58	44.13

TABLE III

OBSERVING STATIONS FOR COMPLEMENTARY ARRAYS WITH 24 ELEMENTS

(Distances are in meters from the center)

ARRAY 1			
Resolution in Seconds			
1"	3"	9"	27"
2625	875	292	97
5250	1750	583	194
7875	2625	875	292
10500	3500	1167	389
13125	4375	1458	486
15810	5270	1757	586
18375	6125	2042	681
21000	7000	2333	778

ARRAY 2			
Resolution in Seconds			
1"	3"	9"	27"
2500	833	278	93
5000	1667	556	185
7500	2500	833	278
10000	3333	1111	370
12500	4167	1389	463
15000	5000	1667	556
17500	5833	1944	648
20000	6667	2222	741

Total number of observing stations per arm = 53

TABLE IV

OBSERVING STATIONS FOR COMPLEMENTARY ARRAYS WITH 36 ELEMENTS

(Distances are in meters from the center)

ARRAY 1			
Resolution in Seconds			
1"	3"	9"	27"
700	233	78	26
1400	467	156	52
2100	700	233	78
4200	1400	467	156
6300	2100	700	233
8400	2800	933	311
10500	3500	1167	389
12600	4200	1400	467
14700	4900	1633	544
16800	5600	1867	622
18900	6300	2100	700
21000	7000	2333	778

ARRAY 2			
Resolution in Seconds			
1"	3"	9"	27"
1667	556	185	62
3333	1111	370	123
5000	1667	556	185
6667	2222	741	247
8333	2778	926	309
10000	3333	1111	370
11667	3889	1296	432
13333	4444	1481	494
15000	5000	1667	556
16667	5556	1852	617
18333	6111	2037	679
20000	6667	2222	741

Total number of observing stations per arm = 68

TABLE V

VLA PERFORMANCE USING COMPLEMENTARY ARRAY CONFIGURATIONS

Model	N	Decli-nation	Tracking Range	Holes (%)	Half-Power Beamwidth	Relative Gain	Maximum Sidelobe Level *					RMS Sidelobe Level				
							Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
1861006	18	30°	±6.0	20.80	9.8 x 10.8	294	-15.8	-17.3	-17.4	-18.7	-19.9	-20.1	-23.8	-24.9	-26.3	-28.5
1860006	18	0°	±6.0	40.76	9.2 x 13.8	273	-12.5	-17.3	-12.8	-14.3	-11.2	-20.4	-24.9	-26.5	-29.5	-31.3
1862006	18	-15°	±5.3	29.14	10.9 x 11.5	285	-17.7	-18.3	-16.9	-19.7	-20.4	-23.7	-26.3	-27.7	-28.9	-30.4
2461023	24	30°	±6.0	6.94	9.5 x 9.8	358	-14.3	-19.6	-14.0	-19.0	-20.5	-21.3	-25.4	-25.0	-28.2	-29.6
2460023	24	0°	±6.0	19.63	9.4 x 11.6	376	-15.0	-16.9	-17.6	-17.5	-13.9	-23.7	-27.2	-30.0	-32.7	-34.3
2462023	24	-15°	±5.3	14.81	10.7 x 11.1	386	-21.4	-24.7	-19.1	-22.5	-21.8	-28.1	-31.0	-29.9	-31.9	-32.9
3661026	36	30°	±6.0	0.97	10.2 x 10.3	650	-23.5	-24.4	-25.9	-26.6	-25.8	-28.4	-29.4	-31.5	-33.9	-34.7
3660026	36	0°	±6.0	8.63	9.8 x 10.7	672	-22.2	-24.9	-24.4	-23.4	-23.5	-30.1	-33.4	-36.0	-38.1	-40.2
3662026	36	-15°	±5.3	9.43	10.3 x 12.2	669	-23.4	-26.2	-28.8	-28.2	-26.3	-29.4	-34.5	-36.8	-37.5	-38.4
Performance of 36-element supplemented wye with single observation:																
3621111	36	30°	±6.0	6.08	10.0 x 10.4	419	-20.6	-21.6	-21.7	-22.1	-21.8	-26.2	-27.7	-28.5	-30.6	-31.3
3621222	36	0°	±6.0	18.32	9.6 x 11.4	420	-18.0	-20.1	-19.2	-14.5	-17.1	-26.0	-29.0	-31.1	-33.0	-35.4
3621333	36	-15°	±5.3	16.56	10.7 x 11.3	427	-20.3	-24.9	-21.0	-21.8	-21.8	-28.0	-31.5	-31.1	-32.4	-33.5

* The field of view has been divided into five zones. The zones are square annuli lying at the following distances from the center of the beam: Zone 1, 21".8 - 41"; Zone 2, 41" - 80"; Zone 3, 80" - 155"; Zone 4, 155" - 310"; Zone 5, 310" - 600"

TABLE VI

COMPARISON OF THE ACTUAL PERFORMANCE OF COMPLEMENTARY ARRAYS WITH CHOW'S STATISTICAL CRITERION

Model	Declination	Tracking Range (Hrs)	Percentage of Holes				
			Array 1 p	Array 2 q	Random pq	Ideal p+q-1	Actual H
1861006	30°	±6.0	49.13	41.76	20.64	0.0	20.80
1860006	0°	±6.0	67.76	53.90	36.52	21.66	40.76
1862006	-15°	±5.3	58.37	39.72	23.18	0.0	29.14
2461023	30°	±6.0	22.62	22.87	5.17	0.0	6.94
2460023	0°	±6.0	34.72	36.15	12.55	0.0	19.63
2462023	-15°	±5.3	27.77	30.25	8.40	0.0	14.81
3661026	30°	±6.0	5.20	6.08	0.32	0.0	0.97
3660026	0°	±6.0	14.55	18.32	2.66	0.0	8.63
3662026	-15°	±5.3	16.31	16.56	2.70	0.0	9.43

