NATIONAL RADIO ASTRONOMY OBSERVATORY

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IONOSPHERIC EFFECTS AT 11 cm WAVELENGTH

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INTRODUCTION

The propogation of radio waves through the earth's atmosphere is influenced mainly by two regions of the atmosphere - the troposphere and the ionosphere. In the troposphere, the index of refraction is greater than unity and decreases monotonically with altitude to a value of unity at approximately 30 km. The ionosphere commences at about 70 km height and has a refractive index less than unity. The refractive index depends upon the frequency of the wave and the electron density of the ionosphere and is minimum where the electron density reaches a peak. Random fluctuations in the properties of the atmosphere result in random fluctuations in the amplitude and phase of the wave. In the ionosphere, the randomness is caused by the presence of irregularities in the form of "blobs" of enhanced ionization floating about at random in background of uniform electron density.

In this report an estimate is made of the effect of ionospheric irregularities on the propagation of a wave of wavelength 11 cm. The values of mean squared fluctuations in phase and angle of arrival have been computed and a comparison is made between the ionospheric and tropospheric effects at this wavelength. The gross refraction suffered by the wave is also computed.

THEORY:

The theory of the influence of irregularities on the propagation of radio waves has been discussed by Muchmore and Wheelon (1955) and Booker (1958), and is briefly reviewed below. In the ionosphere, the refractive index n is related to the electron density N through

$$n^2 = 1 - \frac{\omega_N^2}{\omega^2} \tag{1}$$

where $\omega_N^2 = \frac{Ne^2}{m\varepsilon_0}$, e and m are the charge and mass of an electron, ε_0 is the permittivity of free space and ω is the wave angular frequency. The mean squared fluctuations in refractive index and electron density are related through

$$\overline{\left(\frac{\Delta n}{n}\right)^2} = \left(\frac{\omega_N}{\omega}\right)^4 \overline{\left(\frac{\Delta N}{N}\right)^2} \qquad (2)$$

Assuming the irregularities to be spherical of scale L, the mean square fluctuation in the phase of a wave is given by

$$\overline{(\Delta\phi)^2} = \frac{2\pi^2 DL \text{ Sec } \chi}{\lambda^2} \overline{\left(\frac{\Delta n}{n}\right)^2}$$

or

$$\overline{(\Delta\phi)^2} = \frac{2\pi^2 DL \text{ Sec } \chi}{\lambda^2} \left(\frac{\omega_N}{\omega}\right)^4 \overline{\left(\frac{\Delta N}{N}\right)^2} \qquad (3)$$

The scale L is defined as in Booker (1958) and is the distance at which the correlation of scintillations falls to a value 0.5. D is the thickness of the region containing the irregularities through which the wave passes, λ is the wavelength of the wave and χ is the zenith angle of the source.

Here $\Delta\phi$ is the deviation of the phase from the value it will have in the absence of irregularities. In interferometery, we are concerned with fluctuations in the difference in phase between signals received at two stations separated by a distance d. If the correlation coefficient of the phase fluctuations between the two stations is ρ then the mean square fluctuation in the phase difference between the two stations is

$$\overline{[\Delta(\phi_1-\phi_2)]^2} = 2\,(1-\rho)\ \overline{(\Delta\phi)^2}$$

The correlation coefficient ρ may be taken as gaussian,

$$\rho = e^{-d^2/L^2}$$

If d<<L, we have

$$\overline{[\Delta(\phi_1-\phi_2)]^2} = 2\left(\frac{d}{L}\right)^2 \overline{(\Delta\phi)^2}$$

or

$$\overline{[\Delta(\phi_1-\phi_2)]^2} = \left(\frac{2\pi d}{\lambda}\right)^2 \frac{D}{L} \sec \chi \left(\frac{\omega_N}{\omega}\right)^4 \overline{\left(\frac{\Delta N}{N}\right)^2} \qquad (4)$$

The fluctuation in the phase difference can be interpreted as a fluctuation in the angle of arrival of the wave, θ, which is given by

$$\overline{(\Delta\theta)^2} = \frac{D \sec \chi}{L} \overline{\left(\frac{\Delta n}{n}\right)^2}$$

or

$$\overline{(\Delta\theta)^2} = \frac{D \sec \chi}{L} \left(\frac{\omega_N}{\omega}\right)^4 \overline{\left(\frac{\Delta N}{N}\right)^2} \qquad (5)$$

We see from (3) and (5) above that the rms phase fluctuation varies inversely as the frequency and the rms fluctuation in the angle of arrival varies inversely as the square of the frequency.

