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NOTES ON THE OVERALL CALIBRATION OF THE VLA (CONTINUUM SYSTEM)

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Introduction

Each interferometer in the VLA attempts to measure the temporal cross-correlation function of the electromagnetic fields at two points on the earth's surface. More precisely, it tries to measure the cross-correlation of a particular portion of these fields, namely that portion due to sources in directions within a particular solid angle (defined by the antenna beams), and within a particular frequency band (defined by the electronics).

Such a cross-correlation function is plotted in Figure 1. This plot applies to a system in which the two receivers have identical, rectangular passbands whose bandwidth B is much less than the center frequency f_o, and in which the correlated signal has a flat spectrum (continuum observations). The correlation function then consists of an envelope sinc $B(\tau-\tau_o)$ multiplied by a sinusoid of period $1/f_o$. In this case, it has long been recognized (e.g. Bracewell 1958) that the entire cross-correlation function can be specified by giving only two real numbers, which together may be viewed as a single complex number, specifying the magnitude and phase of the sinusoidal factor. Provided that the source size (as delimited by the antenna beam) is small enough*, the envelope shape depends mainly on the passband shape and the source spectrum; and the envelope position (value of delay τ_o at its center) depends mainly on the direction in which the antennas are pointed. These three quantities may all be assumed known, leaving only the sinusoidal factor to be measured. This is the basis of concept of the "complex visibility" as the basic quantity measured by an interferometer. Note that if the assumptions mentioned here are not all satisfied, then a single complex number is not enough to specify the response of an interferometer.

* $(\sin \theta)$ S B/c << 1, where θ is the angular source size, S is the spacing between the antennas, and c is the speed of light.

Instrumental Effects and Measurement Errors

One way to obtain the desired complex number - and the method employed in most telescopes, including the VLA - is to measure the correlation function at two values of τ separated by one-quarter period of f_o, and to regard these as the real and imaginary parts of the complex visibility. The latter is usually defined so that the real part is taken at delay $\tau_R = \vec{s}_o \cdot \vec{S}/c$, where $\vec{s}_o$ is a unit vector in a reference direction, $\vec{S}$ is the vector spacing between the antennas, and the antennas are both pointed near $\vec{s}_o$.

In practice, if f_o is in the microwave region, then the computation of the two correlations is difficult to do directly. The signals are therefore usually translated to a lower-frequency IF band for transmission and processing. This results in a system like that shown in Figure 2, which shows the essential features of the VLA implementation. The figure shows only one local oscillator per antenna; its instantaneous phase is the sum of that of all L.O.'s in a multiple-L.O. system, such as the VLA.

The delay τ_R is done at the I.F., necessitating a similar delay in the local oscillator signal for Antenna 1 (this LO phase shift is usually called "fringe rotation"). The additional delay $1/4f_o$ needed to compute the imaginary part of the visibility is obtained at the IF without an additional delay of the LO by a linear filter Q. This filter cannot be a simple delay because it must account for the LO; it is nominally a 90° phase shift at each frequency in the IF passband.

In the VLA, the IF signals are sampled and digitized (to 3 levels), and the delay and correlation are performed digitally. The quantization produces a slight non-linearity in the correlator, which we shall ignore here since it is in principle computer-correctable (Cooper 1970).

We can enumerate the following instrumental effects upon the measured visibility $(\hat{R}, \hat{I})$:

1. Gain errors. This includes errors in our knowledge of the antenna gains, the gains through each receiver's electronics (in the VLA, deduced through measurements of the system temperatures T_1 and T_2 and the use of ALC), the A/D converter thresholds, and the gain through the quadrature filter Q. Because of Q and the separate A/D converters, the real and imaginary channel gains may not be the same.

2. LO phase error. The relative phase between the two LO's may differ from the desired value by an arbitrary amount.

3. RF delay error. The delays between reference points on the antennas and the corresponding mixers may not be equal or known.

4. IF delay error. The digital delays have non-zero resolution and may not be set exactly.

5. Quadrature error. The filter Q may deviate from its nominal transfer function.

Possibly there are other effects, including non-linear ones; if so, we ignore them in this analysis. Note that 1-5 above are all linear. The effects of each on the measurements are noted in Figure 1.

