February 4, 1975

VLA SCIENTIFIC MEMORANDUM #117

THE INTERIM VLA SPECTRAL LINE SYSTEM

Eric W. Greisen

Dr. Heeschen has requested observing proposals for the Interim VLA Spectral Line System. These proposals should both define the scientific usefulness of this system and define the nature of the data processing desired. This memo is intended to summarize the properties of the proposed interim line system as I currently understand them.

Digital correlators of the Papadopoulos-Ball variety will be provided for a maximum of 10 telescopes. These correlators have the property that the product of the number of possible channels times the total input bandwidth, B, is a constant. Thus the single-channel bandwidth, $\Delta\nu$, is given by

$$\Delta\nu = B^2 / K$$

where K is a constant. The planned correlators have K = 320 MHz. The planned digital hardware and software will limit the number of recorded channels to 64 with a smaller number of channels available in the range $5 \leq B \leq 40$ MHz. This limitation means that the total observed bandwidth is the smaller of B and 64 $\Delta\nu$. For B > 4.5 MHz, some of the channels will be made unreliable by edge effects. Only one frequency and one polarization may be recorded at a time. However any of the available polarizations and frequencies may be selected under computer control.

The VLA antenna location lists are reproduced in Appendix A. It is hoped that the spectral line system will become available in the middle of 1977. At this time, all C and D array stations should be completed. In addition, all A and B array stations on the southwest arm (except AW8 and AW9) should be completed. Within the limitations of scheduling considerations, the 10 telescopes may be placed at any of the completed stations.

The data for the interim spectral line system will be processed by standard digital methods. The output will consist of un-cleaned maps with one map per frequency and some minimal (e.g. printer) display capability. Additional map analysis, combination, and display routines may be available if there is general agreement on a very limited set of such routines. The map arrays will not be allowed to exceed 128 array points on a side except in very special circumstances.

The table given below summarizes the parameters of the VLA line system. The brightness sensitivity formula, derived in Appendix B, may be written as

$$\sigma(T_B) = c\,\frac{\kappa\, T_S}{\sqrt{\Delta\nu t}}\left(\frac{\lambda}{\beta_o}\right)^2 G(\gamma/\beta) \qquad (1)$$

where $c = 1.77$ for 10 telescopes, T_S is the system temperature, κ is a correction factor dependent on the taper, $\Delta\nu$ the single channel bandwidth in kHz, t the observing time in hours, λ the wavelength in centimeters, β_o the synthesized beamwidth for zero taper in seconds of arc, and $G(\gamma/\beta)$ a slowly-varying function of the array spacing γ and synthesized beamwidth β. The σ values in the table below are computed for zero taper, 1 kHz bandwidth, 1 hour observing time, and $\gamma/\beta = 0.5$. Equation (1) does not include the effects of non-random sources of error such as the sidelobes of the synthesized beam pattern.

The values of β_o and σ are computed for the C and D array dimensions.

"λ" cm	ν GHz	T_S °K	$\beta_o(D)$ "	$\sigma(D)$ °K	$\beta_o(C)$ "	$\sigma(C)$ °K
21	1.35- 1.72	40	52	9.2	16	100
6	4.5- 5.0	40	15	9.2	4.5	100
2	14.4-15.14	250	4.9	58	1.5	624
1.3	22-24	400	3.2	92	1.0	998

Note that the 2- and 1.3-cm systems may be available on only 6 of the telescopes. The effects of tapering may be estimated using equation (1) and the table below.

Taper db	κ	(β/β_o)
0	1.00	1.00
5	0.57	1.28
10	0.40	1.52
15	0.33	1.72
20	0.28	1.90
25	0.25	2.06
30	0.23	2.21
35	0.21	2.36

The computations presented in these tables are only approximate. They assume that the synthesized aperture is circular and fully sampled with those samples lying outside the largest possible circle being ignored. They also assume that calibration uncertainties may be ignored. These assumptions are reasonable for a wide range of possible observations with the VLA. The values of σ given above contain the assumption that the system temperature is roughly the receiver temperature. Observations of strong sources will have higher values of σ.