IONOSPHERIC PARAMETERS

The ionospheric irregularities have been the subject of exhaustive research and although the exact nature of the irregularities is still not well understood, considerable amount of information is available, for example, in Yeh and Swenson (1964) and Titheridge (1963). Irregularities vary in size from a few kilometers all the way up to several hundred kilometers. The largest irregularities are essentially travelling ionospheric disturbances [see Rao and Yeh (1967)]. Most of the irregularities are, however, in the 5 - 100 km range. The heights at which irregularities occur vary from the E-region to almost 1000 km. Mostly the irregularities are in the F-region at a height of 300 - 400 km. The shape of the irregularities is generally assumed spheroidal with a constriction perpendicular to the earth's magnetic field; however, the field alignment has not been well established [Brown and Chapman (1967)]. For the purpose of getting an order of magnitude of phase fluctuations, the assumption of a spherical shape is good enough. The electron density of the irregularities varies as their size. It is maximum at the center of the irregularity and may have a value as much as 10% higher than the background values for irregularities of size 100 km. A typical irregularity can, therefore, be assumed to be a prolate spheroid with the longer dimension 50 km, shorter dimension 15 km, and an electron content variation of about 5%, located at about 300 - 400 km [Titheridge (1963)].

Phase Fluctuations:

The mean square fluctuation in phase can be computed from (3). Taking irregularity size L = 10 km, λ = 11 cm, $\omega_N/\omega = 1/300$, $\overline{\left(\frac{\Delta N}{N}\right)^2} = 10^{-7}$ and turbulent path length D = 100 km, we find that, for vertically incident radiation,

$\overline{(\Delta\phi)^2} = 2.01 \times 10^{-5}$ $(\mathrm{rad})^2$, or $(\Delta\phi)_{\mathrm{rms}} = 0.258°$. For L = 100 km, this becomes 2.01×10^{-4} $(\mathrm{rad})^2$. This result is in good agreement with the results quoted by Kaydanoviskiy and Smirnova (1965) and Millman (1966). The variation of $(\Delta\phi)_{\mathrm{rms}}$ with Zenith angle is shown in Figure 1.

The fluctuation in the phase difference between signals received at two stations separated by a distance d is given by equation (4). The rms fluctuation is directly proportional to the separation d and the wavelength. For very small values of d, $[\Delta(\phi_1-\phi_2)]$ rms is negligible. For very large values of d, it approaches $\overline{2(\Delta\phi)^2}$. For the same assumed values of various parameters, the phase difference fluctuation as a function of baseline is shown in Figure 2. The irregularity size is taken as 10 km. It should be noted that for d<<L, the rms phase difference fluctuation varies inversely as the square root of irregularity size but for d>>L, it varies directly as $\sqrt{L}$.

Angle of Arrival Fluctuations:

For vertically incident radiation of 11 cm wavelength, the rms fluctuation in the angle of arrival is obtained from equation (5). Putting D = 100 km, L = 10 km, $\frac{\omega_N}{\omega} = \frac{1}{300}$ and $\overline{\frac{\Delta N}{N}}^2 = 10^{-7}$, we get $(\Delta\theta)_{\mathrm{rms}} = 2.3 \times 10^3$ seconds of arc. The rms angle of arrival fluctuation varies directly as the square of the wavelength. Measurements of $(\Delta\theta)_{\mathrm{rms}}$ have been made at lower frequencies by several investigators. Hewish (1952) has reported values of 2-3 minutes of arc at 8m wavelength and 10 seconds of arc at 3.4m wavelength. Smyth (1964) reports a value of 10 milliradians (0.57 degrees) at 300 MHz. This would correspond to about 2 seconds of arc at 3000 MHz, a figure greater by a factor of 10^3 compared to the figure computed above. This would possibly be due to measurements made under highly disturbed conditions. Results quoted by Lawrence et el (1960) and Little et el (1962) for $(\Delta\theta)_{\mathrm{rms}}$ also bear out the inverse squared frequency relationship and agree with the results computed here to within an order of magnitude.