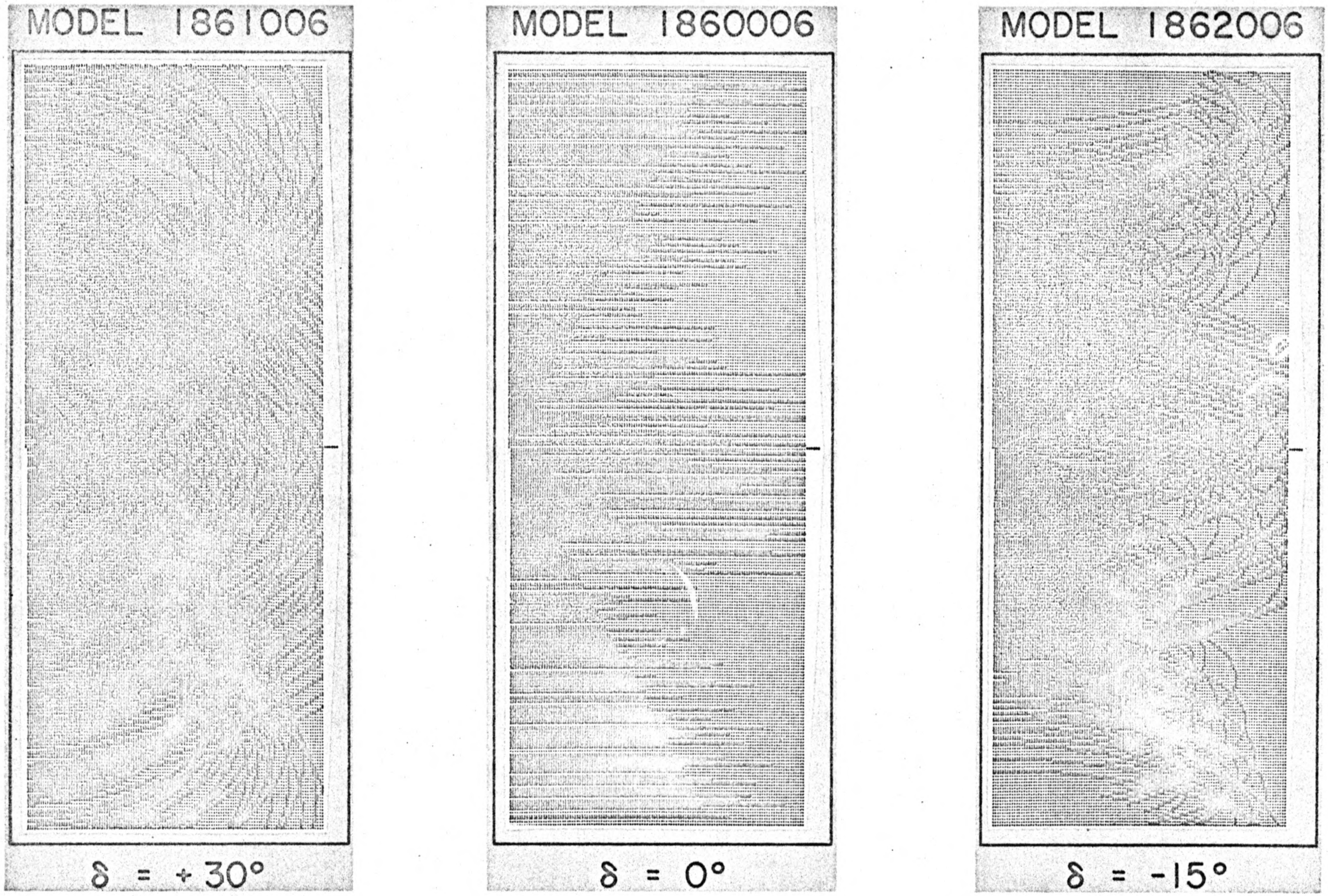


Figure 1. Transfer Functions for Complementary Arrays with 18 Elements

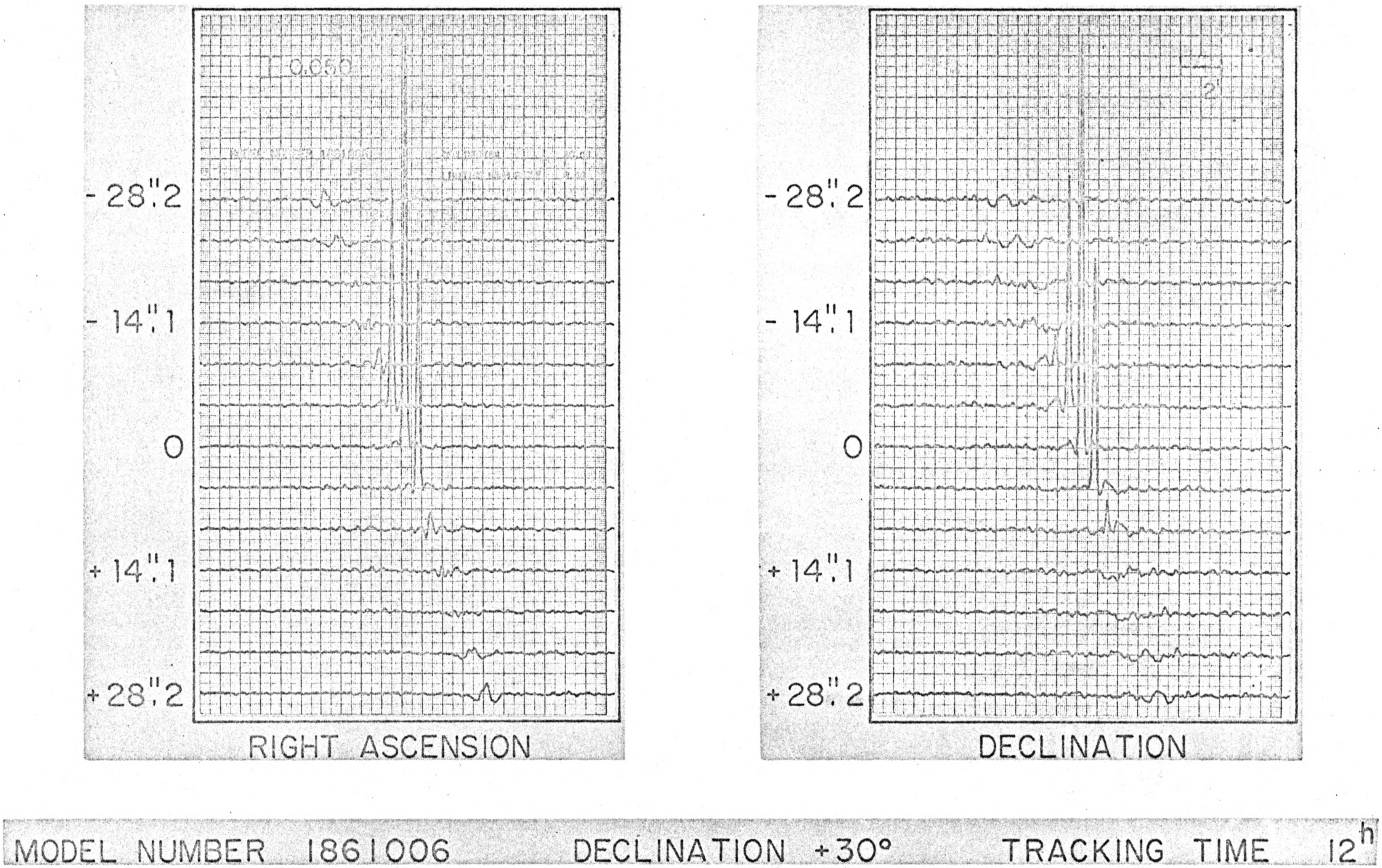


Figure 2. Beam Pattern Corresponding to the Transfer Function of Figure 1, $\delta = 30°$