Smaller synthesized beamwidths may be obtained by including all possible data points. However, the improved resolution will be obtained at the expense of significant increases in the noise and in the sidelobe levels and complexity of the synthesized beam pattern.

The area which may be mapped is limited by the single dish beamwidth given by

$$\theta'_{SD} = 1.5\ \lambda$$

where λ is in cm, by the so-called delay beam given by

$$\theta'_{DB} = 1250\ /\ (LB)$$

where L is the effective arm length in km and B the single-channel bandwidth in MHz, and by the map array size limitation

$$\theta'_{MA} = c\ \left(\frac{\beta}{\beta_o}\right)\ \left(\frac{\gamma}{\beta}\right)\ \lambda$$

where c = 5.3 and 1.6 for the D and C configurations, respectively, and λ is in cm.

ANTENNA LOCATIONS

WEST ARM			EAST ARM			[redacted]NOR[redacted]RM		
STATION	DIST. FROM ORIG.		STATION	DIST. FROM ORIG.		STATION	DIST. FROM ORIG.	
	M	FT.		M	FT.		M	FT.
DW1*	-80.00*	-262.47*	DE1*	-40.00*	-131.23*	DN1	0	0.00
DW2 - CW1	44.85	147.15	DE2 - CE1	44.85	147.15	DN2 - CN1	54.86*	180.00
DW3	89.93	295.05	DE3	89.93	295.05	DN3	94.86*	311.22
DW4 - CW2 - BW1	147.33	483.37	DE4 - CE2 - BE1	147.33	483.37	DN4 - CN2 - BN1	134.86*	442.45
DW5	216.07	708.89	DE5	216.07	708.89	DN5	194.82	639.17
DW6 - CW3	295.43	969.26	DE6 - CE3	295.43	969.26	DN6 - CN3	266.38	873.95
DW7	384.89	1262.76	DE7	384.89	1262.76	DN7	347.04	1138.58
DW8 - CW4 - BW2 - AW1	484.00	1587.93	DE8 - CE4 - BE2 - AE1	484.00	1587.93	DN8 - CN4 - BN2 - AN1	436.40	1431.76
DW9	592.40	1943.57	DE9	592.40	1943.57	DN9	534.15	1752.46
CW5	709.79	2328.71	CE5	709.79	2328.71	CN5	639.99	2099.70
CW6 - BW3	970.50	3184.06	CE6 - BE3	970.50	3184.06	CN6 - BN3	875.07	2870.96
CW7	1264.35	4148.13	CE7	1264.35	4148.13	CN7	1140.03	3740.26
CW8 - BW4 - AW2	1589.92	5216.27	CE8 - BE4 - AE2	1589.92	5216.27	CN8 - BN4 - AN2	1433.58	4703.35
CW9	1946.03	6384.61	CE9	1946.03	6384.61	CN9	1754.67	5756.79
BW5	2331.65	7649.77	BE5	2331.65	7649.77	BN5	2102.37	6897.54
BW6 - AW3	3188.09	10459.61	BE6 - AE3	3188.09	10459.61	BN6 - AN3	2874.59	9431.07
BW7	4153.40	13626.64	BE7	4153.40	13626.64	BN7	3744.98	12286.68
BW8 - AW4	5222.90	17135.50	BE8 - AE4	5222.90	17135.50	BN8 - AN4	4709.31	15450.49
BW9	6392.69	20973.39	BE9	6392.69	20973.39	BN9	5764.08	18911.02
AW5	7659.48	25129.53	AE5	7659.48	25129.53	AN5	6906.29	22658.43
AW6	10472.87	34359.81	AE6	10472.87	34359.81	AN6	9443.03	30981.07
AW7	13643.92	44763.52	AE7	13643.92	44763.52	AN7	12302.27	40361.78
AW8	17157.23	56290.12	AE8	17157.23	56290.12	AN8	15470.10	50754.92
AW9	21000.00	68897.64	AE9	21000.00	68897.64	AN9	18935.00	62122.70
*On South Extension of North Arm			*On South Extension of North Arm			*Adjusted to allow tracks to pass through.		