Comparison with Tropospheric Effects

Scintillation of radio signals is caused primarily by the ionosphere at lower frequencies and by the troposphere at higher frequencies. In the frequency range above 1000 MHz, the waves are rarely affected by the ionosphere and arrive at the lower atmospheric level essentially as plane waves [Aarons (1962)]. The magnitudes of the phase and angle of arrival fluctuations due to the troposphere have been estimated by Millman (1966) and Kaydanovskiy and Smirnova (1965). A comparison between ionospheric and tropospheric effects for a vertically incident 11 cm wave is made below in Table I.

TABLE I

		Ionosphere	Troposphere
Phase	$(\Delta\phi)_{rms}$	0.258° $(\sim f^{-1})$	5.9° $(\sim f)$
Angle of Arrival	$(\Delta\theta)_{rms}$	0.002" $(\sim f^{-2})$	7.21" (independent of f)

Ionospheric Refraction

As the radio wave travels through the earth's atmosphere, refractive bending takes place both in the ionosphere and the troposphere. Consequently, the apparent angle of arrival is not the true direction of the source. Figure 3 shows the ray path and the refractive bending. ΔE is the refractive error. This has been computed for realistic models of the ionosphere and the troposphere by Millman (1966) and Weisbrod and Colin (1960). For the troposphere, the error depends upon the water vapor content and the elevation angle. It decreases rapidly with increasing elevation angles. Figure 4 shows the variation of refraction error with elevation angle for 0% relative humidity. In the ionosphere, the refractive index depends upon the frequency of the wave and the electron density. Figure 4 shows the ionospheric refraction error for 100 MHz and 2695 MHz for a day-time ionosphere. The 100 MHz curve has been computed by Millman (1966) assuming a Chapman model for the ionosphere. The 2695 MHz curve has been computed from the 100 MHz curve assuming an inverse square frequency dependence. The refraction error at night is smaller than that for daytime for the ionosphere. It is clear from Figure 4 that the ionospheric error at 2695 MHz is negligible compared to the tropospheric error.

CONCLUSIONS:

The influence of the ionosphere on extra terrestrial radio waves at 11 cm wavelength has been considered in this report. The regular ionosphere causes refractive bending of the wave resulting in a refractive error. The ionospheric irregularities cause a fluctuation in the amplitude and phase of the signal received. These effects are frequency dependent since the ionospheric refractive index is frequency dependent. It is shown that rms fluctuation in the phase due to the ionosphere at 11 cm is of the order of 0.30° and is an order of magnitude less than that due to the troposphere. Under highly disturbed conditions the error may be of the same order of magnitude as the tropospheric error. The fluctuations in the angle of arrival due to the ionosphere are negligible compared to those due to the troposphere. The total refraction error due to the ionosphere is only a few microradians compared to a few milliradians due to the troposphere. It is, therefore, concluded that the ionospheric effects can be ignored at 11 cm.