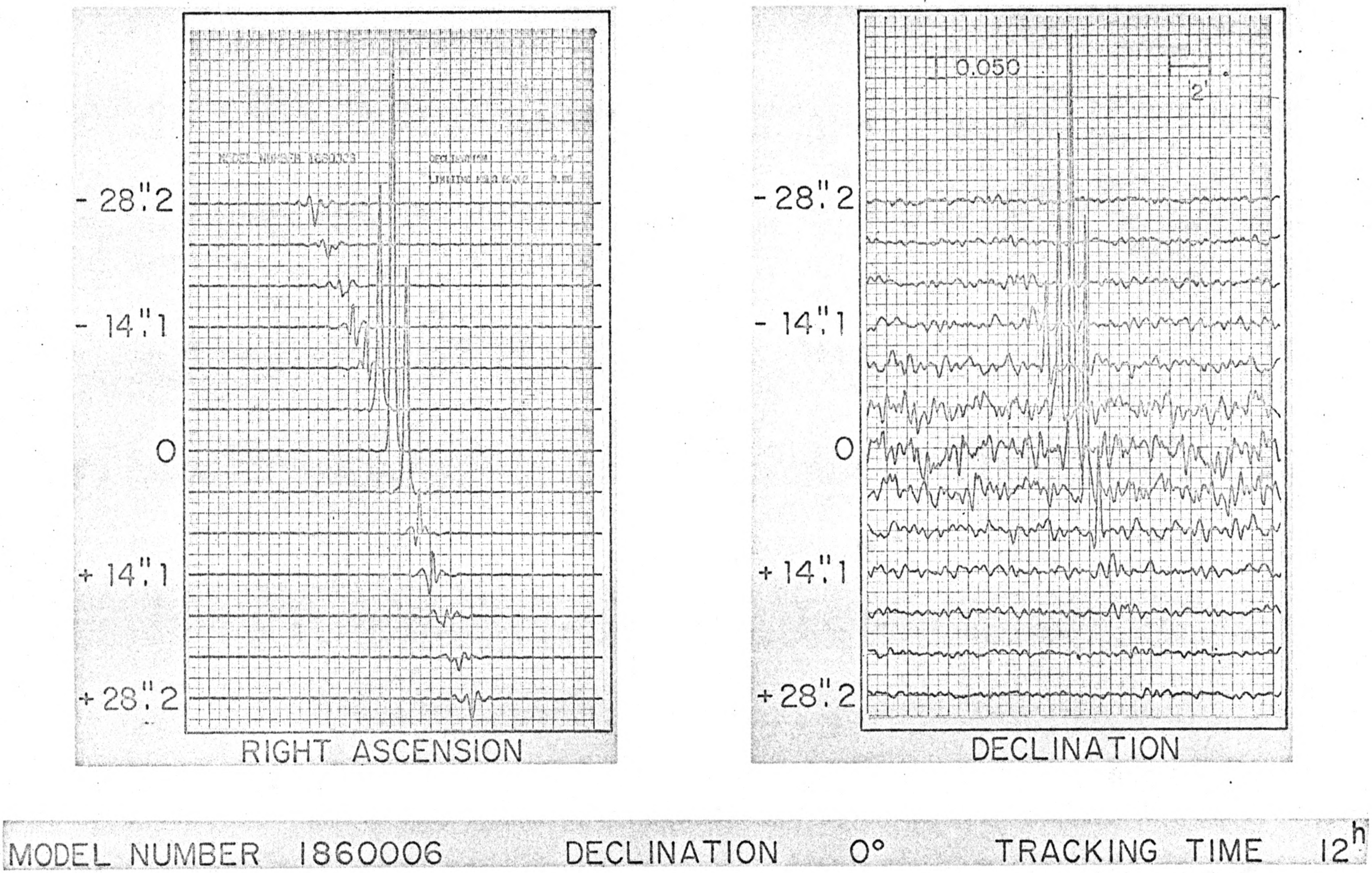


Figure 3. Beam Pattern Corresponding to the Transfer Function of Figure 1, $\delta = 0°$

