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THE VLA "FIELD OF VIEW"

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1. DEFINITION

Consider the following alternative definitions of the field of view of a telescope:

A. The portion of the sky to which the telescope is sensitive.

B. The portion of the sky which can be mapped by the telescope without significant distortion.

C. The maximum source size which can be mapped without significant distortion.

We would be happiest if all these definitions were equivalent. For optical telescopes they usually are, or nearly so; but for aperture synthesis telescopes they generally are not. Definition A apparently depends on the telescope's sensitivity and the strengths of the sources; most telescopes are "sensitive" in any direction, if only there is a strong enough source there (e.g. a 100 kW local radar transmitter 120° from the main beam!). To make the definition practical, one can specify a source strength and ask, in what portion of the sky can such a source be located and still be detectable?

However, definition A is still inadequate because a source, even though detectable, may not be "mappable" without unacceptable distortion.

Thus, we have definition B. With synthesis telescopes, the distortion may be due to the primary beam pattern, bandwidth effects, noncoplanar baselines, or other effects.

Definition B may seem to be satisfactory, but actually it can rarely be applied. There may well be <u>no</u> portion of the sky which can be accurately mapped without specifying what is in the <u>rest</u> of the sky, especially those parts of it adjacent to the field being mapped. That is, a strong source just outside the "field of view" may cause serious distortion inside the field of view. For linear data processing schemes, the distortion may be viewed as "sidelobes" of the outlying source. Thus, we have definition C: it states implicitly that no sources may be outside the field of view if we are to insure a low-distortion map. I consider this definition the most satisfactory one, and I therefore shall adopt it in this discussion. To be sure, it is not a pleasant choice from the telescope user's point of view; he usually cannot be sure that there are no sources outside a field of interest. But the definition takes a realistic view of the problems, whereas B does not.[1]

Another alternative definition might be mentioned:

D. The portion of the sky which the astronomer is interested in mapping.

This is a parameter of the astronomer, rather than of the instrument. The latter might be analytically or empirically tractable, while the former certainly is not.

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[1] Definition C implicitly assumes there exists a small enough source size for which distortion will not be "significant"; but sometimes even a centered point source will have bad sidelobes. Nevertheless, the analysis which follows will show that there is usually a source size beyond which the map rapidly gets worse.

2. THE CLASSICAL SYNTHESIS TELESCOPE

Our definitions have implicitly considered the "telescope" to include all of the data processing which goes into making a map. To proceed further, we shall have to specify the telescope more precisely; we shall restrict ourselves to aperture synthesis telescopes which:

a) measure interferometric visibilities at a finite number of points in uvw-space.

b) multiply each measurement by a real beam-shaping coefficient ("weight"),

c) perform some averaging or smoothing of the weighted measurements,

d) sample the smoothed, weighted measurements on a rectangular grid in the uv-plane (ignoring w), and

e) compute the discrete Fourier transform of the latter samples.

This might be called the "classical" aperture synthesis telescope. If the map is produced by some other data processing scheme, especially if it is non-linear, then our conclusions about the field of view may not apply.

Operations (a) through (e) above are summarized by the following equations:

$$V(\underline{r}) = \int_{-\infty}^{\infty} \oint A(\underline{s},f)\, B(\underline{s},f)\, e^{i2\pi(f/c)(\underline{s} - \underline{s}_o) \cdot \underline{r}}\, dA(\underline{s})\, df \qquad (1)$$

$$\hat{B}(\underline{s}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{[S(\underline{r})\; V(\underline{r})] * C(u,v)\right\} Ш \left(\frac{u}{\Delta u}, \frac{v}{\Delta v}\right) \sqcap \left(\frac{u}{u_{max}}, \frac{v}{v_{max}}\right)$$

$$\cdot\; e^{-i2\pi(f_o/c)\,(\underline{s} - \underline{s}_o) \cdot (w,v,o)}\, du\, dv \qquad (2)$$

where (1) describes the visibility measured on baseline $\underline{r} = (u,v,w)$ due to a source whose spectral brightness is $B(\underline{s},f)$ at frequency f in the direction of unit vector $\underline{s}$, and (2) describes the computation of the map $\hat{B}(\underline{s})$ from the measured visibilities. In (1), the inner integral is a surface integral over the unit sphere; $A(\underline{s},f)$ is the power gain function of the antennas and receivers; and $\underline{s}_o$ is a unit vector in the delay-tracking direction. In (2), the notation is taken mostly from E. Greisen's report (VLA Scientific Report No. 110): $S(\underline{r})$ is the array sampling function, including the beam-shaping coefficients; $C(u,v)$ is the smoothing function; Ш$(\cdot,\cdot)$ is the 2-dimensional regular sampling function (array of delta functions); $\sqcap(\cdot,\cdot)$ is the 2-dimensional rectangle function; and f_o is a reference frequency, normally taken to be the center of the band passed by $A(\underline{s},f)$. The astrisk denotes two-dimensional convolution.