Calibration

The traditional method of calibrating an interferometer involves observing an unresolved source located exactly at $\vec{s}_0$. The visibility is then known to be (R,I) = (F,0) where F is the flux density. This is compared with the measured value $(\hat{R},\hat{I})$. If we assume that the instrument is fully described by an instrumental gain G and phase ϕ, then we have just enough information to determine both of these: In matrix notation, assume

$$\begin{pmatrix}\hat{R}\\ \hat{I}\end{pmatrix} = G\begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}R\\ I\end{pmatrix} = G\begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}F\\ 0\end{pmatrix} \qquad (1)$$

Then

$$\tan\phi = \hat{I}/\hat{R}$$

$$G = (\hat{R}^2 + \hat{I}^2)^{1/2}/F \ . \qquad (2)$$

Actually, however, quadrature error and the use of separate A/D converters in the real and imaginary channels mean that the instrument cannot be so simply described. When these effects are significant, (1) must be replaced by

$$\begin{pmatrix}\hat{R}\\ \hat{I}\end{pmatrix} = \begin{pmatrix}G_R & 0\\ 0 & G_I\end{pmatrix}\begin{pmatrix}1 & 0\\ \sin q & \cos q\end{pmatrix}\begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}R\\ I\end{pmatrix} \triangleq \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}R\\ I\end{pmatrix} \qquad (3)$$

It should be clear that there are now 4 parameters to be calibrated, rather than 2.

One technique for calibrating all 4 parameters is based on the fact that in the VLA we can accurately change ϕ by means of the fringe rotation system, under computer control. Thus, if we make two calibrator observations, one with the normal L.O. phase and one with the phase offset a known amount (preferably 90°), we obtain the necessary 4 measurements. For 90° offsets, the results are

$$\begin{aligned} a &= \hat{R}/F \\ c &= \hat{I}/F \\ b &= -\hat{R}'/F \\ d &= -\hat{I}'/F \end{aligned} \qquad (4)$$

where $\hat{R}'$, $\hat{I}'$ are the measured values when the phase is offset. This technique will be successful only if the programmed phase offset is much more precise than the quadrature errors.

In the VLA, additional correlators are provided to monitor the quadrature error of each antenna, resulting in output $\hat{E}$ in Figure 2. This additional measurement allows 3 of the 4 parameters to be found with a single calibrator observation, but another measurement is still needed to obtain complete calibration.

References

Bracewell, 1958, Proc. IRE, **46**, 97.

Cooper, 1970, Aust. J. Phys., **23**, 521.