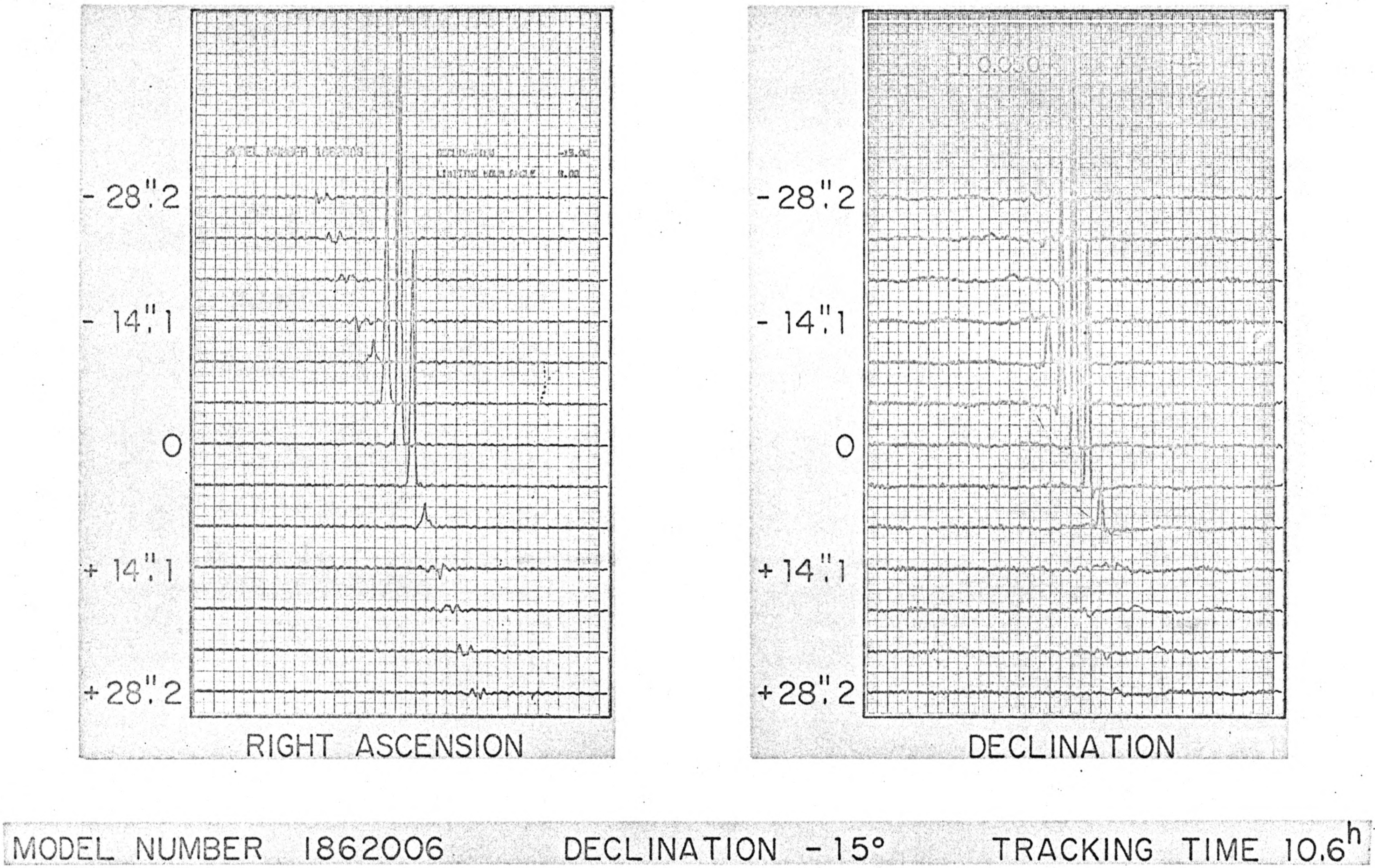


Figure 4. Beam Pattern Corresponding to the Transfer Function of Figure 1, δ = -15°