Appendix B. Derivation of noise on synthesized maps

The prediction of noise on synthesized maps is a controversial business, but I will attempt to derive some formulae suitable for the VLA. I will assume a DFT approach since the FFT, when properly handled, is equivalent.

The maps are produced using the formula

$$T_B(x,y) = \kappa \sum_{j=1}^{N} \omega_j F_j \cos(\phi_j + 2\pi (u_j x + v_j y)) \tag{B-1}$$

where Fj and ϕj are the measured amplitude and phase of the j'th observation, ωj is the assigned weight including taper, and κ is a constant to be determined below. Converting (B-1) using the Hermitian property to

$$T_B(x,y) = \frac{\kappa}{2} \sum_{j=-N}^{N} \omega_j \underset{\sim}{V}_j \, \ell^{-2\pi i(u_j x+v_j y)}$$

and using the usual formula for the propagation of errors

$$\sigma^2 (T_B) = \left\langle \left| \sum_{j=1}^{N} \frac{\partial T_B}{\partial V_j} \Delta V_j \right|^2 \right\rangle$$

we obtain

$$\sigma^2(T_B) = \sum_{-N}^{N} (\kappa\omega_j/2)^2 \left\langle |\Delta V_j|^2 + \Delta V_j \Delta V_j e^{-4\pi i(u_j x+v_j y)} \right\rangle \tag{B-2}$$

where we assume $<\Delta V_j \; \Delta V_k^* > = 0$ for $j \neq \pm k$. The second portion of (B-2) contains the expressions

$$(<\Delta R_j^2> - <\Delta I_j^2>)$$

and

$$<\Delta R_j \Delta I_j>$$

where R_j and I_j are the real and imaginary parts of V_j. In the absence of abnormal contributions to the noise, both these expressions are zero. Hence

$$\sigma^2 \ (T_B) = \kappa^2 \sum_{j=1}^{N} \omega_j^{\ 2} \ <|\Delta V_j|^2>/2$$

We need to obtain expressions for $|\Delta V_j^{\ 2}|$ and κ. Assuming that calibration is not a source of error, $|\Delta V_j|^2$ is determined by the system temperature as

$$\frac{<|\Delta V\ |^2>}{2} = \frac{<|\Delta R|^2> + <|\Delta I|^2>}{2} = <|\Delta R|^2>$$

$$\sqrt{\frac{<|\Delta V\ |^2>}{2}} = 2k \ \frac{T_S'}{\sqrt{2\Delta\nu t}} \ \frac{1}{(\epsilon\pi d^2/4)}$$

where $\Delta\nu$ = bandwidth
Δt = integration time per sample
ϵ = antenna aperture efficiency
d = antenna diameter
T_S' = system temperature corrected for 2-bit sampling
k = Boltzman's constant

The simplest method to determine κ is to consider what results one would obtain for a one flux unit source at the origin. Equation (B-1) yields

$$T_B(o) = \kappa \ \Sigma \ \omega_j$$

but

$$T_B(o) = \frac{\lambda^2}{2k} \ \frac{\Delta S}{\Delta\Omega} = \frac{\lambda^2}{2k} \ \frac{\Delta S}{\gamma 2}$$

where ΔS is the flux contained in the cell of size $\Delta\Omega$ at the origin. The distribution of the intensity should follow a Gaussian of the form

$$B(x,y) = \exp \left\{ - \frac{4 \ln 2}{\beta^2} (x^2+y^2) \right\}$$

where β is the full half-power synthesized beamwidth. With this beam pattern,

$$\Delta S = \frac{\iint_{-\gamma/2}^{\gamma/2} B(x,y)\, dxdy}{\iint_{-\infty}^{\infty} B(x,y)\, dxdy}$$

$$\Delta S = \left[\text{erf.} \left(\frac{\gamma \sqrt{\ell n\ 2}}{\beta} \right) \right]^2 \equiv \left(\frac{\gamma}{\beta}\right)^2 G(\gamma/\beta)$$