3. PARAMETERS AFFECTING THE FIELD OF VIEW

3.1 The Primary Beam

The beam pattern of the individual antennas (assumed identical) is given by $A(\underline{s},f)$. If we define $B'(\underline{s},f) = A(\underline{s},f)B(\underline{s},f)$ to be the modified brightness of the source, then $\hat{B}(\underline{s})$ attempts to estimate $\int B'(\underline{s},f)df$ [$\approx B'(s,f_o)$ for narrow bandwidths]. Apparently, then, $B(\underline{s},f)$ is distorted by the primary beam $A(\underline{s},f)$. One might consider correcting this distortion by computing

$$\hat{B}_{corrected} = \frac{\hat{B}(\underline{s})}{A(\underline{s},f_o)} \tag{3}$$

provided $A(\underline{s},f_o)$ is known. But it should be apparent from substituting (1) into (2) that $A(\underline{s},f_o)$ does not divide through. Nevertheless, (3) provides a useful approximate correction in many practical cases. But when $\underline{s}$ is sufficiently far from the beam center, the gain-to-noise ratio will be too low and/or $A(\underline{s},f_o)$ will be too poorly known to make any corrections useful. The distortion thus introduced by the primary beam provides our first limit to the field of view; we shall call this the primary field of view, recognizing that the actual field of view may be smaller because of other effects.

With well-designed antennas, for which $A(\underline{s},f_o)$ decreases monotonically from the beam center to the edge of the primary field, the effect of the primary beam is simply to attenuate the response to the outlying parts of $B(\underline{s},f)$. Notice that, for example, if there is a point source at $\underline{s} = \underline{s}_1$, then the response $\hat{B}(\underline{s})$ is proportional to $A(\underline{s}_1,f)$ for ALL $\underline{s}$, not just for $\underline{s} = \underline{s}_1$; i.e., the sidelobes of outlying sources are attenuated too. For some other kinds of distortion, this will not be ture.

3.2 Sampling Effects (Sidelobes and Aliasing)

In this section we make two assumptions which may not be entirely true for the VLA:

$$(f_o/c)(z-1)\,w \ll 1 \text{ for all } \underline{s},w \text{ [co-planar baselines]} \qquad (4)$$

and $$(\Delta f/c)(\underline{s} - \underline{s}_o)\cdot\underline{r} \ll 1 \text{ for all } \underline{s},\ \underline{r} \text{ [narrow band]} \qquad (5)$$

where $\underline{r}=(u,v,w)$, $\underline{s}=(x,y,z)$, $\underline{s}_o = (0,0,1)$, $|\underline{s}| = |\underline{s}_o| = 1$,

and Δf is the effective bandwidth of $A(\underline{s},f)$. The situation when these conditions are not satisfied will be discussed in Sections 3.4 and 3.5. When (4) and (5) are satisfied we can approximate $w \approx 0$ and

$A(\underline{s},f) \approx A_o(\underline{s})\, \delta(f-f_o)$; then (1) simplifies to

$$V(u,v,w) = \iint_{x^2+y^2<1} \frac{B'(x,y,f_o)}{\sqrt{1-x^2-y^2}}\, e^{i2\pi(f_o/c)(ux+vy)}\, dxdy \qquad (6)$$

which is precisely a (inverse) Fourier transform (FT) of the modified brightness.[2] This FT relation between V(w,v) and $B'_o(x,y) \triangleq B'(\underline{s},f_o) \cdot (1-x^2-y^2)^{-1/2}$ allows us to make use of the smapling theorem of Shannon:

If a function $B'_o(x,y)$ is confined so that $B'_o = 0$ for all $|x| > X/2$, $|y| > Y/2$, then it is fully described by a discrete set of <u>samples</u> of its FT, namely by $\{V(u_i,v_j)\}$

where $u_i = \frac{c}{f_o}\frac{i}{X}$, $v_j = \frac{c}{f_o}\frac{i}{Y}$, $-\infty \le (i,j) \le \infty$.

Thus, to describe B'_o fully, we need not measure V(u,v) everywhere--only at the sampling points. In practice, of course, we cannot do even this, because there are infinitely many sampling points and they extend to infinite (u,v). In addition, the sampling <u>interval</u> $\underline{\Delta r} \triangleq \frac{c}{f_o}(\frac{1}{X}, \frac{1}{Y})$ depends on the source size; conversely, the available sampling interval of the telescope may be thought of as a second parameter limiting the field of view.

Actually, the sampling is rarely done on a regular grid of spacing $\underline{\Delta r}$, and unfortunately no irregular-spacing sampling theorem is known in two dimensions. But we can proceed by using the regular-spacing sampling theorem to guide our intuition, and we will be able to come to quite definite - though numerically approximate -- conclusions.