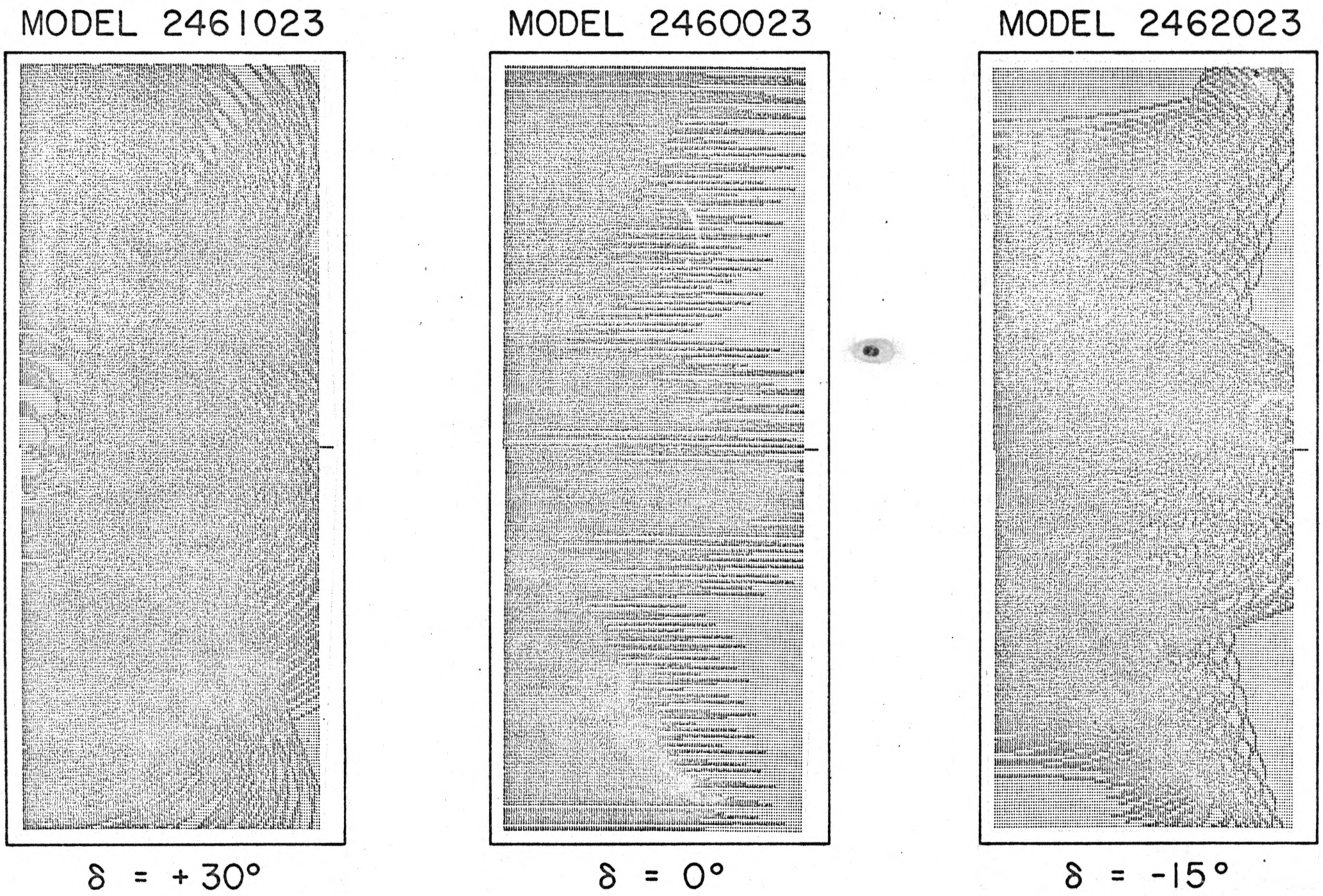


Figure 5. Transfer Functions for Complementary Arrays with 24 elements

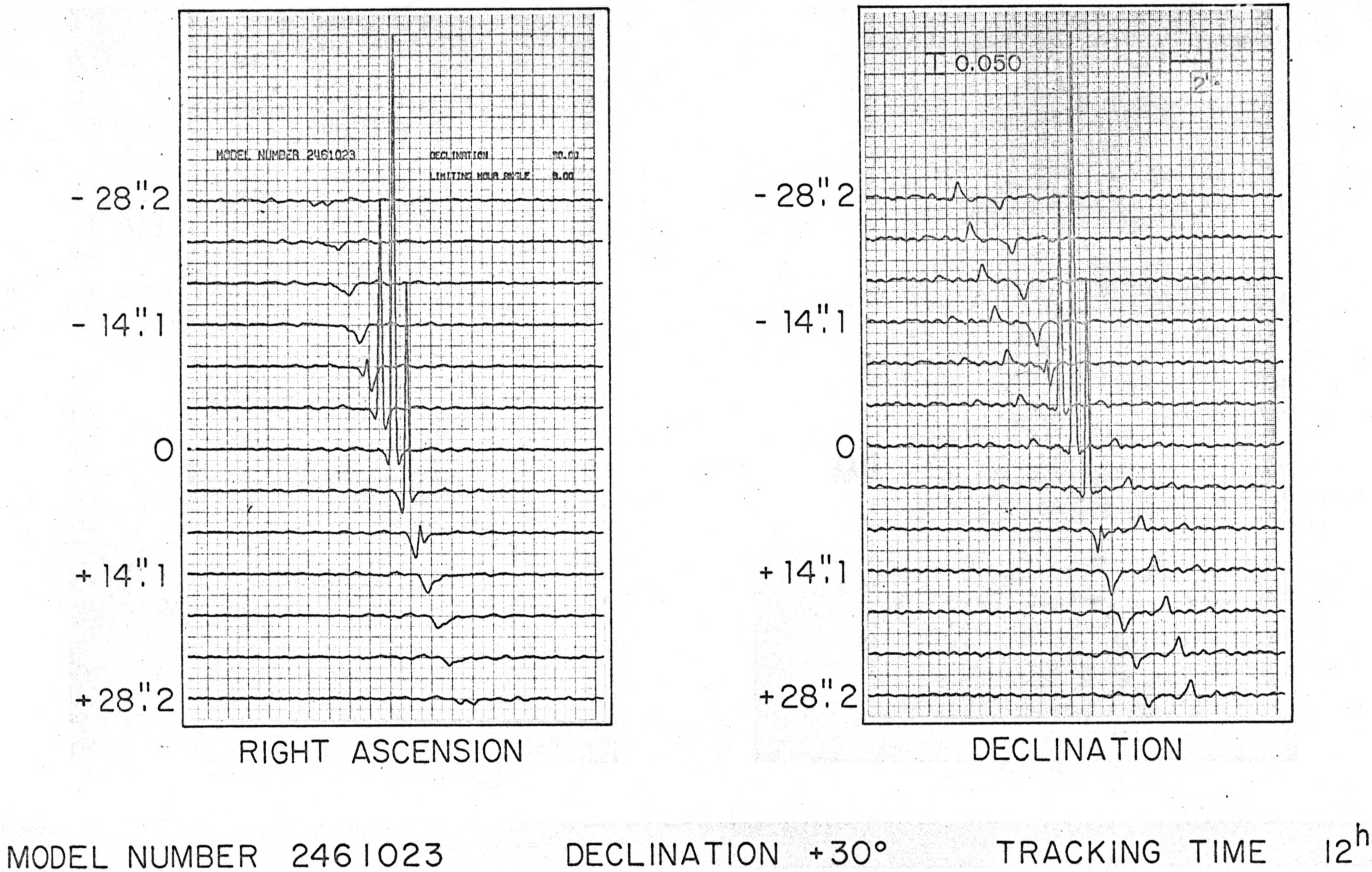


Figure 6. Beam Pattern Corresponding to the Transfer Function of Figure 5, $\delta = 30°$

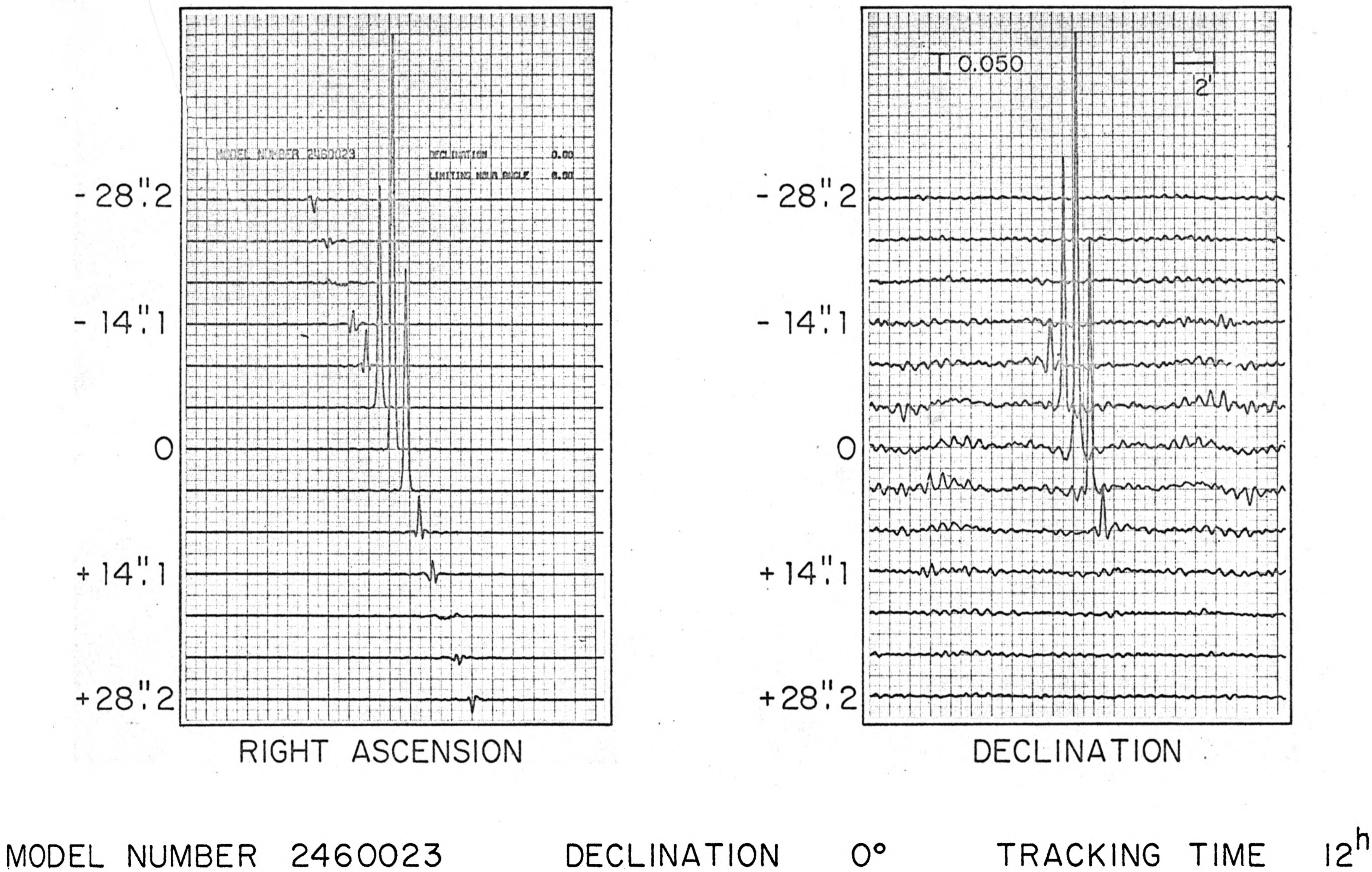


Figure 7. Beam Pattern Corresponding to the Transfer Function of Figure 5, $\delta = 0°$

