National Radio Astronomy Observatory
Charlottesville, Virginia

VLA SCIENTIFIC MEMO 118 March 25, 1975

BANDWIDTH CORRECTION BY DECONVOLUTION

B. G. Clark

There has been considerable discussion of the deleterious effects of bandwidth on the VLA observations. As well as noting the effects, we need a (necessarily imperfect) correction procedure for the area in which the effects are serious but not fatal. Simply dividing the apparent map by a "Delay Beam" is insufficient. We must deconvolve the delay beam. Because a minor manipulation is required to put the deconvolution in familiar form, it is worth discussing the process in some detail. The discussion below embodies a two dimensional discussion--that is either the sky or the VLA is assumed flat. No significant alteration in the discussion appears to be required to take sky curvature into account--the two effects may, to sufficient precision, be handled separately.

The observations consist of the visibility function V(u,v) measured at various points in the u,v plane. We, straightforwardly, fourier transform them to get an uncorrected map, using some canonical frequency ω_o.

$$B(x,y) = \iint_{-\infty}^{\infty} V(u,v)\, e^{i\omega_o(ux+vy)}\, du dv$$

(u and v measured in nanoseconds).

$$B(x,y) = \int_0^{\infty} P\left(\frac{\omega}{\omega_o}\right) B_o\left(\frac{\omega}{\omega_o} x, \frac{\omega}{\omega_o} y\right) d\omega$$

where $P(\omega)$ is the instrument bandpass normalized to

$$\int_0^\infty P\left(\frac{\omega}{\omega_0}\right) d\omega = 1$$

We know from array theory that $B_0(x,y)$ is the convolution of the array beam with the sky brightness, and is the quantity of interest to us here.

Since the smearing is entirely radial, it is profitable to express it in polar coordinates, defining

$$B'(r,\theta) = B(r \cos \theta,\ r \sin \theta)$$

and B_0' in a similar fashion. Then,

$$B'(r,\theta) = \int_0^\infty P\left(\frac{\omega}{\omega_0}\right) B_0' \left(\frac{\omega}{\omega_0} r,\theta\right) d\omega \ .$$

To put this integral into the familiar convolution form, we make the substitutions

$$\omega = \omega_0 \, e^s \quad , \quad r = e^t$$

Getting

$$B'(e^t,\theta) = \int_{-\infty}^{\infty} P(e^s) \ \omega_0 \ e^s \ B_0' \ (e^{s+t},\theta) \ ds$$

To solve this, we substitute

$$f(t) = B' \ (e^t,\theta), \ f_0(t) = B_0' \ (e^t,\theta), \ g(t) = \omega_0 \ P(e^t) \ e^t$$

$$f(t) = \int_0^\infty g(s) \ f_0(s+t) \ ds$$

Whose solution is

$$F_0(\nu) = \frac{F(\nu)}{G(\nu)}$$

where capitalization denotes fourier transforms.

The mechanical steps necessary to produce the bandwidth effect corrected maps are thus as follows:

(1) Calculate the uncorrected map $B(x,y)$ by the usual procedures, using a NxN FFT.

(2) Sort and interpolate to calculate $B'(r,\theta)$. This must be done at intervals in r equal to the original interval in x or y, and at intervals in θ of about $\frac{2}{N}$. Thus an original map of N rows and N columns is transformed into an array of πN radial strips, each of length $\frac{N}{2}$.

(3) Calculate f(t), and interpolate to equal intervals. Since, at the outer end of the strip, a step of one unit in r corresponds to a step of $\frac{2}{N}$ in t, the interval length for the sampled f must be $\frac{2}{N}$. Since the point $r = 0$ is mapped into the point $t = -\infty$, we clearly cannot carry out the process all the way. Because the bandpass cuts off rather quickly, we know that the outer reconstructed map will be quite uninfluenced by the inner part, and we can ignore all of the map inside a radius of some M units, where M is just a little smaller (by about the factor $(1 - \Delta\omega/\omega_o)$) than that central part of the map that we feel we do not need to correct for bandwidth at all. Therefore, we may set f(t) to zero for t less than log M. The total number of samples we have in f(t) is therefore $\frac{N}{2} \log \frac{N}{M}$.

(4) Use an FFT to transform f(t) into $F(\nu)$.

(5) Divide $F(\nu)$ by $G(\nu)$ to produce $F_o(\nu)$. $G(\nu)$ can easily be calculated from the instrumental bandpass functions for the various IF filters.

(6) Fourier transform again to produce $f_o(t)$.

(7) Calculate $B_o'(r,\theta)$, and interpolate to equal intervals. The portion of the restored map for $r \lesssim M\ (1 + \Delta\omega/\omega_o)$ units will have been seriously distorted by our procedure. This inner part of B_o' should be discarded and replaced by the original B' (which, by hypothesis, is nearly unaffected by bandwidth).

(8) After doing steps (3)-(7) above for all radial strips, interpolate the maps back to equal intervals in x and y, producing the restored map $B_o(x,y)$.

The procecure clearly becomes very sensitive to noise and to the instrumental bandpass function if restoration is attempted in the region where the sausage pattern for the longest baseline involved is approaching a null ($G(\nu)$ is very nearly the sausage pattern).