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2 Note to curvature freaks: if $w \equiv 0$, then it remains precisely a FT no matter how big the source is, even if "curvature is important"!

Notice from (2) that the computation of $\hat{B}(\underline{s})$ involves <u>two separate</u> sampling operations: there is the sampling function $S(\underline{r})$ which is determined by the geometry of the telescope, and there is the rectangular-grid sampling function $Ш(\frac{u}{\Delta u}, \frac{v}{\Delta v})$ which is applied to the telescope samples after smoothing by $C(u,v)$ [the latter might be merely the pill-box function $\sqcap(\frac{u}{\Delta u}, \frac{v}{\Delta v})$]. Fortunately, we can treat each of these sampling operations separately in order to understand the distortion each introduces. Since (2) is a FT, let us re-write it as follows:

$$\hat{B}(\underline{s}) = \left\{[\tilde{S}(\underline{s}) * B'_o(\underline{s})]\ \tilde{C}(\underline{s})\right\} * F\left\{Ш(\frac{u}{\Delta u}, \frac{v}{\Delta v})\ \sqcap(\frac{u}{u_{max}}, \frac{v}{v_{max}})\right\}. \quad (7)$$

Here $\tilde{f}$ denotes the FT of any function f, and $F\{\cdot\}$ is the FT operator; note that in the present approximations [(4),(5)], $\tilde{V}(\underline{r}) = B'_o(\underline{s})$.

3.2.1 The Telescope Beam

To examine the effects of the telescope sampling, consider

$$\hat{B}_1(\underline{s}) \triangleq \tilde{S}(\underline{s}) * B'_o(\underline{s}) \quad (8)$$

This has been called the "direct FT" brightness estimate, because it avoids the "indirect" steps of smoothing and re-sampling on a rectangular grid before computing the FT; our telescope might have computed $\hat{B}_1$ rather than $\hat{B}$ if it were not for our desire to take advantage of the FFT algorithm. Anyway, the smoothing and re-sampling will operate upon B_1, so we should first understand the latter.

Let us call $\tilde{S}(\underline{s})$ - the FT of the telescope sampling function - the <u>telescope beam</u>. What properties will it have? For any specific $S(u,v)$,

$\tilde{S}(\underline{s})$ can certainly be computed; but is there anything that can be said in general? Let us be guided by the sampling theorem: consider the regular $S(u,v)$ of Fig. 1a, and the corresponding telescope beam of Fig. 1b. Notice that the "main beam" at the

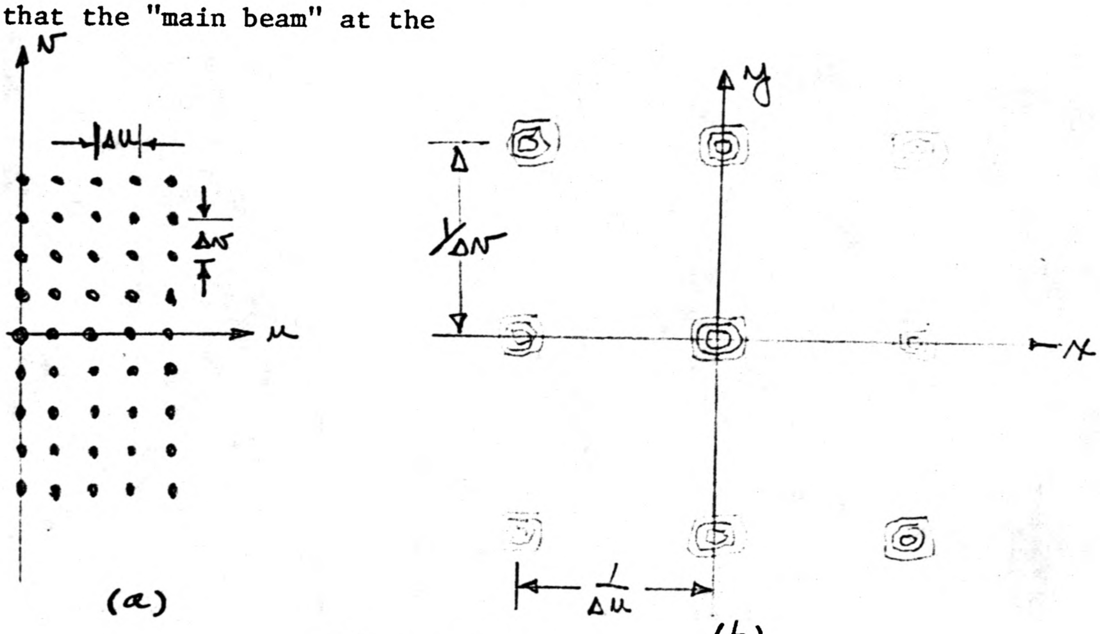

FIGURE 1

center of Fig. 1b is replicated on a grid of spacing[3] $(\frac{1}{\Delta u}, \frac{1}{\Delta v})$. This is in accord with the sampling theorem which says that a source extending beyond $|x| \le \frac{1}{2\Delta w}$, $|y| \le \frac{1}{2\Delta v}$ cannot be represented correctly by samples spaced $(\Delta u, \Delta v)$; what happens, as we see here, is that the outlying parts of such a source appear near the center, distorting the image of whatever was there. This effect is usually called <u>aliasing</u>.