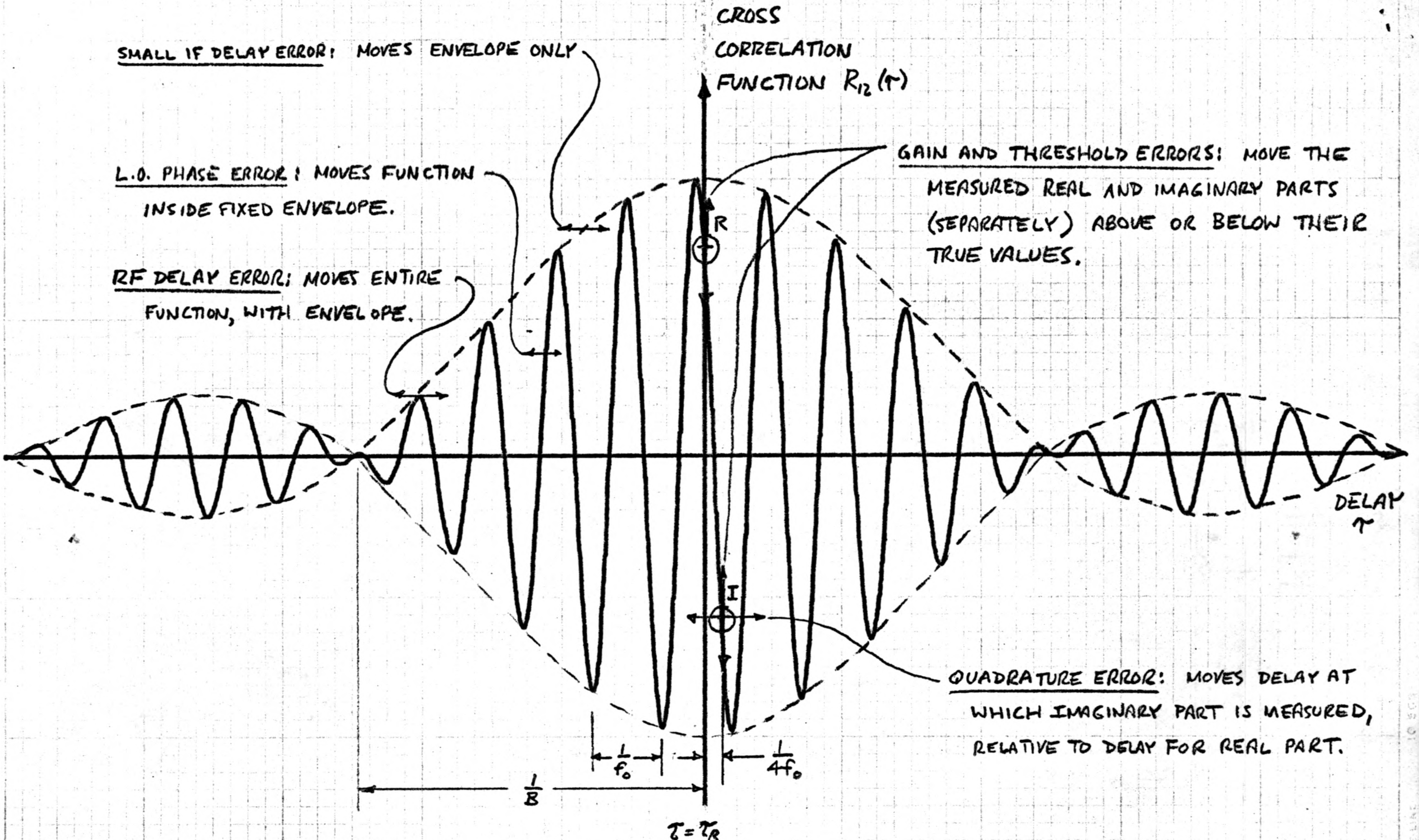

FIGURE 1 CROSS-CORRELATION OF EM FIELDS AT TWO ANTENNAS VS. DELAY, WITH NOTES ON INSTRUMENTAL EFFECTS ON ITS MEASUREMENT WITH AN INTERFEROMETER.

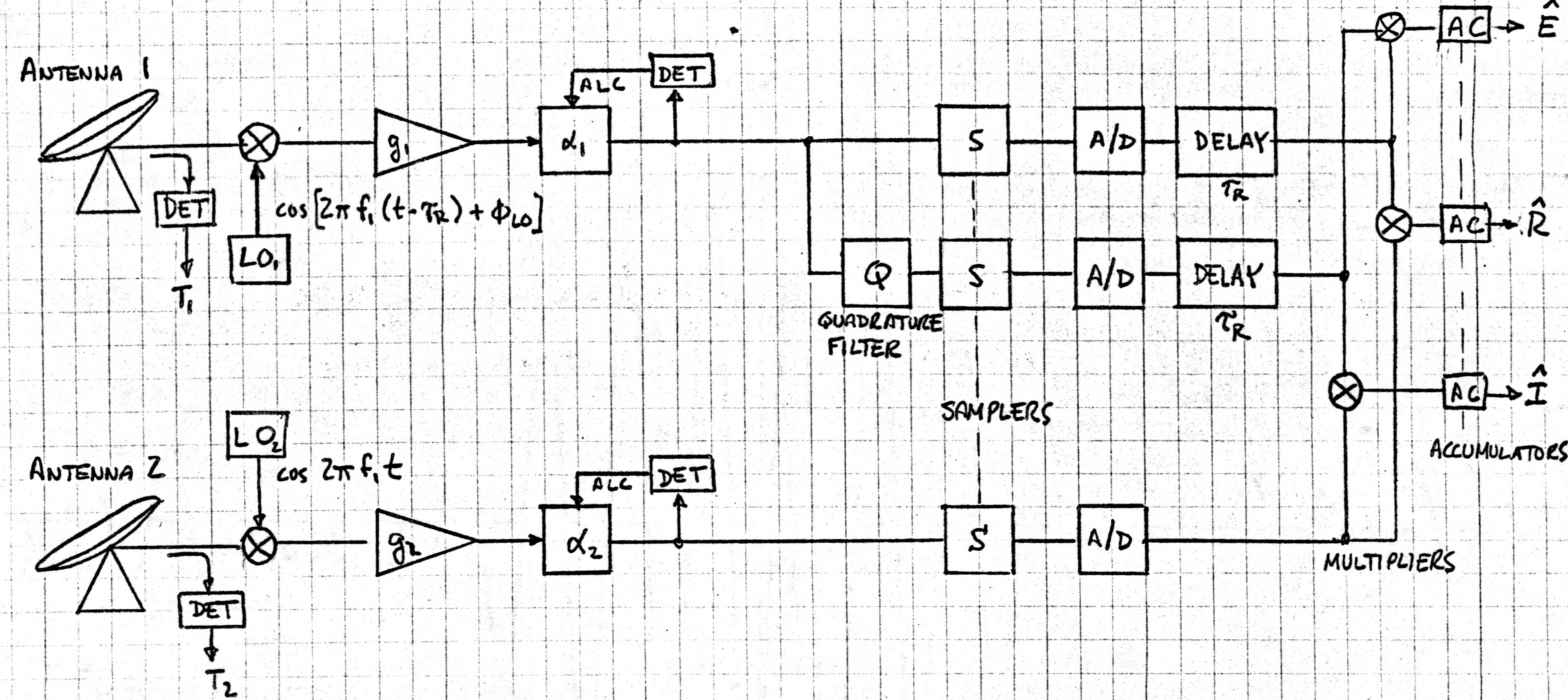

Figure 2: Simplified block diagram of a practical interferometer, showing essential features of the VLA implementation.