Thus

$$\kappa = \frac{T_o}{\Sigma \omega i} = \left(\frac{\lambda}{\beta}\right)^2 \frac{1}{2k} \frac{G(\gamma/\beta)}{\Sigma \omega i}$$

which yields

$$\sigma(T_B) = \frac{2^{3/2}}{\pi} \frac{G(\gamma/\beta)}{\varepsilon\ d^2} \left(\frac{\lambda}{\beta}\right)^2 \frac{T_S'}{\sqrt{\Delta\nu\ t}} \sqrt{\frac{\Sigma \omega_j{}^2}{(\Sigma \omega_i)^2}}$$

To put this in a more convenient notation, we note that

$$\Sigma \omega_i = \bar{\omega}\ P\ M(M-1)/2\ N$$

$$\Sigma \omega_i{}^2 = \overline{\omega^2}\ P\ M(M-1)/2\ N$$

where M is the number of telescopes, N the number of samples per inteferometer pair and P is the fraction of the samples included in the map-making process. Thus

$$\sigma(T_B) = \frac{2^{4/2}}{\pi} \frac{G(\gamma/\beta)}{\varepsilon\ d^2} \left(\frac{\gamma}{\beta}\right)^2 \frac{T_S'}{(P\ M(M-1)Nt\ \Delta\nu)^{1/2}} \frac{1}{E}$$

or $$\sigma\ (T_B) = 2^{1/2} \frac{G(\gamma/\beta)}{A_e} \left(\frac{\gamma}{\beta}\right)^2 \frac{T_S'}{\sqrt{\Delta\nu\ T}}$$

where

$$E^2 \equiv (\bar{\omega})^2 / (\overline{\omega^2})$$

$$T \equiv Nt$$

and

$$Ae \equiv 2^{-5/2} \pi d^2 \varepsilon \sqrt{P M(M-1)}\ E$$

is the effective area of the array.

The function E and beamwidth β depend on the chosen taper and, of course, on which data samples are included. In the following, we assume a circular effective aperture and gaussian tapering. Studies of the data sampling by the VLA show that the radius of the circular aperture is essentially equal to an arm length - at least for a declination of 30°. These studies show that, at least roughly, the probability that a sample will occur at u and v may be represented as

$$\begin{aligned} P(u,v) &= 1 / (2\pi R r) && r \leq R \\ &= 0 && r > R \end{aligned}$$

where $r \equiv \sqrt{u^2+v^2}$. Letting the weighting function be

$$w = (2\pi R) r\ \ell^{-b^2 r^2}$$

we find

$$\overline{W} = \pi(1 - \ell^{-b^2R^2}) / b^2$$

$$\overline{W^2} = \frac{\pi^2 R}{\sqrt{2} b^3} \frac{\sqrt{\pi}}{2}\left(\mathrm{erf}(\sqrt{2}bR) - \sqrt{2}bR\ell^{-2b^2R^2}\right)$$

$$E^2 = \frac{1}{Rb} \frac{(1 - \ell^{-b^2R^2})^2}{\frac{\sqrt{\pi}}{2} \mathrm{erf}\ (\sqrt{2}bR) - \sqrt{2}bR\ \ell^{-2b^2R^2}} .$$

For numerical purposes we may now specify a number of parameters as

$$\begin{aligned} T_S' &= 1.235\ T_S \\ E_o &\equiv E(\text{o taper}) = 0.866 \\ \beta_o &\equiv \beta(\text{o taper}) \\ \kappa &\equiv (E_o\beta_o)^2/(E\beta^2) \\ B &\equiv 10^3\ \Delta\nu = \text{bandwidth in kHz} \end{aligned}$$

$H \equiv 3600\ T$ = observing time in hours

$P = 0.6$ (actually $P > 0.605$ for the full array)

$M = 10$

$\varepsilon = 0.5$

$d = 2500$ cm

and express wavelength in cm and βo in seconds of arc. Then we obtain

$$\sigma(T_B) = 1.77 \frac{\kappa \quad T_S}{\sqrt{BH}} \left(\frac{\lambda}{\beta o}\right)^2 G(\gamma/\beta)$$

The numerical constant is 0.63 for 27 antennas. The function $G(\gamma/\beta)$ may be tabulated as

γ/β	$G(\gamma/\beta)$
0.0	0.883
0.2	0.845
0.5	0.786
1.0	0.579
2.0	0.241