Now, the VLA -- like other synthesis telescopes -- does not take its samples on the grid of Fig. 1a. The VLA, in fact, might be initially modeled as taking a random sampling of points over a certain portion of the uv-plane. Suppose, then, that we randomly perturb the positions of the sampling points in Fig. 1a, keeping the same total number of points and keeping them all within $|u| \le u_M$, $|v| \le v_M$. Then their average density

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[3] Here u, v are in units such that $c/f_o = 1$.

will be the same as before, although there may be some clumping. Still, viewed on a scale several times larger than (Δu, Δv), the sampling will remain fairly uniform. Fluctuations in the sampling function on such scales correspond to structure in the central part of the telescope beam. Thus we conclude that random perturbations of the sampling points do not change the central part of the beam very much; in fact, the shape of the main beam is mainly determined by the outer boundary of the sampled region of the uv plane, provided that the number of sampling points is large. On the other hand, when viewed on a scale ~(Δu, Δv), the perturbed sampling function looks much grainier than before; fluctuations can be found on a variety of scales smaller and larger than (Δu, Δv). This means that the energy in the telescope beam formerly concentrated at $(\frac{n}{\Delta u}, \frac{m}{\Delta v})$ will now be randomly scattered about in the outlying parts of the beam, perhaps something like Fig. 2. These portions of the beam are usually called the "far sidelobes", and they can serve to limit the field of view. From this discussion it appears that the far sidelobes will become strong at distances from the main beam ~(average sampling density)$^{1/2}$, for random sampling.

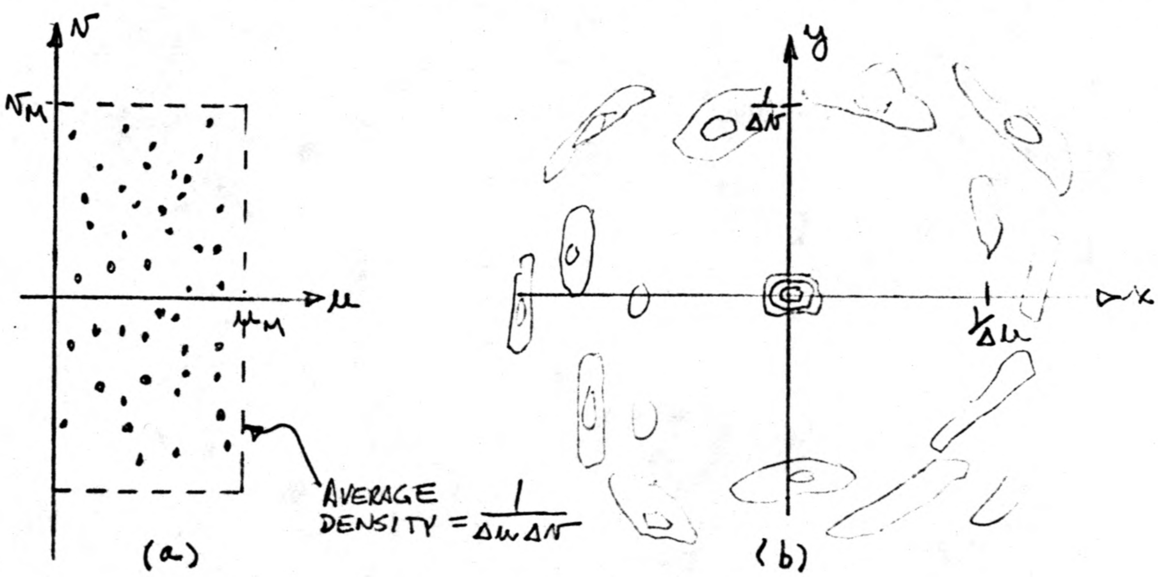

Figure 2

Table 1 applies this result to the VLA. It appears that the average sampling density is sufficient to make the sampling field of view larger than the primary field of view for all configurations except the largest. Table 1 should be taken with a couple of grains of salt, however: first, the VLA sampling is not really random; it is systematically much more dense near the center of the uv-plane than near the edges. Second, we have not considered the effects of weighting the samples non-uniformly in an attempt to reduce the effects of clumping and/or to smooth out close-sidelobes by tapering; both may improve the sampling field of view.