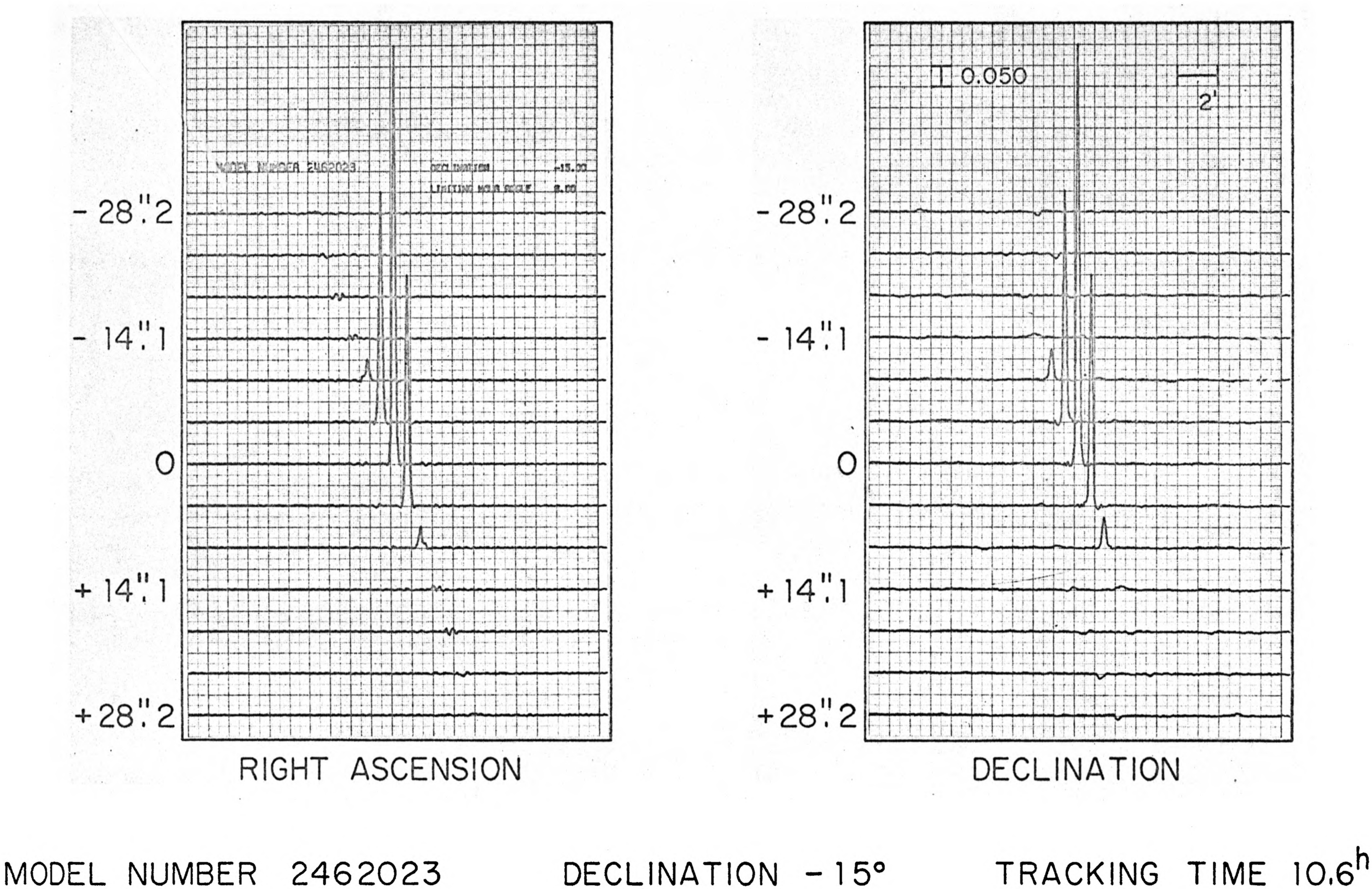


Figure 8. Beam Pattern Corresponding to the Transfer Function of Figure 5, $\delta = -15°$