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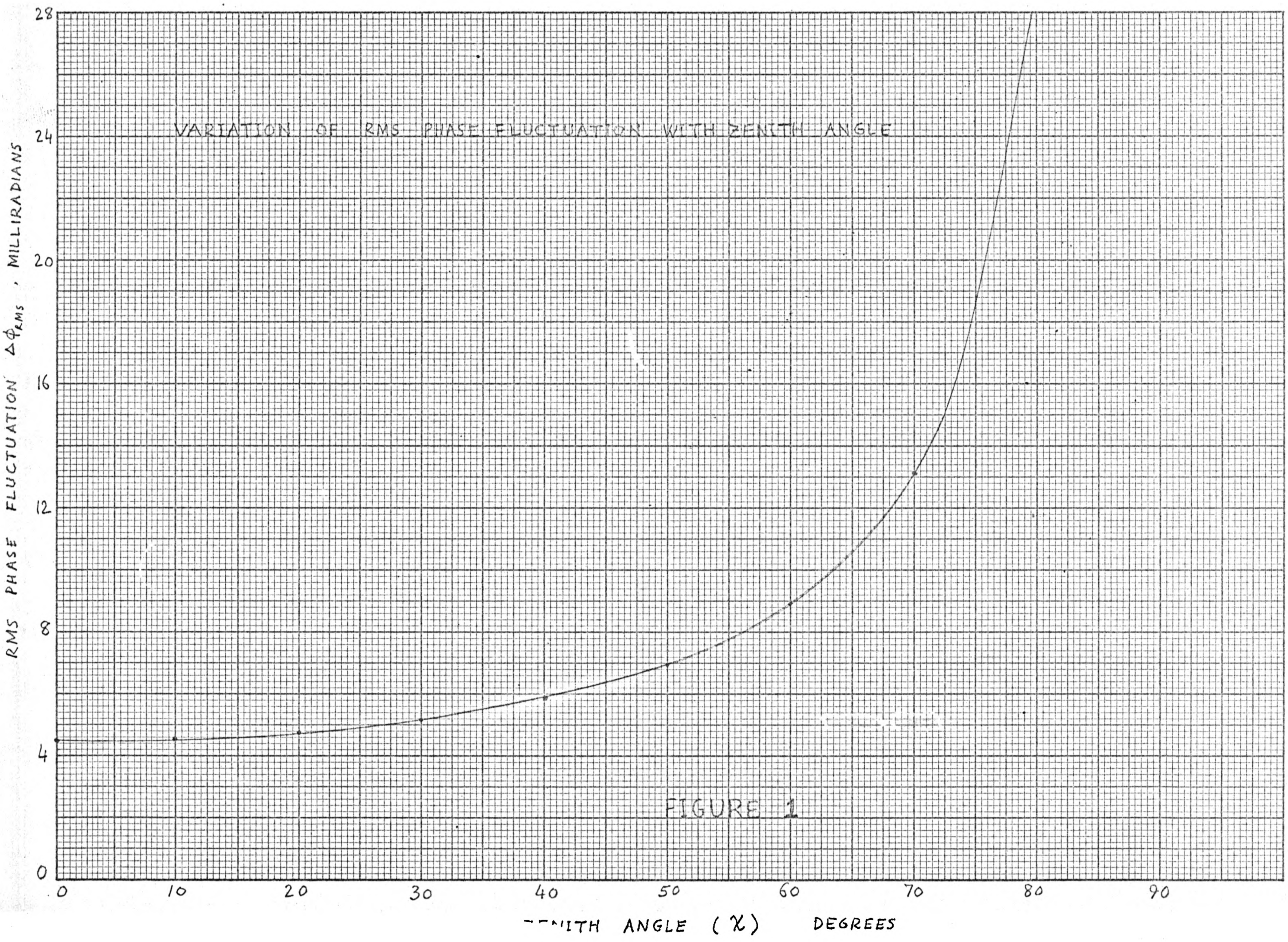
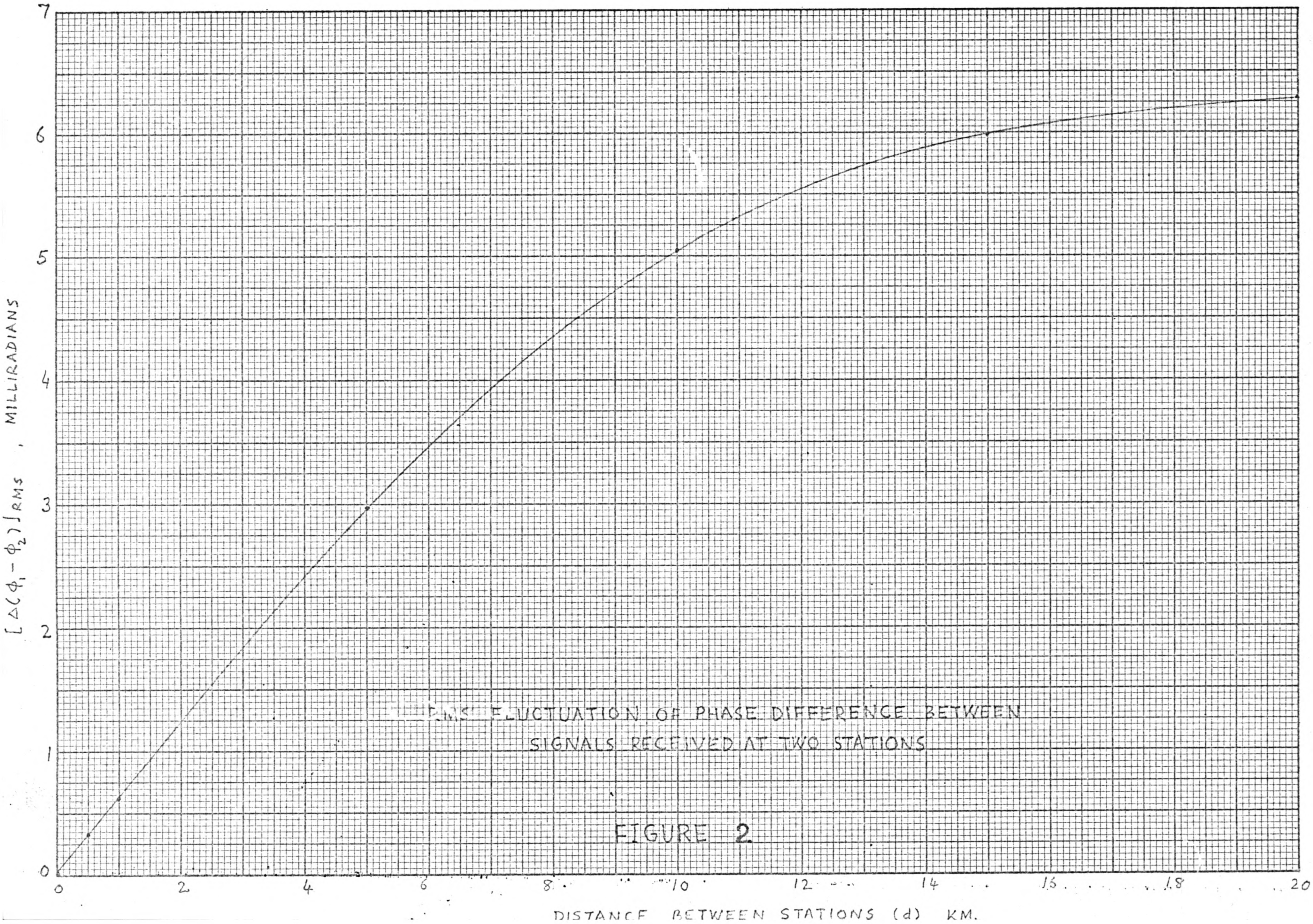