Table 1. VLA Sampling Considerations

Config.	Area of uv Plane Covered* at $\delta \approx 30°$	Avg. Sampling Density* σ	$\sigma^{-1/2}$	Primary HPBW
A: 21 km	4.19×10^{13} cm$^2/\lambda^2$	3.60×10^{-8} cm$^{-2}\lambda^2$	.65 arcmin cm^{-1} λ	1.70 arcmin cm^{-1} λ
B: 6.4km	3.8×10^{12}	3.97×10^{-7}	2.2	"
C: 19.km	3.63×10^{11}	4.16×10^{-6}	7.0	"
D: 0.6km	3.14×10^{10}	4.81×10^{-5}	23.5	"

* Based on 12^h observing on all 351 baselines @ 10^s/sample $\Rightarrow 1.51 \times 10^6$ samples

3.2.2 Gridding Effects

Once we have understood the properties of the telescope beam and the corresponding direct FT map, it is a simple matter to understand the remaining operations of Eqn (2) or (7) which lead to the final map $\hat{B}(\underline{s})$.

In the uv-domain, the telescope samples are smoothed by convolving with C(u,v), and the resulting function is sampled on the rectangular gird; the FT of the samples is then computed, possibly using the FFT [the latter, however, computes $\hat{B}$ only at discrete values of $\underline{s}$, whereas we are thinking of the map as a continuous function of $\underline{s}$; cf (2)].

In the xy-domain, the direct FT map $\hat{B}_1(\underline{s})$ is multiplied by $\tilde{C}(\underline{s})$ so as to avoid aliasing from the sampling which follows. Since the sampling will be on a rectangular grid, we can apply the sampling theorem directly: aliasing will be completely avoided if $\hat{B}_1(\underline{s})\,\tilde{C}(\underline{s})$ is zero for $|x| > \frac{1}{2\Delta u}$, $|y| > \frac{1}{2\Delta v}$. Call the latter region the <u>gridding field of view</u>. Ideally, then, $\tilde{C}(\underline{s})$ should be a rectangle function and Δu, Δv should be chosen so that the region of interest is included in the field. In practice, some less-than-ideal $\tilde{C}$ is usually chosen for computational efficiency.

In general, then, sampling on the rectangular grid in the uv-domain corresponds to convolution in the xy-domain with a function just like Fig. 1b. Note well that the function being convolved is $\hat{B}_1(\underline{s}) \cdot \tilde{C}(\underline{s})$ and NOT the true sky brightness $B(\underline{s},f_o)$, nor even the modified brightness $B'(\underline{s},f_o)$ (modified by the primary beam). If $\hat{B}_1(\underline{s})\,\tilde{C}(\underline{s})$ extends beyond the gridding field of view, then the outlying parts will be aliased into the center; <u>this can happen even if $B(\underline{s},f_o)$ or $B'(\underline{s},f_o)$ lies entirely</u>

within the gridding field of view. The sidelobes of the telescope beam $\tilde{S}(\underline{s})$ appear in $\hat{B}_1(\underline{s})$, and these will always extend out to infinity (although for most practical samplings they decrease in strength as one goes out farther). To avoid trouble even on compact sources, these sidelobes must either be suppressed by $\tilde{C}(\underline{s})$, or the gridding $(\Delta u, \Delta v)$ must be fine enough to make the gridding field of view so large that there are no significant telescope sidelobes outside it.

3.3 Summary of Results so Far

We began by restricting our attention to what we called the "classical" synthesis telescope; i.e., one which processes the measurements by a certain commonly-used procedure to obtain a brightness map. We then further specialized the analysis by assuming that the baselines are nearly co-planar [Inequality (4)] and that the bandwidth is narrow [Inequality (5)], because only then can we say that the measurements are samples of the Fourier transform of the brightness distribution (modified by the primary beam).

We then found that, under these conditions, various effects limit the field of view. The presence of significant brightness sufficiently far from the field center leads to a distorted map because

(a) the primary beam attenuation is not exactly correctable, and the approximate corrections become very noisy far from the beam center; and/or

(b) the synthesized beam arising from the telescope sampling of the uv-plane causes "aliasing" of the outlying brightness into the central part of the field (due to far sidelobes of the telescope beam); and/or

(c) the smoothing function C(u,v) (typically a rectangle function; representing cell-averaging) attenuates the response to the outlying brightness (although in practice this is often correctable); and/or

(d) the grid-sampling causes aliasing of the outlying brightness and the outlying sidelobes of the central brightness into the central part of the field.

It should be noted that the final map is not related to either the true brightness or the modified brightness by convolution with a "beam" even in the approximations so far considered. Only the direct FT map has this property, by Eqn (8). If the final map is to approximate the direct FT map, then C(u,v), Δu, and Δv must be chosen to make effects (c) and (d) negligible. Usually this will require setting the grid sampling rate $(\frac{1}{\Delta u}, \frac{1}{\Delta v})$ much larger than the average sampling rate of the telescope.