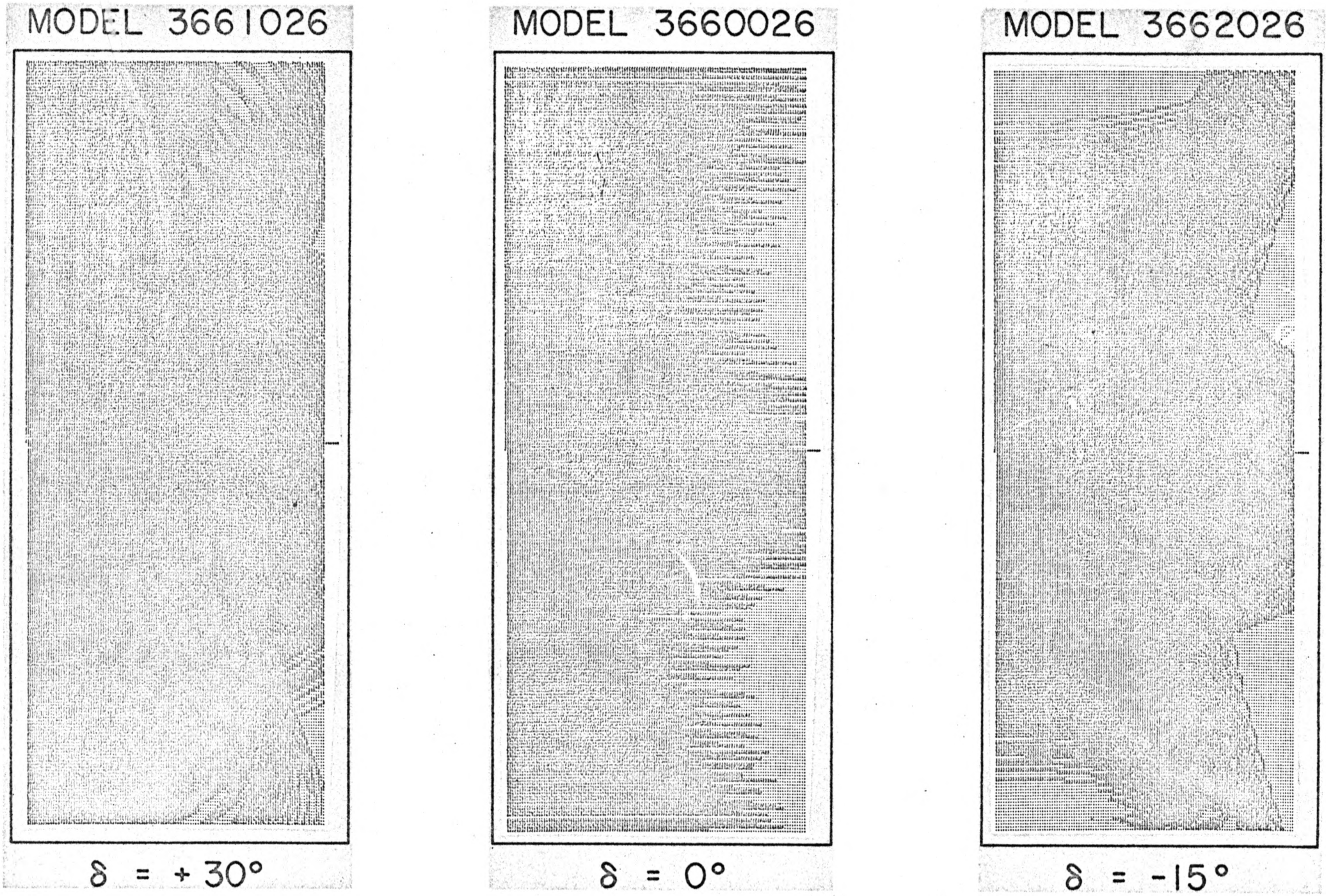


Figure 9. Transfer Functions for Complementary Arrays with 36 Elements

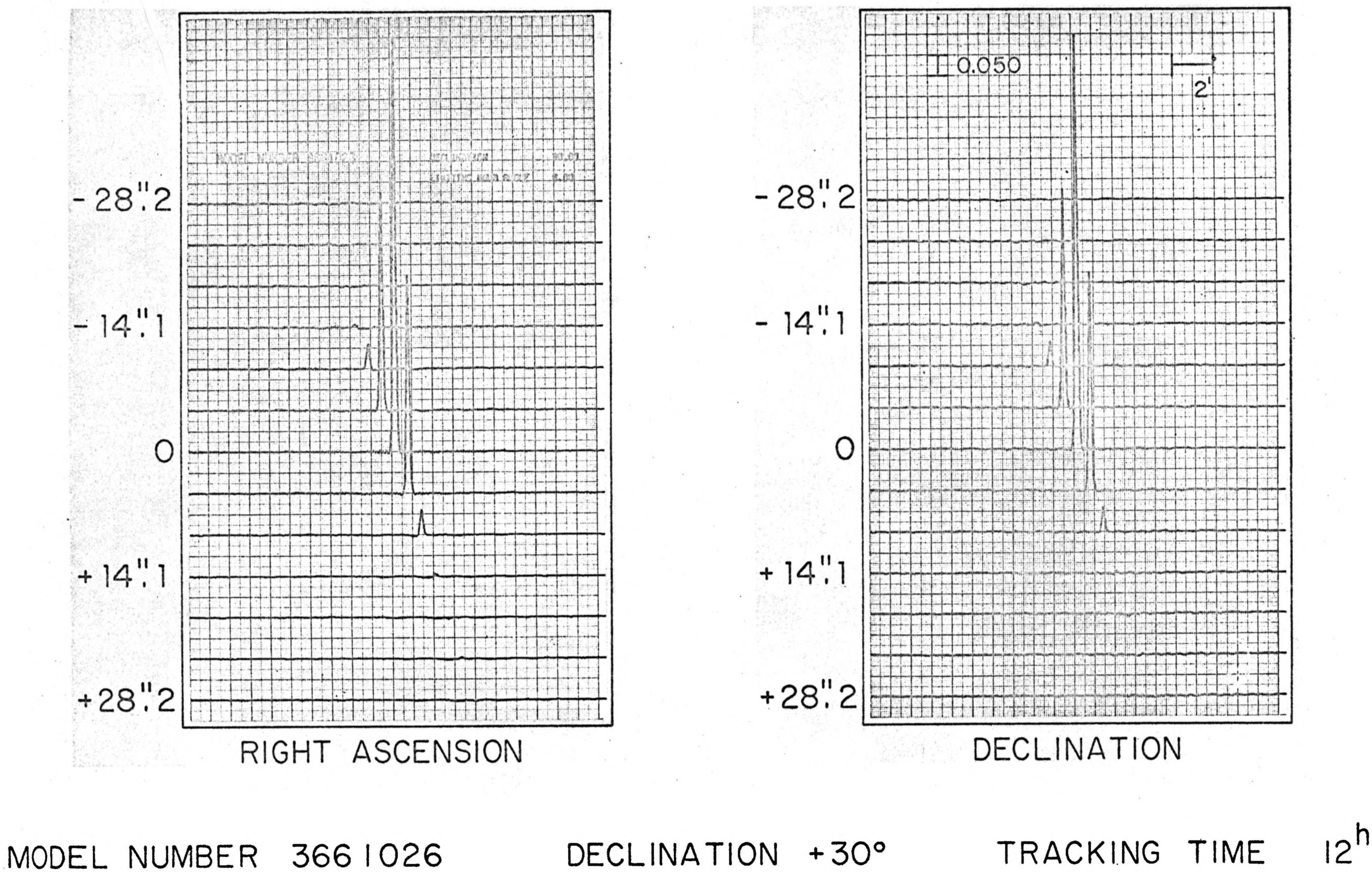
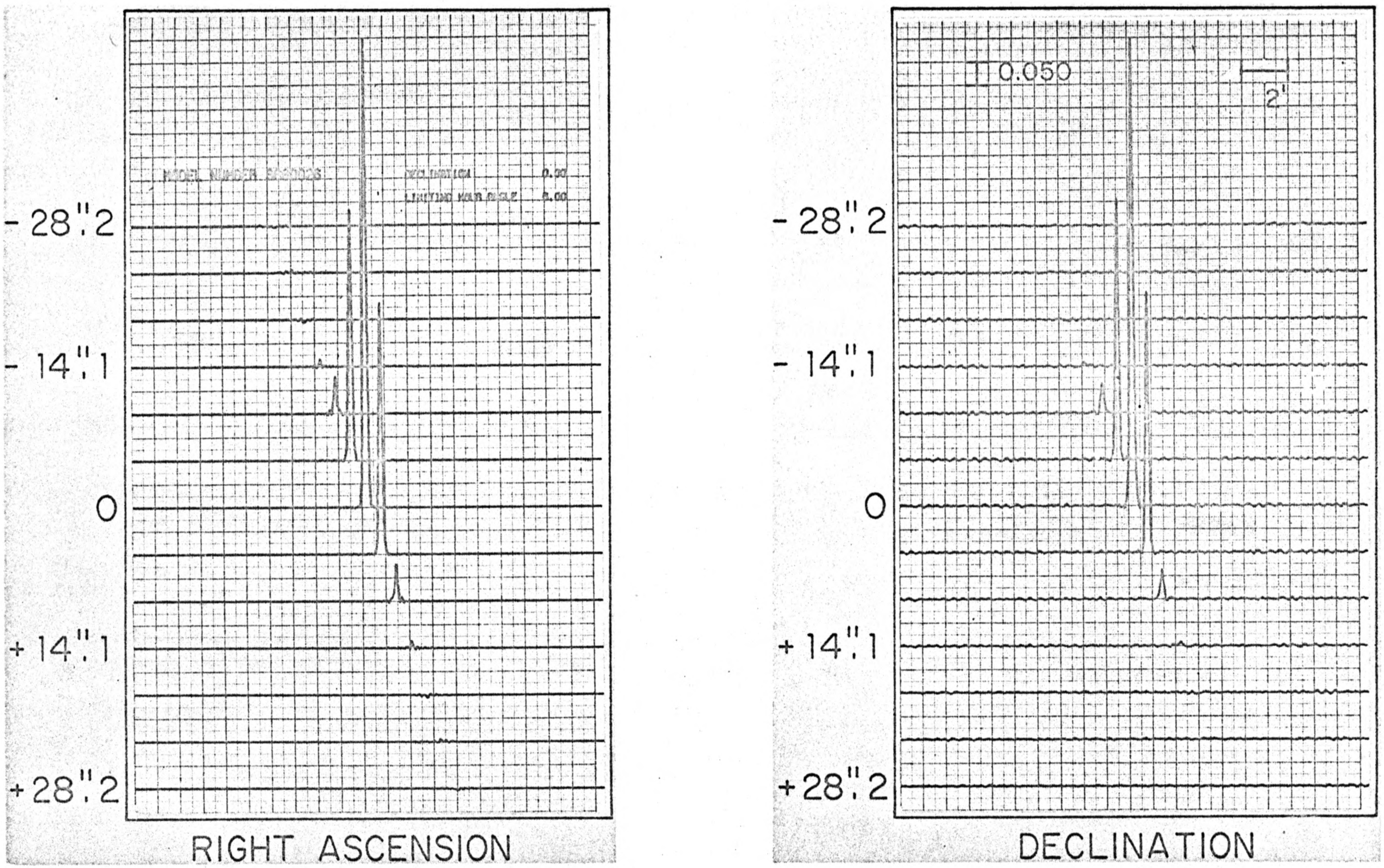