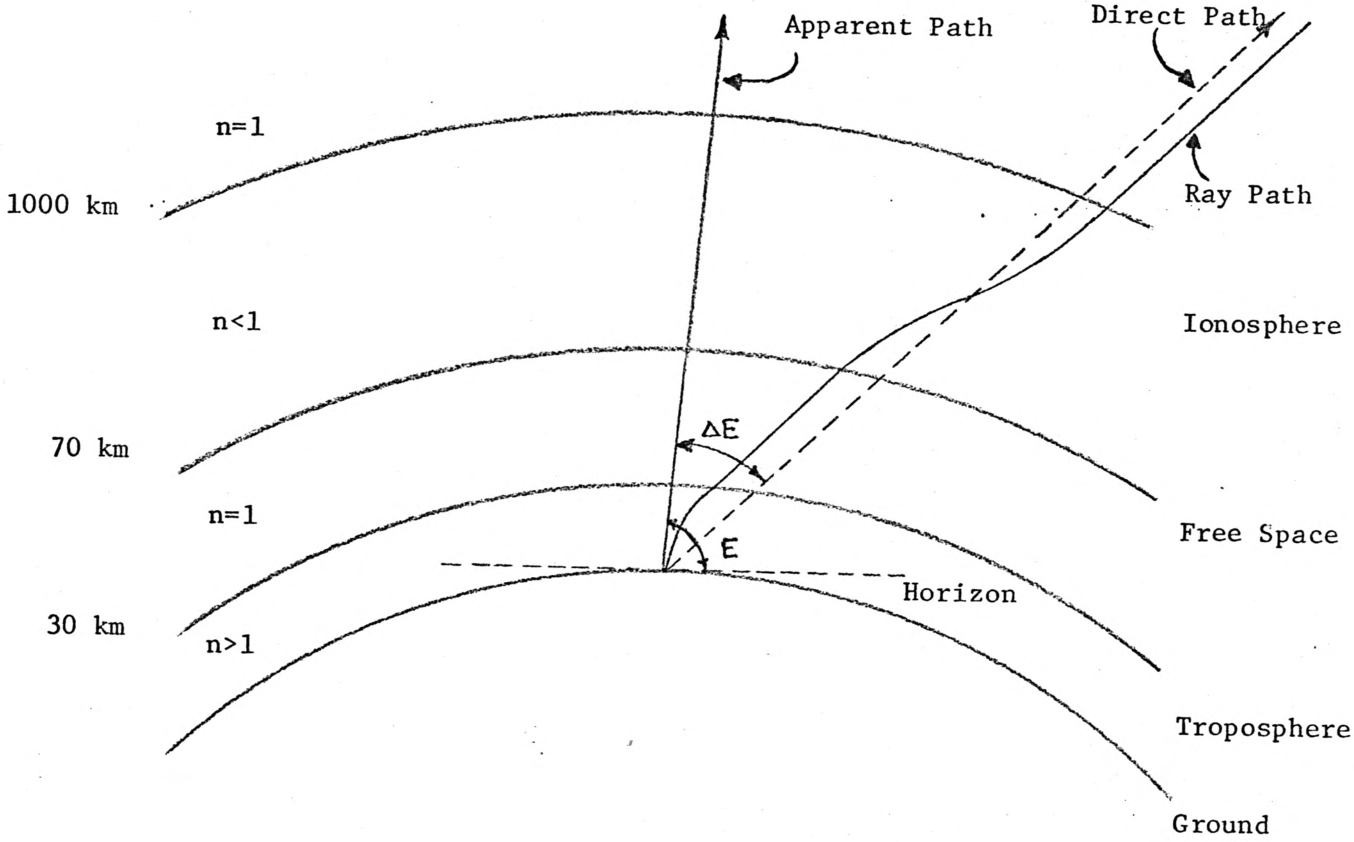