3.4 Bandwidth Effects

Let us now consider the effect of relaxing the assumption (5), namely

$$\frac{\Delta f}{c}\,(\underline{s} - \underline{s}_o)\cdot\underline{r} << 1 \text{ for all } \underline{s},\ \underline{r}, \qquad (9)$$

which will be true only for sufficiently small bandwidths. Re-writing (9) as

$$\frac{\Delta f}{f_o} << \left[\frac{1}{(\underline{s}-\underline{s}_o)\cdot(\underline{r}/\lambda_o)}\right]_{max} \qquad (10)$$

we see that it means that the fractional bandwidth must be much less than the reciprocal number of resolution elements across the source. For the

VLA, where we desire $\sim 10^3$ resolution elements across the field of view, this implies very narrow bandwidths (e.g., $\Delta f \ll 5$ MHz when $f_o = 5$ GHz). Much larger bandwidths will actually be available, and therefore it is important to understand what will happen when (9) or (10) is not satisfied.

3.4.1 Analysis

In this case the basic observation equation (1) does not reduce to the inverse Fourier integral (6); nevertheless, the classical telescope continues to processes the measurements using equation (2), which is a Fourier transform of a certain sampled, smoothed, and resampled function of u and v. It is therefore meaningful to rewrite (2) in a way similar to (7):

$$\hat{B}(\underline{s}) = [\hat{B}_1(\underline{s})\ \tilde{C}(\underline{s})] * F\left\{ ш(\frac{u}{\Delta u}, \frac{v}{\Delta v})\ \Pi(\frac{u}{u_{max}}, \frac{v}{v_{max}}) \right\} \tag{11}$$

where

$$\hat{B}_1(\underline{s}) = F\left\{ S(u,v)\ V(u,v,w) \right\}$$

is the direct FT map; the difference is that before we could write $\hat{B}_1 = \tilde{S} * B_o'$, whereas now we cannot. It should be evident from (7) and (11) that the smoothing and gridding operations are independent of the bandwidth, in that they always affect the direct FT map in the same way, namely as described in section 3.2.2. Therefore in this and the following section we consider only the direct FT map.

Putting (1) into (12) gives

$$\hat{B}_1(x,y) = \int_v \int_u S(u,v) \int_f F(f) \int_{y'} \int_{x'} B_o'(s',y') \exp \frac{i2\pi}{c} [f(ux'+vy') - f_o(ux+vy)]\ dx'dy'\ df\ dudv \tag{13}$$

where we have retained assumption (4), coplanar baselines, and we have assumed that $A(\underline{s},f) = A(\underline{s},f_o)G(f)$. Re-ordering the integrals,

$$\hat{B}_1(x,y) = \int_{y'}\int_{x'} B_o'(x',y') \int_f G(f) \int_v \int_u S(u,v) \exp \frac{i2\pi}{c} (f-f_o)(ux'+vy')] \exp \frac{-i2\pi f_o}{c} [u(x-x')+v(y-y')]\, du dv\, df\, dx'dy' \tag{14}$$

$$\triangleq \int_{y'}\int_{x'} B_o'(s',y')\, P(x',y',x,y)\, dx'dy' \tag{15}$$

where $P(x',y',x,y)$ may be interpreted as the response at (x,y) to a point source at (x',y'). In the narrow-band case, when $G(f) \approx \delta(f-f_o)$, the integrals over u,v in (14) become a FT, and $P(x',y',x,y) = \tilde{S}(x-x',y-y')$; then (15) becomes a convolution integral, in agreement with (8). With a little re-arranging of the definition of P implied by (14) and (15), it is not hard to show that

$$P(x',y',x,y) = \int_f G(f)\, \tilde{S}\left(\frac{f}{f_o} x' - x, \frac{f}{f_o} y'-y\right) df. \tag{16}$$

Careful study of (16) shows that it represents a radial smearing of the point source response (along a line through x = 0, y = 0) which increases in proportion to the fractional bandwidth and the distance $\|(x',y')\|$ of the point source from the origin. This result has been derived elsewhere, so we shall not consider it much more here. For a detailed derivation see A. R. Thompson, VLA Electronics Memo #118; and for a concise treatment see B. Clark, VLA Scientific Memo #113. Both of these reports consider implications for the VLA, and they are strongly recommended to the interested reader.

3.4.2 Discussion

It should be emphasized, as Thompson points out, that the effect of increasing the bandwidth is a loss of radial resolution for outlying sources. One does not, in general, get a reduction in the integrated flux contributed by such a source; the peak response for an unresolved source will be reduced, but not for a resolved source. On the other hand, Clark (Scientific Memo #113) shows that the far sidelobes of an unresolved source tend to be reduced as the bandwidth is increased. Quantitavely, the important parameter is $n = (r'/b)(\Delta f/f_o)$, where $r' = (x'^2+y'^2)^{1/2}$ and b is the half-power width of $\tilde{S}(x,y)$. Then the radial width of the point source response $P(x',y',x,y)$ is about (n+1) times larger than the width of $P(0,0, x,y) = \tilde{S}(x,y)$.