Figure 10. Beam Pattern Corresponding to the Transfer Function of Figure 9, δ = 30°



MODEL NUMBER 3660026 DECLINATION 0° TRACKING TIME 12^h

Figure 11. Beam Pattern Corresponding to the Transfer Function of Figure 9, $\delta = 0°$

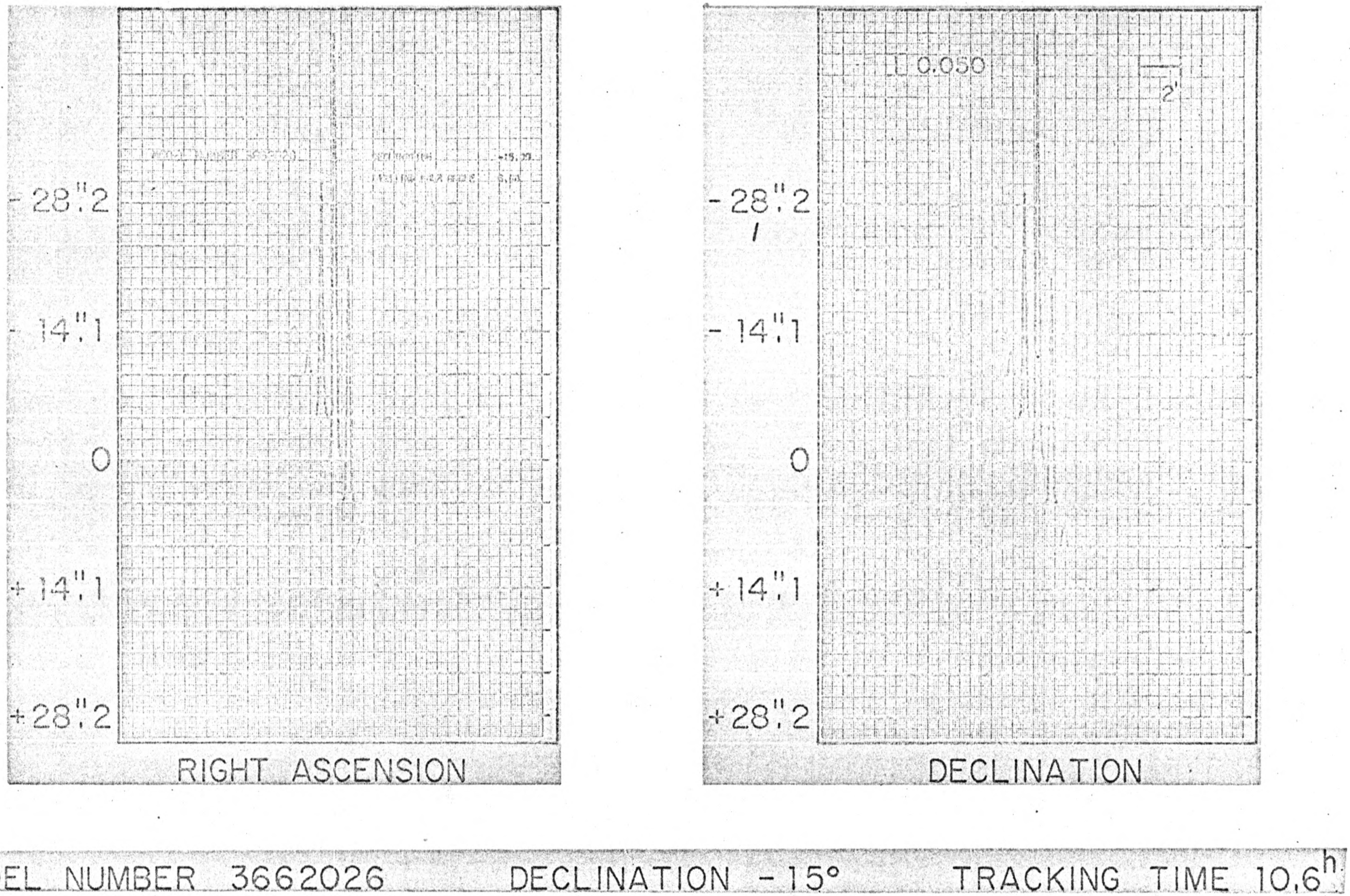


Figure 12. Beam Pattern Corresponding to the Transfer Function of Figure 9, $\delta = -15°$