VARIATION OF RMS PHASE FLUCTUATION WITH ZENITH ANGLE
RMS PHASE FLUCTUATION $\Delta\phi_{RMS}$, MILLIRADIANS
0
4
8
12
16
20
24
28
0
10
20
30
40
50
60
70
80
90
ZENITH ANGLE (χ) DEGREES

FIGURE 1



RMS FLUCTUATION OF PHASE DIFFERENCE BETWEEN SIGNALS RECEIVED AT TWO STATIONS

FIGURE 2



Typical Ray Path Trajectory

Figure 3

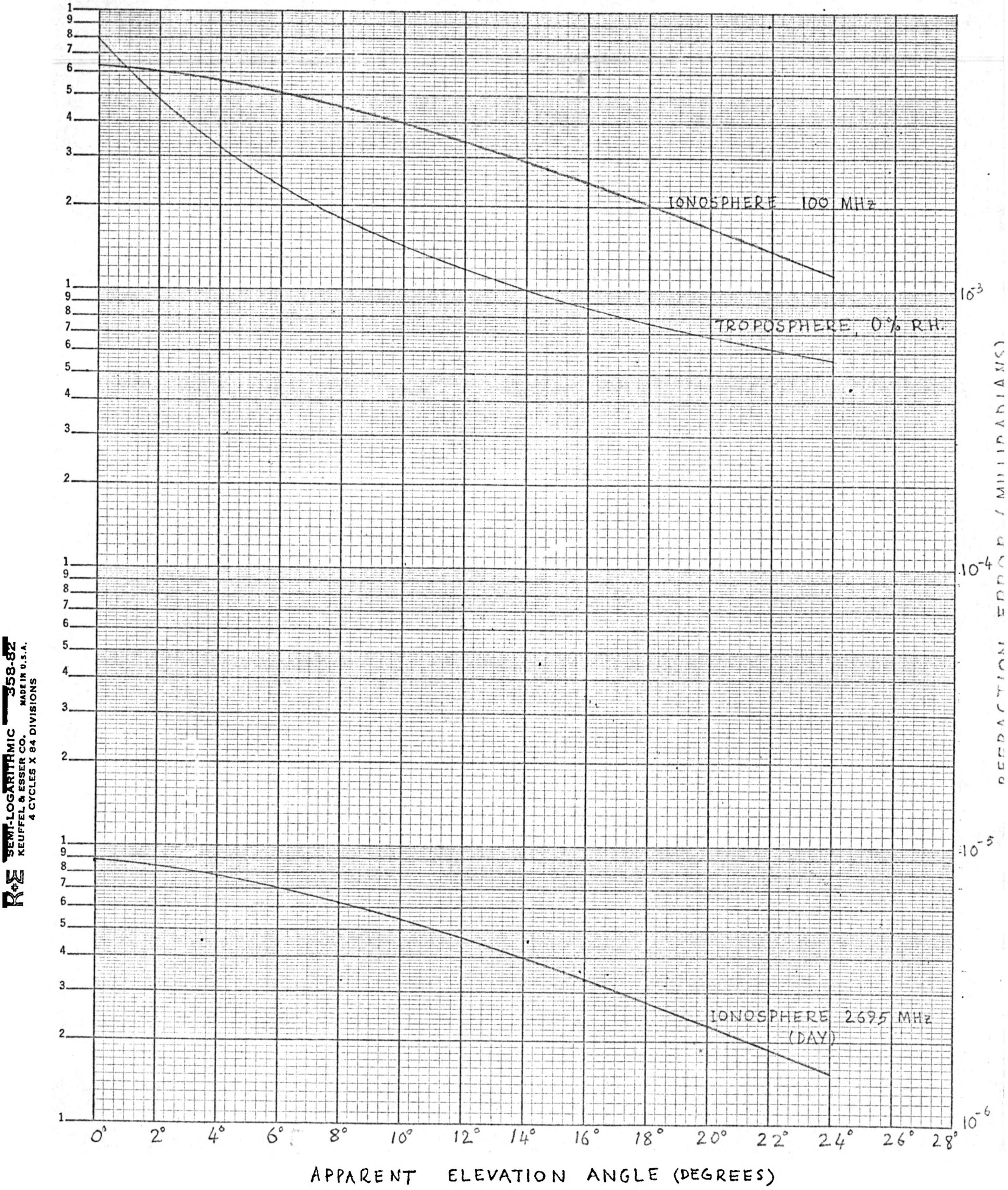

IONOSPHERE 100 MHz
TROPOSPHERE, 0% R.H.
IONOSPHERE 2695 MHz (DAY)
10^{-3}
10^{-4}
10^{-5}
10^{-6}
REFRACTION ERROR (MILLIRADIANS)
0° 2° 4° 6° 8° 10° 12° 14° 16° 18° 20° 22° 24° 26° 28°
APPARENT ELEVATION ANGLE (DEGREES)
K+E SEMI-LOGARITHMIC 358-82
KEUFFEL & ESSER CO. MADE IN U.S.A.
4 CYCLES X 84 DIVISIONS

FIGURE 4