In the present context - that of understanding the effect on the field of view - we must consider the distortion introduced when a source is larger than the field. This distortion may be of two kinds: distortion of that part of the map which is inside the field of view, and of that part which is outside. We have seen that the primary beam distorts only the outside part, attenuating the source brightness by a (possibly poorly known) amount, which is not precisely correctable due to subsequent convolution with the point source response. We have also seen that sampling effects tend to cause distortion of the inside part of the map, by aliasing of the outlying brightness through sidelobes. The bandwidth effects which we are now considering can conceivably result in both types of distortion. The outside part of the map will suffer the loss of radial resolution discussed above, but sidelobes of the outlying brightness appearing in the inside part will also be affected by the bandwidth smearing. Thompson

(Electronics Memo #118) suggests that such sidelobes might be separated from the response to in-field brightness by comparing maps made with different bandwidths. The exact effect of non-zero bandwidth on the in-field sidelobe responses will depend on the sampling distribution, but there is some indication (Clark, Scientific Memo #113) that the sidelobe responses almost always get smaller as the bandwidth is increased.

3.4.3 Conclusions

Non-zero bandwidth limits the field of view primarily by distorting the outlying parts of the map. If a source extends beyond the sampling field of view, the distortion inside the sampling field of view can usually be reduced by increasing the bandwidth. If the primary and sampling fields of view are much larger than the source, the distortion caused by large bandwidth consists only of a loss of radial resolution away from the field center. If such distortion is considered significant, it then limits the field of view.

In Table 2 we give the numerical field of view which results in a factor of two loss in radial resolution at the edge, for the various wavelengths and bandwidths which will be available in the VLA. By the numerical field of view I mean the ratio of the field diameter to the half-power width of $\tilde{S}(x,y)$. The latter depends on the weighting applied, as well as on the array geometry.

Table 2. Numerical Field of View for Doubling of Radial Beamwidth at Edge.[1]

λ_o \ Δf	49 MHz	24	12	4	1.5	0.5
20 cm	122	250	500	1499	3997	11,990
6.3 cm	388	793	1586	4759	12,690	38,070
2.0 cm	1223	2498	4997	14,990	39,970	$>10^5$
1.3 cm	1883	3847	7687	23,060	61,500	$>10^5$

NOTES: 1. Diffraction sidelobes at center due to pt. source at edge are down 8 db compared to monochromatic.

2. Numerical field of view for VLA limited by sampling to ~1400.

3.5 Non-Coplanar Baselines

Finally, we consider the effect of relaxing assumption (4), namely

$$(f_o/c)(z-1)\, w \ll 1 \quad \text{[co-planar baselines]}, \qquad (4)$$

which will be true if all the baselines are in the uv-plane (w=0), or if the source is sufficiently small, since

$$z-1 = \sqrt{1-x^2-y^2} - 1 \approx \frac{1}{2}(x^2+y^2), \quad x^2+y^2 \ll 1.$$

However, it is also possible to satisfy (4) whenever all the baselines are co-planar by defining coordinates (u'v'w') so that the baselines are all in the u'v' plane; then (4) is satisfied with w replaced by w'. This may result in the source not being centered at the corresponding x' = 0, y' = 0, but (1) will still reduce to a FT in the new coordinates.

When all the baselines are geographically east-west to high precision -- so that they reamin coplanar under Earth rotation -- there may be an advantage in working in (u',v',w')space. On the other hand, when the baselines are not coplanar, then uvw-space has the advantage that the (z-1) factor in (4) is very small for small source sizes, so we have a chance of satisfying the inequality if the source is small enough. Therefore, we'll stick to the usual (u,v,w) system here [where, we recall, $\underline{s}_o=(0,0,1)$ is a unit vector in the reference direction (cf. (2)), best chosen near the source center].

3.5.1 Analysis

What happens, then, if (4) is not satisfied? As in section 3.4.1, we consider just the direct FT map, because the effects of gridding on the latter can always be considered separately. Substituting (1) into (12) and re-arranging, we find that the direct FT map, assuming neither co-planar baselines nor narrow bandwidth, is

$$\hat{B}_1(x,y) = \int_{x'} \int_{y'} B_o'(x',y') \int_f G(f) \int\int_{u,v} S(u,v,w)$$
$$\exp\, i2\pi\frac{f_o}{c}\left[w(\frac{f_{x'}}{f_o} - x)+v(\frac{f_{y'}}{f_o} - y)+w\frac{f}{f_o}(z'-1)\right] \text{dudvdf dx'dy'} \tag{17}$$

$$= \int_{x'} \int_{y'} B_o'(x',y')\, P(x',y',x,y)\, dx'dy'. \tag{18}$$

From (18) it seems[4] that we once again have a superposition integral

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[4] It might be noted that (17) appears to depend on w, but (18) does not. But since S(u,v,w) is not a continuous function due to the discreteness of the measurements, we let w = w(u,v). Remember also that $z' = (1 - x'^2 - y'^2)^{1/2}$.

in the point-source response P; the map is linear in the modified brightness, but it is not shift-invariant. The form of P is slightly different than before because of the term $w \frac{f_o}{c} (z-1)$ in the exponential; this will generally prevent P from being shift-invariant even in the narrow-band limit $G(f) \rightarrow \delta(f-f_o)$.

Study of (17) also reveals that bandwidth effects and non-coplanar baseline effects cannot be treated separately when both are significant. For example, when $w(z'-1)$ is significant then the wideband point source response does not reduce to a superposition of narrow-band responses with varying radial scale (as in (16)); this is because $z'-1$ is not linear in (x',y'). In order to gain some insight into the effects of the non-coplanar baselines alone, we'll consider only the narrow-band limit, keeping in mind that the full story is more complex.

Putting $G(f) = \delta(f-f_o)$, we get from (17) and (18):

$$P(x',y',x,y) = \int_u \int_v S(u,v,w) \exp i2\pi \frac{f_o}{c} [u(x'-x)+v(y'-y)+w(z'-1)] \, du dv. \quad (19)$$

It is hard to say much about the character of this function without detailed knowledge of $S(u,v,w)$. One simple conclusion, however, is that

$$P(x,y,x,y) = \int_u \int_v S(u,v,w) \, e^{i2\pi \frac{f_o}{c} w(z-1)} \, du \, dv$$

$$\leq \int_u \int_v \left| S(u,v,w) \, e^{i2\pi \frac{f_o}{c} w(z-1)} \right| du \, dv = P(0,0,0,0); \quad (20)$$

that is, the response at the position of an outlying point source ("peak response", roughly) is always less than it would be if the source were centered. The extra exponential term apparently scatters the source flux about the map; exactly how it is scattered -- into near sidelobes or far sidelobes, for example -- is an open question, and a general answer may not be possible. Thus we may need to depend on simulations to calculate $P(x',y',x,y)$ for each specific $S(u,v,w)$.

3.5.2 Discussion

For the VLA, it has been pointed out that $(f_o/c)\, w(z'-1)$ reaches several cycles on the worst-case baseline when x' is at the edge of the primary field of view. Perhaps it seems that measurements made on such baselines are useless because of this large "phase error". On the other hand, consider that (i) the response to sources near the center of the field on the very same baselines has no phase error, and (ii) the response to outlying sources depends on many baselines with small w in addition to the few "bad" ones with large w. Thus, it is not yet clear what effect the non-coplanar baselines will have on classical-processing maps produced by the VLA.

4. SUMMARY

Table 3 summarizes all the results we have discussed, and gives some typical numbers for the VLA.

5. CAVEAT

The following cannot be said too strongly.

In all this discussion, we have considered only the classical synthesis telescope; that is, one which produces a map by means of equation (2). It is very important to realize that this is not the only way to do things; in fact, $\hat{B}(\underline{s})$ is by some measures a very poor estimate the brightness, compared with what might be done with the same data. On the other hand, the classical processing is expected to be computationally feasible (though difficult) for the VLA; hence it is of interest to understand it, in case it turns out to be the <u>only</u> computationally feasible approach.

But it is very dangerous to think of the results given here as fundamental to synthesis telescopes. To do so would stifle creative thinking. The results apply only to a specific method of data processing, one which is fairly arbitrary and which has only a weak theoretical basis.

Table 3. Summary

(1) Field of View Limitation	(2) Parameter Proportional To Field of View	(3) Value for VLA: Configuration A, $\lambda = 20$ cm	(4) Assumptions For Column (3)	(5) Response to Source Outside the Field
Primary Beam	Primary HPBW	34 arcmin	fov = HPBW	Response Attenuated.
Telescope Sampling	(Avg. u-v density of samples)$^{1/2}$	13		Sidelobes appear within field.
Gridding	(Cell size)$^{-1}$	20	1024 x 1024 points, All Baselines Covered	"Alias" source appears within field; also telescope sidelobes of central source may be aliased into center.
Bandwidth	$\left(\frac{\Delta f}{c} \lvert \underline{r} \rvert_{max}\right)^{-1}$	4	Δf = 49 MHz; Center Beamwidth=2"; Edge Beamwidth=4".	Radial Resolution degraded for the outlying source, Sidelobes of outlying source modified.
Non-coplanar Baselines	$\left(\frac{f_o}{c} w_{max}\right)^{-1/2}$	11	12^h tracking; worst baseline produces 2 radians extra phase in integrand.	Unknown